OCR Maths C4

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1	(Quotient =) $x^2 + 2x + 2$ (Remainder =) $0x – 3$ Allow without working	B1 M1 A1 A1 **4**	For correct leading term x^2 in quotient For evidence of division/identity process For correct quotient For correct remainder. The '$0x$' need not be written but must be clearly derived. **4**
2	 $x \sin x – \int \sin x \, dx$ (= $x \sin x + \cos x$) Answer = ½ π – 1	M1 A1 B1 M1 A1 **5**	For attempt at parts going correct way (u = x, dv = cos x and f(x) +/–∫g(x) (dx) For both terms correct Indic anywhere that $\int \sin x \, dx = – \cos x$ For correct method of limits For correct exact answer ISW **5**
3	(i) **r** = (2**i**-3**j**+**k** or –**i**–2**j**–4**k**) + t(3**i**–**j**+5**k**) (ii) *L*(2) (**r**) = 3**i**+2**j**–9**k**+s(4**i** – 4**j** + 5**k**) *L(1)&L(2)* must be of form **r** = **a** + t**b** 2+3t=3+4s, –3–t=2–4s,1+5t= – 9+5s or suitable equivalences (t,s) = (+/-3,2) or (–/+1,1) or (–/+9,-7) or (+/–4,2) or (0,1) or (–/+8,-7) Basic check other eqn & interp √	M1 A1 **2** M1 M1 M1 A1 B1 **5**	For (either point) + t(diff betw vectors) Completely correct including **r** =. AEF For point + (s or t) direction vector For 2/3 eqns with 2 different parameters For solving any relevant pair of eqns For both parameters correct **7**
4	(i) $dx = \sec^2\theta \, d\theta$ AEF Indefinite integral = $\int \cos^2\theta \, d\theta$ (ii) = k∫ +/– 1 +/– cos 2θ dθ ½[θ + ½ sin 2θ] Limits = ¼π(accept 45) and 0 (π + 2)/8 AEF	M1 A1 A1 **3** M1 A1 M1 A1 **4**	Attempt to connect dx,dθ (not dx = dθ) For $dx = \sec^2\theta \, d\theta$ or equiv correctly used With at least one intermed step **AG** "Satis" attempt to change to double angle Correct attempt + correct integration New limits for θ or resubstituting Ignore decimals after correct answer **7** Single 'parts' + $\sin^2\theta = 1 – \cos^2\theta$ acceptable
5	(i)**OD**=**OA**+**AD** or **OB**+**BC**+**CD** AEF **AD** = **BC** or **CD** = **BA** (**a** + **c** – **b**) = 2**j** + **k** (ii) **AB.CB** = |**AB**||**CB**| cos θ Scalar product of <u>any</u> 2 vectors Magnitude of <u>any</u> vector 94°(94.386…) or 1.65 (1.647…)	M1 A1 A1 **3** M1 M1 M1 A1 **4**	Connect **OD** & 2/3/4 vectors in their diag Or similar ,from their diag [i.e.if diag mislabelled, M1A1A0 possible] Or **AB.BC** i.e.scalar prod for correct pair 2 + 3 – 6 = –1 is expected √19 or 3 expected Accept 86°(85.614…) or 1.49(424..) **7**
6	(i) For $d/dx\,(y^2) = 2y\,dy/dx$ Using d(uv) = u dv + v du $2xy\,dy/dx + y^2 = 2 + 3\,dy/dx$ $dy/dx = (2 – y^2)/(2xy – 3)$	B1 M1 A1 M1 A1 **5**	 Solving an equation,with at least 2 dy/dx terms, for d*y*/d*x*; d*y*/d*x* on one side, non d*y*/d*x* on other. **AG**

