4724 Core Mathematics 4

1	Method for finding magnitude of any vector	M1		Expect $\sqrt{14}$ and $\sqrt{6}$
	Method for finding scalar prod of any 2 vectors	M1		Expect $1.2+(-2).1+3.1=3$
	Using $\cos\theta = \dfrac{\mathbf{i}-2\mathbf{j}+3\mathbf{k}\,.\,2\mathbf{i}+\mathbf{j}+\mathbf{k}}{\lvert\mathbf{i}-2\mathbf{j}+3\mathbf{k}\rvert\lvert 2\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert}$	M1		Correct vectors only. Expect $\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}$
	70.9 (70.89, 70.893) WWW; 1.24 (1.237)	A1	**4**	Condone answer to nearest degree (71)
2 (i)	Correct format $\dfrac{A}{x+1}+\dfrac{B}{x+2}$	M1		stated or implied by answer
	$-\dfrac{1}{x+1}$ or $A=-1$	A1		
	$+\dfrac{2}{x+2}$ or $B=2$	A1	**3**	
(ii)	$\int \dfrac{1}{x+1}\,dx = \ln(x+1)$ or $\ln\lvert x+1\rvert$ or $\int \dfrac{1}{x+2}\,dx = \ln(x+2)$ or $\ln\lvert x+2\rvert$	B1		
	$A\ln\lvert x+1\rvert + B\ln\lvert x+2\rvert + c$ ISW	√A1	**2**	Expect $-\ln\lvert x+1\rvert + 2\ln\lvert x+2\rvert + c$
3	Method 1 (Long division)			
	Clear correct division method at beginning	M1		x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4)=0$]
	Correct method up to & including x term in quot	M1		[At subtraction stage, cf $(x^3)=0$]
	Method 2 (Identity)			
	Writing $(x^2+2x-1)(x^2+bx+2)+cx+7$	M1		Probably equated to $x^4-2x^3-7x^2+7x+a$
	Attempt to compare cfs of x^3 or x^2 or x or const	M1		
	Then:			
	$b=-4$	A1		
	$c=-1$	A1		
	$a=5$	A1	**5**	
4	$\dfrac{d}{dx}(x^2y) = x^2\dfrac{dy}{dx}+2xy$	B1		s.o.i.;
	$\dfrac{d}{dx}(y^3) = 3y^2\dfrac{dy}{dx}$	B1		
	Substitute (x,y) = (1,1) and solve for $\dfrac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
		M1		dep on either/both B1 earned]
	$\dfrac{dy}{dx} = -\dfrac{11}{7}$ WWW	A1		Implied if grad normal = $\dfrac{7}{11}$
	Gradient normal = $-\dfrac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
	$7x-11y+4=0$ AEF	A1	**6**	No fractions in final answer.

