

4724 Core Mathematics 4

- 1 Attempt to factorise numerator and denominator M1 $\frac{A}{f(x)} + \frac{B}{g(x)}$; fg = $6x^2 - 24x$
- Any (part) factorisation of both num and denom A1 Corres identity/cover-up
- Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{5}{6x}$ A1

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- 2 Use parts with $u = x, dv = \sec^2 x$ M1 result $f(x) + / - \int g(x) dx$
- Obtain correct result $x \tan x - \int \tan x dx$ A1
- $\int \tan x dx = k \ln |\sec x|$ or $k \ln |\cos x|$, where $k = 1$ or -1 B1 or $k \ln |\sec x|$ or $k \ln |\cos x|$
- Final answer = $x \tan x - \ln |\sec x| + c$ or $x \tan x + \ln |\cos x| + c$ A1

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- 3 (i) $1 + \frac{1}{2} \cdot 2x + \frac{\frac{1}{2} \cdot -\frac{1}{2}}{2} (4x^2 \text{ or } 2x^2) + \frac{\frac{1}{2} \cdot -\frac{1}{2} \cdot -\frac{3}{2}}{6} (8x^3 \text{ or } 2x^3)$ M1
- = $1 + x$ B1
- ... $-\frac{1}{2}x^2 + \frac{1}{2}x^3$ (AE fract coeffs) A1 (3) For both terms

- (ii) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3$ B1 or $(1+x)^3 = 1 + 3x + 3x^2 + x^3$
- Either attempt at their (i) multiplied by $(1+x)^{-3}$ M1 or (i) long div by $(1+x)^3$
- $1 - 2x \dots \quad \sqrt{1 + (a-3)x}$ A1 f.t. (i) = $1 + ax + bx^2 + cx^3$
- ... $+\frac{5}{2}x^2 \dots \quad \sqrt{(-3a+b+6)x^2}$ A1
- ... $-2x^3 \quad \sqrt{(6a-3b+c-10)x^3}$ A1 (5) (AE fract.coeffs)

- (iii) $-\frac{1}{2} < x < \frac{1}{2}$, or $|x| < \frac{1}{2}$ B1 (1)

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- 4 Attempt to expand $(1 + \sin x)^2$ and integrate it *M1 Minimum of $1 + \sin^2 x$
 Attempt to change $\sin^2 x$ into $f(\cos 2x)$ M1
 Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ A1 dep M1 + M1
 Use $\int \cos 2x \, dx = \frac{1}{2} \sin 2x$ A1 dep M1 + M1
 Use limits correctly on an attempt at integration dep* M1 Tolerate $g\left(\frac{1}{4}\pi\right) - 0$
 $\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F A1 WW 1.51... → M1 A0

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- 5 (i) Attempt to connect du and dx , find $\frac{du}{dx}$ or $\frac{dx}{du}$ M1 But not e.g. $du = dx$
 Any correct relationship, however used, such as $dx = 2u \, du$ A1 or $\frac{du}{dx} = \frac{1}{2}x^{-1/2}$
 Subst with clear reduction (≥ 1 inter step) to **AG** A1 (3) WWW

- (ii) Attempt partial fractions M1
 $\frac{2}{u} - \frac{2}{1+u}$ A1
 $\sqrt{A \ln u + B \ln(1+u)}$ √A1 Based on $\frac{A}{u} + \frac{B}{1+u}$
 Attempt integ, change limits & use on $f(u)$ M1 or re-subst & use 1 & 9
 $\ln \frac{9}{4}$ AEexactF (e.g. $2 \ln 3 - 2 \ln 4 + 2 \ln 2$) A1 (5) Not involving $\ln 1$

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6 (i) Solve $0 = t - 3$ & subst into $x = t^2 - 6t + 4$ M1
 Obtain $x = -5$ A1 (2) $(-5, 0)$ need not be quoted
 N.B. If (ii) completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)

(ii) Attempt to eliminate t M1
 Simplify to $x = y^2 - 5$ ISW A1 (2)

(iii) Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form M1 Award anywhere in Que

Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or $(-)\frac{1}{2}(x+5)^{-\frac{1}{2}}$ A1

If $t = 2$, $x = -4$ and $y = -1$ B1 Awarded anywhere in (iii)

Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn M1

$x + 2y + 6 = 0$ AEF(without fractions) ISW A1 (5)

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7 (i) Attempt direction vector between the 2 given points M1
 State eqn of line using format $(\mathbf{r}) = (\text{either end}) + s(\text{dir vec})$ M1 's' can be 't'
 Produce 2/3 eqns containing t and s M1 2 different parameters
 Solve giving $t = 3$, $s = -2$ or 2 or -1 or 1 A1
 Show consistency B1
 Point of intersection = $(5, 9, -1)$ A1 (6)

(ii) Correct method for scalar product of 'any' 2 vectors M1 Vectors from this question
 Correct method for magnitude of 'any' vector M1 Vector from this question

Use $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$ for the correct 2 vectors $\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ & $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ M1 Vects may be mults of dvs

62.2 (62.188157...) 1.09 (1.0853881) A1 (4)

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- 8 (i) $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ B1
- Consider $\frac{d}{dx}(xy)$ as a product M1
- $= x \frac{dy}{dx} + y$ A1 Tolerate omission of '6'
- $\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ ISW AEF A1 (4)

- (ii) $x^3 = 2^4$ or 16 and $y^3 = 2^5$ or 32 *B1
- Satisfactory conclusion dep* B1 AG
- Substitute $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$ M1 or the numerator of $\frac{dy}{dx}$
- Show or use calc to demo that num = 0, ignore denom AG A1 (4)

- (iii) Substitute (a, a) into eqn of curve M1 & attempt to state 'a = ...'
- $a = 3$ only with clear ref to $a \neq 0$ A1
- Substitute $(3,3)$ or (their a , their a) into their $\frac{dy}{dx}$ M1
- 1 only WWW A1 (4) from (their a , their a)
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- 9 (i) $\frac{d\theta}{dt} = \dots$ B1
- $k(160 - \theta)$ B1 (2) The 2 @ 'B1' are indep
- (ii) Separate variables with $(160 - \theta)$ in denom; or invert *M1 $\int \frac{1}{160 - \theta} d\theta = \int k, \frac{1}{k}, 1 dt$
- Indication that LHS = $\ln f(\theta)$ A1 If wrong ln, final 3@A = 0
- RHS = kt or $\frac{1}{k}t$ or t (+ c) A1
- Subst. $t = 0, \theta = 20$ into equation containing 'c' dep* M1
- Subst $t = 5, \theta = 65$ into equation containing 'c' & 'k' dep* M1
- $c = -\ln 140$ (-4.94) ISW A1
- $k = \frac{1}{5} \ln \frac{140}{95}$ (≈ 0.077 or 0.078) ISW A1
- Using their 'c' & 'k', subst $t = 10$ & evaluate θ dep* M1
- $\theta = 96(95.535714)$ $\left(95 \frac{15}{28}\right)$ A1 (9)
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