	(ii) Stating/using $2xy - 3 = 0$ Attempt to eliminate x or y $8x^2 = -9$ or $y^2 = -2$	B1 M1 A1 **3**	No use of 2 - y^2 in this part. Between $2xy$ - 3 = 0 & eqn of curve Together with suitable finish **8**
7	(i)dy / dx = (dy/dt) / (dx/dt) = $(-1/t^2) / 2t$ as unsimplified expression = $-1 / 2t^3$ as simplified expression (ii) (4,–1/2) → $t = -2$ only Satis attempt to find equation of tgt $x - 16y = 12$ only (iii) $t^3 - 12t - 16 = 0$ or $16y^3 + 12y^2 - 1 = 0$ or $x^3 - 24x^2 + 144x - 256 = 0$ $t = 4$ (only) ISW giving cartesian coords	M1 A1 A1 **3** B1 M1 A1 **3** M1 A1 B2 **4**	(S.R.Award M1 for attempt to change to cartesian eqn & differentiate + A1 for dy/dx or dx/dy in terms of x or y) Not $1/-2t^3$. Not in terms of x &/or y. Using $t = -2$ or 2 **AG** For substituting (t^2,1/t) into tgt eqn or solving simult tgt & their cartes eqns For simplified equiv non-fract cubic S.R. Award B1 for "4 or –2". S.R. If B0, award M1 for clear indic of method of soln of correct eqn. **10**
8	(i) $3x+4 \equiv A(2+x)^2+B(2+x)(1+x) + C(1+x)$ $A = 1$ $C = 2$ $A+B = 0$ or $4A+3B+C=3$ or $4A+2B+C = 4$ $B = -1$ (ii) $1 - x + x^2$ $1 - ½ x + ¼ x^2$ $1 - x$ $+ ¾ x^2$ $1 - 5/4\, x + 5/4\, x^2$ (iii) $-1 < x < 1$ AEF	M1 A/B1 A/B1 A1 A1 **5** B1 B1 B1 B1 B1 **5** B1 **1**	Accept ≡ or = If identity used, award 'A' mark, if cover-up rule used, award 'B' mark. Any correct eqn for B from identity Expansion of $(1 + x)^{-1}$ Expansion of $(1 + ½ x)^{-1}$ First 2 terms of $(1 + ½x)^{-2}$ Third term of $(1 + ½x)^{-2}$ Complete correct expansion If partial fractions not used Award B1 for expansion of $(1+x)^{-1}$ B1+B1 for expansion of $(1 + ½x)^{-2}$, and B1 for 1–5/4x… & B1 for…+5/4x² Or if denom expanded to give $a+bx+cx^2$ with a=4.b=8,c=5,award B1 Expansion of $[1+(b/a\}x+(c/a)x^2]^{-1}$ = $1 - (b/a)x + \ldots (-c/a + b^2/a^2)x^2$ B1+B1 Final ans = $(1 - 5/4x\ldots + 5/4x^2)$B1+B1 Other inequalities to be discarded. **11**
9	k = const of proportionality – = falling, dθ/dt = rate of change θ – 20 = diff betw obj & surround temp (ii) ∫ 1/(θ – 20) dθ = – k ∫ dt ln(θ – 20) = – kt + c Subst (θ,t) = (100,0) or (68,5)	B2 **2** M1 A1A1 M1 A1	All 4 items (first two may be linked) S.R. Award B1 for any 2 items For separating variables For integ each side (c not essential) Dep on 'c' being involved [or M2 for limits (100,0) (68,5) + A1 for

	c = ln 80 k = 1/5 ln 5/3 $\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}$ (iii) Substitute θ = 68 – 32 *t* = 15.75 Extra time = 10.75, √their 15.75 – 5	A1 M1 A1 **8** M1 A1 B1 **3**	k] *AG* Subst into AEF of given eqn & solve Accept 15.7 or 15.8 f.t. only if θ = their (68 – 32) or 32 **13**