5	**(i)** Use 3**i** – 4**j** + 2**k** and 2**i** – **j** – 5**k** only	M1	
	Use correct method for scalar prod of <u>any</u> 2 vectors	M1	(indep) May be as part of $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$
	Obtain 6 + 4 – 10, state = 0 & deduce perp **AG**	A1 **3**	
	(ii) Produce 3 equations in s and t	*M1	of the type $5+3s=2+2t$, $-2-4s=-2-t$ and $-2+2s=7-5t$
	Solve 2 of the equations for s and t	dep*M1	*<u>Or</u> Eliminate s (or t) from 2 pairs dep*M1*
	Obtain (s,t) = $\left(\frac{3}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{22},\frac{18}{11}\right)$ or $\left(\frac{3}{19},\frac{33}{19}\right)$	A1	*(5t=12,11t=18,19t=33) <u>or</u> (5s=3,22s= 9,19s=3) A1,A1*
	Substitute their values in 3rd equation	dep*M1	*State/show inconsistency <u>& state non-parallel</u>*
	State/show inconsistency <u>& state non-parallel</u> ∴ skew	A1 **5**	*∴ skew WWW* A1
6	**(i)** $1-4ax+...$	B1	
	$\frac{-4.-5}{1.2}(ax)^2$ or $\frac{-4.-5}{1.2}a^2x^2$ or $\frac{-4.-5}{1.2}ax^2$	M1	Do not accept $\binom{-4}{2}$ unless 10 also appears
	$...+10a^2x^2$	A1 **3**	
	(ii) f.t. (their cf x) + b(their const cf) = 1	√B1	Expect $b-4a=1$
	f.t. (their cf x^2) + b(their cf x) $=-2$	√B1	Expect $10a^2-4ab=-2$
	Attempt to eliminate 'b' and produce equation in 'a'	M1	Or eliminate 'a' and produce equation in 'b'
	Produce $6a^2+4a=2$ AEF	A1	Or $6b^2+4b=42$ AEF
	$a=\frac{1}{3}$ and $b=\frac{7}{3}$ only	A1 **5**	Made clear to be only (final) answer
7	**(i)** Perform an operation to produce an equation connecting A and B (or possibly in A or in B)	M1	Probably substituting value of θ, or comparing coefficients of sin x, and/or cos x
	$A=2$	A1	
	$B=-2$	A1 **3**	WW scores 3
	(ii) Write $4\sin\theta$ as $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)$ and re-write integrand as $A+\frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}$	M1	A and B need not be numerical – but, if they are, they should be the values found in (i).
	$\int A\,d\theta = A\theta$	√B1	general or numerical
	$\int \frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}d\theta = B\ln(\sin\theta+\cos\theta)$	√A2	general or numerical
	Produce $\frac{1}{4}A\pi+B\ln\sqrt{2}$ f.t. with their A,B	√A1 **5**	Expect $\frac{1}{2}\pi-\ln 2$ (Numerical answer only)
8	**(i)** $\frac{dx}{dt}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1	k non-numerical; i.e. 1 side correct
	$\frac{dx}{dt}=-kx^{\frac{1}{2}}$ or $\frac{dx}{dt}=kx^{\frac{1}{2}}$	A1 **2**	i.e. both sides correct
	(ii) Separate variables or invert, + attempt to integrate *	M1	Based <u>only</u> on above eqns or $\frac{dx}{dt}=x^{\frac{1}{2}}$, $-x^{\frac{1}{2}}$
	Correct result for their equation after integration	A1	Other than omission of 'c'
	Subst $(t,x)=(0,2)$ into eqn containing k &/or c dep*	M1	or substitute (5,1)
	Subst $(t,x)=(5,1)$ into eqn containing k <u>&</u> c dep*	M1	or substitute (0,2)
	Subst $x=0.5$ into eqn with their k & c subst dep*	M1	
	$t=8.5(8.5355339)$	A1 **6**	[1 d.p. requested in question]

9	**(i)** Use $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{\frac{dy}{dp}}{\frac{dx}{dp}}$	M1		Or conv to cartes form & att to find $\frac{dy}{dx}$ at P
	$=\frac{2t}{3t^2}$ or $\frac{2p}{3p^2}$	A1		
	Find eqn tgt thro (p^3, p^2) or (t^3, t^2) ,their gradient	M1		Using $y-y_1=m(x-x_1)$ or $y=mx+c$
	$3py-2x=p^3$ **AG**	A1	**4**	Do not accept t here
	(ii) Substitute $(-10,7)$ into given equation *	M1		to produce a cubic equation in p
	Satis attempt to find at least 1 root/factor dep*	M1		Inspection/factor theorem/rem theorem/t&i
	Any one root	A1		-1 or -4 or 5
	All 3 roots	A1		$-1, -4$ and 5
	$(-1,1)$, $(-64,16)$ and $(125,25)$	A1	**5**	All 3 sets; no f.t.
10	**(i)** $(1-x^2)^{\frac{3}{2}}\to\cos^3\theta$	B1		May be implied by $\int\sec^2\theta\,d\theta$
	$dx\to\cos\theta\,d\theta$	B1		
	$\frac{1}{(1-x^2)^{\frac{3}{2}}}dx\to\sec^2\theta\,(d\theta)$ or $\frac{1}{\cos^2\theta}(d\theta)$	B1		
	$\int\sec^2\theta\,(d\theta)=\tan\theta$	B1		
	Attempt change of limits (expect 0 & $\frac{1}{6}\pi/30$)	M1		Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$
	$\frac{1}{\sqrt{3}}$ AEF	A1	**6**	Obtained with no mention of 30 anywhere
	(ii) Use parts with $u=\ln x$, $\frac{dv}{dx}=\frac{1}{x^2}$	*M1		obtaining a result $f(x)+/-\int g(x)(dx)$
	$-\frac{1}{x}\ln x+\int\frac{1}{x^2}(dx)$ AEF	A1		Correct first stage result
	$-\frac{1}{x}\ln x-\frac{1}{x}$	A1		Correct overall result
	Limits used correctly	dep*M1		
	$\frac{2}{3}-\frac{1}{3}\ln 3$	A1	**5**	
	<u>If substitution attempted in part (ii)</u>			
	$\ln x=t$	B1		
	Reduces to $\int t\,e^{-t}\,dt$	B1		
	Parts with $u=t, dv=e^{-t}$	M1		
	$-te^{-t}-e^{-t}$	A1		
	$\frac{2}{3}-\frac{1}{3}\ln 3$	A1		