1		Attempt to factorise numerator and denominator	M1		
		num = $xx(x-3)$ <u>or</u> denom = $(x-3)(x+3)$	A1		Not num = $x(x^2-3x)$
		<u>Final</u> answer = $\frac{x^2}{x+3}$ [Not $\frac{xx}{x+3}$]	A1	**3**	Do not ignore further cancellation.
2		$\frac{d}{dx}(\sin y) = \cos y.\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1		[SR: If xy taken to LHS, accept $-x\frac{dy}{dx} + y$ as s.o.i.]
		$\cos y.\frac{dy}{dx} = x\frac{dy}{dx} + y + 2x$ AEF	B1		
		[If written as $\frac{dy}{dx} = \cos y\frac{dy}{dx} = x\frac{dy}{dx} + y + 2x$, accept for prev B1 but not for following marks if the $\frac{dy}{dx}$ is used]			
		$f(x,y)\frac{dy}{dx} = g(x,y)$	M1		Regrouping provided > one $\frac{dy}{dx}$ term
		$\frac{y+2x}{\cos y - x}$ or $-\frac{y+2x}{x-\cos y}$ or $\frac{-2x-y}{x-\cos y}$	A1	**5**	ISW Answer could imply M1
3	**(i)**	Quotient = $3x + ...$	B1		For correct leading term in quotient
		For evidence of correct division process	M1		Or for cubic $\equiv (x^2-2x+5)(gx+h)\ (+...)$
		$3x+4$	A1		For correct quotient
		$-6x-13$	A1	**4**	For correct remainder ISW
	(ii)	$a = 7$	B1√		<u>Follow through</u> If rem in (i) is $Px+Q$,
		$b = 20$	B1√	**2**	then B1√ for $a = 1-P$ and B1√ for $b = 7-Q$
		[SR: If B0+B0, award B1√ for $a = 1+P$ AND $b = 7+Q$; also SR B1 for $a = 20, b = 7$]			
4	**(i)**	Parts using correct split of $u = x, \frac{dv}{dx} = \sec^2 x$	M1		1st stage result of form $f(x) +/- \int g(x)\,dx$
		$x\tan x - \int \tan x\,dx$	A1		Correct 1st stage
		$\int \tan x\,dx = -\ln\cos x$ or $\ln\sec x$	B1		
		$x\tan x + \ln\cos x + c$ or $x\tan x - \ln\sec x + c$	A1	**4**	
	(ii)	$\tan^2 x = +/-\sec^2 x +/- 1$	M1		or $\sec^2 x = +/-1 +/- \tan^2 x$
		$\int x\sec^2 x\,dx - \int x\,dx$ s.o.i.	A1		Correct 1st stage
		$x\tan x + \ln\cos x - \frac{1}{2}x^2 + c$	A1√	**3**	f.t. their answer to part $(i) - \frac{1}{2}x^2$

5	**(i)**	$\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$	M1		Used, not just quoted
		$\frac{1}{t}$ or t^{-1}	A1	**2**	Not $\frac{2}{2t}$ as final answer
		SR: M1 for Cart conv, finding $\frac{dy}{dx}$ & ans involv t + A1			M1 is attempt only, accuracy not involved
	(ii)	Finding equation of tangent (using p or t)	M1		
		$py=x+p^2$	A1	**2**	**AG**; p essential; at least 1 line inter working
	(iii)	$(25,-10)\Rightarrow p=-5$ or $-5y=x+25$ seen	B1		$5y=x+25$ seen $\Rightarrow$ B0
		Substitution of their values of p into given tgt eqn	M1		Producing 2 equations
		Solving the 2 equations simultaneously	M1		
		$(-15,-2)$ $x=-15, y=-2$	A1	**4**	Common wrong ans $(15,8)\Rightarrow$ B0, M2, A0
6	**(i)**	Attempt to connect $dx, d\theta$	M1		But not $dx=d\theta$
		$dx=2\sin\theta\cos\theta\, d\theta$	A1		AEF
		$\sqrt{\frac{x}{1-x}}=\frac{\sin\theta}{\cos\theta}$	B1		Ignore any references to $\pm$.
		Reduction to $\int 2\sin^2\theta\, d\theta$	A1	**4**	**AG** WWW
	(ii)	$\sin^2\theta=k(+/-1+/-\cos 2\theta)$	M1		Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$
		$2\sin^2\theta=1-\cos 2\theta$	A1		Correct attempt
		$\int\cos 2\theta\, d\theta=\frac{1}{2}\sin 2\theta$	B1		Seen anywhere in this part
		Attempting to change limits	M1		<u>Or</u> Attempting to resubstitute; Accept degrees
		$\frac{1}{2}\pi$	A1	**5**	
		<u>Alternatively</u> Parts once & use			
		$\cos^2\theta=1-\sin^2\theta$	(M2)		Instead of the M1 A1 B1
		$\frac{1}{2}(\theta-\sin\theta\cos\theta)$	(A1)		Then the final M1 A1 for use of limits
7	**(i)**	$A=3$	B1		For correct value stated
		$C=1$	B1		For correct value stated
		$11+8x\equiv A(1+x)^2+B(2-x)(1+x)+C(2-x)$	M1		AEF; any suitable identity
		e.g. $A-B=0, 2A+B-C=8, A+2B+2C=11$	A1		For any correct (f.t.) equation involving B
		$B=3$	A1	**5**	
	(ii)	$\left(1-\frac{x}{2}\right)^{-1}=1+\frac{x}{2}+\frac{x^2}{4}+\ldots$	B1		s.o.i.
		$(1+x)^{-1}=1-x+x^2-\ldots$	B1		s.o.i.
		$(1+x)^{-2}=1-2x \quad +3x^2-\ldots$	B1,B1		s.o.i.
		Expansion = $\frac{11}{2}-\frac{17}{4}x+\frac{51}{8}x^2+\ldots$	B1	**5**	CAO. No f.t. for wrong A and/or B and/or C

SR(1) If partial fractions not used but product of but $(11+8x)(2-x)^{-1}(1+x)^{-2}$ attempted, then

B1 for $\left(1-\frac{x}{2}\right)^{-1}=1+\frac{x}{2}+\frac{x^2}{4}+\ldots$

B1,B1 for $(1+x)^{-2}=1-2x+\ldots+3x^2+\ldots$

B1,B1 for $\frac{11}{2}-\frac{17}{4}x+\ldots+\frac{51}{8}x^2+\ldots$

SR(2) If partial fractions not used denominator multiplied out, then

B1 for denom = $2+3x(+0x^2)+\ldots$

B1 for $\left(1+\frac{3x}{2}\right)^{-1}=1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots$

B1,B1,B1 for $\frac{11}{2}\ldots-\frac{17}{4}x\ldots+\frac{51}{8}x^2+\ldots$

N.B. In both SR, if final expansion given B0, -------------------allow SR B1 for 22 – 17x + 51/2 x^2

8	**(i)**	$\int(y-3)\,dy=\int(2-x)\,dx$ or equiv	M1		For separation & integration of both sides
		$\frac{1}{2}y^2-3y=2x-\frac{1}{2}x^2$	A1		or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2$
		For an arbitrary const on one/both sides	*B1		} (or + M2 for equiv statement using limits)
		Substituting $(x,y)=(5,4)$ or (4,5) & finding 'c'	dep*M1		}
		$\frac{1}{2}y^2-3y=-\frac{1}{2}x^2+2x-\frac{3}{2}$ AEF ISW	A1	**5**	or $\frac{1}{2}(y-3)^2=-\frac{1}{2}(x-2)^2+5$ AEF
	(ii)	Attempt to clear fracts (if nec) & compl square	M1		
		$a=2, b=3, k=10$	A2	**3**	For all 3; SR: A1 for 1 or 2 correct
	(iii)	Circle clearly indicated in a sketch	B1		
		Centre $(2,3)$ or their (a,b)	B1√		
		Radius $\sqrt{10}$ or their $\sqrt{k}$	B1√	**3**	√ provided $k>0$

9	**(i)**	Using $\begin{pmatrix}-8\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}-9\\2\\-5\end{pmatrix}$ as the relevant vectors	M1		i.e. correct direction vectors
		Using $\cos\theta=\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$ AEF for any 2 vectors	M1		Accept $\cos\theta=\left\lvert\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}\right\rvert$
		Method for scalar product of any 2 vectors	M1		
		Method for finding magnitude of any vector	M1		
		15° (15.38…), 0.268 rad	A1	**5**	
	(ii)	Produce (at least) 2 of the 3 eqns in t and s	M1		e.g. $4-8t=-2-9s$, $-6-2t=-2-5s$
		Solve the (x) and (z) equations	M1		
		$t=3$ or $s=2$	A1		for first value found
		$s=2$ or $t=3$ f.t.	A1√		for second value found
		Substituting their (t,s) into (y) equation	M1		
		$a=1$	A1		
		Substituting their t into l_1 or their (s,a)			

into l_2	M1		
$\begin{pmatrix} -20 \\ 5 \\ -12 \end{pmatrix}$	A1	**8**	Any format but not $\begin{pmatrix} \\ \\ \\ \end{pmatrix} + \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

1		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	**B1**		s.o.i. e.g. $2x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	**B1**		
		Substitute (1,2) into their differentiated equation and attempt to solve for $\frac{dy}{dx}$. [Allow subst of (2,1)]	**M1** dep at least 1 x **B1**		Or attempt to solve their diff equation for $\frac{dy}{dx}$ and then substitute (1,2)
		$\frac{dy}{dx} = -2$	**A1**	**4**	
2	**(i)**	$1 + (-2)(-3x) + \frac{(-2)(-3)}{1.2}(-3x)^2$ (+ … ignore)	**M1**		State or imply; accept $-3x^2$ & $-9x^2$
		= $1 + 6x$	**B1**		Correct first 2 terms
		… $+27x^2$	**A1**	**3**	Correct third term
	(ii)	$(1+2x)^2(1-3x)^{-2}$	**M1**		For changing into suitable form, seen/implied
		Attempt to expand $(1+2x)^2$ & select (at least) 2 relevant products and add	**M1**		Selection may be after multiplying out
		55 (Accept $55x^2$)	**A2**√	**4**	If (i) is $a + bx + cx^2$, f.t. $4(a+b)+c$
		<u>SR 1</u> For expansion of $(1+2x)^2$ with 1 error, A1√			
		<u>SR 2</u> For expansion of $(1+2x)^2$ & > 1 error, A0			
		<u>**Alternative Method**</u>			
		For correct method idea of long division	**M1**		
		1 ….. $+10x$ ….. $+55x^2$	**A1,A1,A1(4)**		
3	**(i)**	$\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$	**M1**		Correct format + suitable method
		$\frac{1}{x}$	**A1**		seen in (i) or (ii)
		$-\frac{1}{3-x}$	**A1**	**3**	ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately
	(ii)	$\int \frac{1}{x}(dx) = \ln x$ or $\ln\lvert x\rvert$	**B1**		
		$\int \frac{1}{3-x}(dx) = -\ln(3-x)$ or $-\ln\lvert 3-x\rvert$	**B1**		Check sign carefully; do not allow $\ln(x-3)$
		Correct method idea of substitution of limits	**M1**		Dep on an attempt at integrating
		ln 2 (+ ln 1 − ln 1) − ln 2 = 0	**A1**	**4**	Clearly seen; WWW **AG**
		<u>**Alternative Method**</u>			
		If ignoring PFs, ln x(3 − x) immediately	**B2**		$\ln x(x-3) \to 0$
		As before	**M1,A1**	**(4)**	
	(iii)	Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)	**B1**	**1**	

4	**(i)** Working out **b** – **a** or **a** – **b** or **c** – **a** or **a** – **c**	**M1**	)		Irrespective of label
	= ± (– 3**i** – **j** – **k**) or ± (– 2**i** + **j** – 2**k**)	**A1**	)		If not scored ,these 1st 3 marks can be
	Method for finding magnitude of <u>any</u> vector	**M1**	)		awarded in part (ii)
	Method for finding scalar product of <u>any</u> 2 vectors	**M1**			
	Using $\cos\theta = \frac{a.b}{\|a\|\|b\|}$ AEF for <u>any</u> 2 vectors	**M1**			
	[<u>Alternative cosine rule method</u> $\|\overrightarrow{BC}\| = \sqrt{6}$	**B1**			
	Cosine rule used	**M1**			'Recognisable' form
	$45.3°, 0.79(0), \frac{\pi}{3.97}$ (45.289378, 0.7904487)	**A1**		**6**	Do not accept supplement (134.7 etc)
	(ii) Use of $\frac{1}{2}\|\overrightarrow{AB}\|\|\overrightarrow{AC}\|\sin\theta$	**M1**			Accept $\left\|\frac{1}{2}\overrightarrow{AB} \times \overrightarrow{AC}\right\|$
	3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$	**A1**		**2**	Accept from correct supp (134.7 etc)
5	**(i)** $\frac{dA}{dt}$ or kA^2 seen	**M1**			
	$\frac{dA}{dt} = kA^2$	**A1**		**2**	
	(ii) Separate variables + attempt to integrate	***M1**			Accept if based on $\frac{dA}{dt} = kA^2$ or A^2
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	**A1**			
	Subst one of (0,0),(1,1000) or (2,2000) into eqn.	**dep*M1**			Equation must contain k and/or c
	Subst another of (0,),(1,1000) or (2,2000) into eqn	**dep*M1**			This equation must contain k <u>and</u> c
	Substitute $A = 3000$ into eqn with k and c subst	**dep*M1**			
	$t = \frac{7}{3}$ ISW	**A1**		**6**	Accept 2.33, 2h 20 m
6	**(i)** Attempt to connect du and dx e.g. $\frac{du}{dx} = e^x$	**M1**			But not $du = dx$
	Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i.	**A1**			
	Simplification to $\int \frac{u-1}{u}(du)$ WWW	**A1**		**3**	**AG**
	(ii) Change $\frac{u-1}{u}$ to $1 - \frac{1}{u}$ or use parts	**M1**			If parts, may be twice if $\int \ln x \, dx$ is involved
	$\int \frac{1}{u} du = \ln u$	**A1**			Seen anywhere in this part
	<u>Either</u> attempt to change limits <u>or</u> resubstitute	**M1** (indep)			Expect new limits e+1 & 2
	Show as $e + 1 - \ln(e+1) - \{2 \text{ or } (1+1)\} + \ln 2$	**A1**			
	WWW show final result as $e - 1 - \ln\left(\frac{e+1}{2}\right)$	**A1**		**5**	**AG**

7	(i)	Produce at least 2 of the 3 relevant eqns in λ and μ	M1		e.g. $1 + 3\lambda = -8 + \mu$, $-2 + \lambda = 2 - 2\mu$
		Solve the 2 eqns in λ & μ as far as $\lambda = \ldots$ or $\mu = \ldots$	M1		
		1st solution: $\lambda = -2$ or $\mu = 3$	A1		
		2nd solution: $\mu = 3$ or $\lambda = -2$ f.t.	A1√		
		Substitute their λ and μ into 3rd eqn and find 'a'	M1		
		Obtain $a = 2$ <u>& clearly state that a cannot be 2</u>	A1	6	
	(ii)	Subst their λ or μ (& poss a) into either line eqn	M1		
		Point of intersection is $-5\mathbf{i} - 4\mathbf{j}$	A1	2	Accept any format <u>No f.t. here</u>
		N.B. In this question, award marks irrespective of labelling of parts			
8	(i)	<u>Integration method</u>			
		Attempt to change $\cos^2 6x$ into $\mathrm{f}(\cos 12x)$	M1		
		$\cos^2 6x = \frac{1}{2}(1 + \cos 12x)$	A1		with $\cos^2 6x$ as the subject of the formula
		$\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$	A1		AG Accept $\frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right)$
		<u>Differentiation method</u>			
		Differentiate RHS producing $\frac{1}{2} + \frac{1}{2}\cos 12x$ ---(E)	B1		
		Attempt to change $\cos 12x$ into $\mathrm{f}(\cos 6x)$	M1		Accept $+/-2\cos^2 6x + /-1$
		Simplify (E) WWW to $\cos^2 6x$ + satis finish	A1	3	

	(ii)	Parts with $u = x, \mathrm{d}v = \cos^2 6x$	*M1		
		$x\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right) - \int\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right)\mathrm{d}x$	A1		Correct expression only
		$\int \sin 12x\,\mathrm{d}x = -\frac{1}{12}\cos 12x$	B1		Clear indication somewhere in this part
		Correct use of limits to <u>whole</u> integral	dep*M1		Accept ( ) (-0)
		$\frac{\pi^2}{288} - \frac{\pi^2}{576} - \frac{1}{288} - \frac{1}{288}$	A1		AE unsimp exp. Accept 12x24,sin π here
		$\frac{\pi^2}{576} - \frac{1}{144}$	+A1	6	Tolerate e.g. $\frac{2}{288}$ here
		S.R. If final marks are A0 + A0, allow SR A1 for			0.01/0.010/0.0101/0.0102/0.0101902

9	**(i)**	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	**M1**		Used, not just quoted
		$\frac{dx}{dt}=-4\sin t$ or $\frac{dy}{dt}=3\cos t$	***B1**		
		$\frac{dy}{dx}=-\frac{3\cos t}{4\sin t}$ or $\frac{3\cos t}{-4\sin t}$ ISW	**dep*A1**	**3**	Also $\frac{-3\cos t}{4\sin t}$ provided B0 not awarded
		SR: M1 for Cartesian eqn attempt + B1 for $\frac{d}{dx}\left(y^2\right)=2y\frac{dy}{dx}$ **+ A1** as before(must be in terms of *t*)			
	(ii)	$y-3\sin p=\left(\text{their }\frac{dy}{dx}\right)(x-4\cos p)$	**M1**		Accept *p* or *t* here
		<u>or</u> $y=\left(\text{their }\frac{dy}{dx}\right)x+\text{c}$ & subst cords to find c			Ditto
		$4y\sin p-12\sin^2 p=-3x\cos p+12\cos^2 p$	**A1**		Correct equation cleared of fractions
		<u>or</u> c = $\frac{12\sin^2 p+12\cos^2 p}{4\sin p}$			
		$3x\cos p+4y\sin p=12$ WWW	**A1**	**3**	**AG** Only *p* here. Mixture earlier $\rightarrow$ A0
	(iii)	Subst *x* = 0 and *y* = 0 separately in tangent eqn	**M1**		to find *R* & *S*
		Produce $\frac{3}{\sin p}$ and $\frac{4}{\cos p}$	**A1**		Accept $\frac{12}{4\sin p}$ and/or $\frac{12}{3\cos p}$
		Use $\Delta=\frac{1}{2}\left(\frac{3}{\sin p}.\frac{4}{\cos p}\right)=\frac{12}{\sin 2p}$ WWW	**A1**	**3**	**AG**
	(iv)	Least area = 12	**B1**		
		$p=\frac{1}{4}\pi$ as final or only answer	**B2**	**3**	These B marks are independent.
		S.R. $45^\circ\rightarrow$ B1;			S.R. [-12 and e.g. $-\pi/4\rightarrow$ B1]

1	Factorise numerator and denominator	M1		or Attempt long division
	Num = $(x+6)(x-4)$ or denom = $x(x-4)$	A1		Result = $1+\frac{6x-24}{x^2-4x}$
	Final answer = $\frac{x+6}{x}$ or $1+\frac{6}{x}$	A1	3	= $1+\frac{6}{x}$
2	Use parts with $u=\ln x, \mathrm{d}v = x$	M1		& give 1st stage in form $\mathrm{f}(x)+/-\int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $\frac{1}{2}x^2\ln x-\int \frac{1}{x}.\frac{1}{2}x^2(\mathrm{d}x)$	A1		or $\frac{1}{2}x^2\ln x-\int \frac{1}{2}x(\mathrm{d}x)$
	= $\frac{1}{2}x^2 \ln x-\frac{1}{4}x^2$ (+c)	A1		
	Use limits correctly	M1		
	Exact answer $2\ln 2-\frac{3}{4}$	A1	5	AEF ISW
3	(i) Find $\boldsymbol{a}-\boldsymbol{b}$ or $\boldsymbol{b}-\boldsymbol{a}$ irrespective of label	M1		(expect $11\mathbf{i}-2\mathbf{j}-6\mathbf{k}$ or $-11\mathbf{i}+2\mathbf{j}+6\mathbf{k}$)
	Method for magnitude of any vector	M1		
	$\sqrt{161}$ or $12.7(12.688578)$	A1	3	
	(ii) Using $(\overline{\text{AO}}$ or $\overline{\text{OA}})$ and $(\overline{AB}$ or $\overline{\text{BA}})$	B1		Do not class angle AOB as MR
	$\cos\theta = \frac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$	M1		
	43 or better (42.967…), 0.75 or better (0.7499218..)	A1	3	If 137 obtained, followed by 43, award A0 Common answer 114 probably → B0 M1 A0
4	Attempt to connect $\mathrm{d}x$ and $\mathrm{d}u$	M1		but not just $\mathrm{d}x=\mathrm{d}u$
	For $\mathrm{d}u=2\,\mathrm{d}x$ AEF correctly used	A1		sight of $\frac{1}{2}$ ($\mathrm{d}u$) necessary
	$\int u^8+u^7\,(\mathrm{d}u)$	A1		or $\int u^7(u+1)(\mathrm{d}u)$
	Attempt new limits for u at any stage (expect 0,1)	M1		or re-substitute & use ($\frac{5}{2}$,3)
	$\frac{17}{72}$	A1	5	**AG** WWW
	S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72}, \frac{34}{36}$ or $\frac{17}{18}$			ISW
5	(i) Show clear knowledge of binomial expansion	M1		$-3x$ should appear but brackets can be missing; $-\frac{1}{3}.-\frac{4}{3}$ should appear, not $-\frac{1}{3}.\frac{2}{3}$
	= $1+x$	B1		Correct first 2 terms; not dep on M1
	$+2x^2$	A1		
	$+\frac{14}{3}x^3$	A1	4	
	(ii) Attempt to substitute $x+x^3$ for x in (i)	M1		Not just in the $\frac{14}{3}x^3$ term
	Clear indication that $(x+x^3)^2$ has no term in x^3	A1		
	$\frac{17}{3}$	√A1	3	f.t. $\mathrm{cf}(x)$ + $\mathrm{cf}(x^3)$ in part (i)
6	(i) $2x+1 = / \equiv A(x-3)+B$	M1		
	$A=2$	A1		
	$B=7$	A/B 1	3	Cover-up rule acceptable for B1
	(ii) $\int \frac{1}{x-3}(\mathrm{d}x)=\ln(x-3)$ or $\ln\lvert x-3\rvert$	B1		Accept A or $\frac{1}{A}$ as a multiplier
	$\int \frac{1}{(x-3)^2}(\mathrm{d}x)=-\frac{1}{x-3}$	B1		Accept B or $\frac{1}{B}$ as a multiplier
	6 + 2 ln 7 Follow-through $\frac{6}{7}B+A\ln 7$	√B2	4	

7	$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1		
	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
	$4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	B1		
	Put $\frac{dy}{dx} = 0$	*M1		
	Obtain $4x + y = 0$ AEF	A1		and no other (different) result
	Attempt to solve simultaneously with eqn of curve	dep*M1		
	Obtain $x^2 = 1$ or $y^2 = 16$ from $4x + y = 0$	A1		
	$(1,-4)$ and $(-1,4)$ and no other solutions	A1	**8**	Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$
8	(i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $-\frac{1}{m}$ for grad of normal	M1		or change to cartesian.,diff & use $-\frac{1}{m}$
	$= -p$ **AG** WWW	A1	**2**	Not $-t$.
	(ii) Use correct formula to find gradient of line	M1		
	Obtain $\frac{2}{p+q}$ **AG** WWW	A1	**2**	Minimum of denom = $2(p-q)(p+q)$
	(iii) State $-p = \frac{2}{p+q}$	M1		Or find eqn normal at P & subst $(2q^2, 4q)$
	Simplify to $p^2 + pq + 2 = 0$ **AG** WWW	A1	**2**	With sufficient evidence
	(iv) (8,8) $\rightarrow$ t or p or $q = 2$ only	B1		No possibility of -2
	Subst $p = 2$ in eqn (iii) to find q_1	M1		Or eqn normal,solve simult with cartes/param
	Subst $p = q_1$ in eqn (iii) to find q_2	M1		Ditto
	$q_2 = \frac{11}{3} \rightarrow \left(\frac{242}{9}, \frac{44}{3}\right)$	A1	**4**	No follow-through; accept (26.9 , 14.7)
9	(i) Separate variables as $\int \sec^2 y \, dy = 2\int \cos^2 2x \, dx$	M1		seen or implied
	LHS = tan y	A1		
	RHS; attempt to change to double angle	M1		
	Correctly shown as 1 + cos 4x	A1		
	$\int \cos 4x \, dx = \frac{1}{4}\sin 4x$	A1		
	Completely correct equation (other than +c)	A1		$\tan y = x + \frac{1}{4}\sin 4x$
	+c on either side	A1	**7**	<u>not</u> on both sides unless c_1 and c_2
	(ii) Use boundary condition	M1		provided a sensible outcome would ensue
	c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
	Substitute $x = \frac{1}{6}\pi$ into their eqn, produce y = 1.05	A1	**3**	or 4.19 or 7.33 etc. Radians only
10	(i) For (either point) + t(diff between posn vectors)	M1		"**r** =" not necessary for the M mark
	r = (either point) + t(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**)	A1	**2**	**…** but it is essential for the A mark
	(ii) **r** = s(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + s(**i** + 2**j** – **k**)	B1		Accept any parameter, including t
	Eval scalar product of **i**+2**j**-**k** & their dir vect in (i)	M1		
	Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3)	A1		This is just one example of numbers involved
	= 0 <u>and</u> state perpendicular **AG**	A1	**4**	
	(iii) For at least two equations with diff parameters	M1		e.g. 5 + t = s, 2 – 2t = 2s, $-9 - 3t = -s$
	Obtain $t = -2$ or s = 3 (possibly -3 or 2 or -2)	A1		Check if $t = 2, 1$ or -1
	Subst. into eqn AB or OT and produce 3**i** + 6**j** – 3**k**	A1	**3**	
	(iv) Indicate that $\lvert\overrightarrow{OC}\rvert$ is to be found	M1		where C is their point of intersection
	$\sqrt{54}$;f.t. $\sqrt{a^2+b^2+c^2}$ from a**i** + b**j** + c**k** in (iii)	√A1	**2**	

<u>In the above question, accept any vectorial notation</u>
t and s may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.

1	(i) Correct format $\frac{A}{x+2}+\frac{B}{x-3}$	M1		s.o.i. in answer
	$A=1$ and $B=2$	A1	2	for both
	(ii) $-A(x+2)^{-2}-B(x-3)^{-2}$ f.t.	√A1		
	Convincing statement that each denom > 0	B1		accept ≥ 0. Do not accept $x^2>0$.
	State whole exp < 0 **AG**	B1	3	Dep on previous 4 marks.
				5
2	Use parts with $u=x^2, \mathrm{d}v=\mathrm{e}^x$	*M1		obtaining a result $\mathrm{f}(x)+/-\int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $x^2\mathrm{e}^x-\int 2x\mathrm{e}^x\,(\mathrm{d}x)$	A1		
	Attempt parts again with $u=(-)(2)x, \mathrm{d}v=\mathrm{e}^x$	M1		
	Final = $(x^2-2x+2)\mathrm{e}^x$ AEF incl brackets	A1		s.o.i. eg $\mathrm{e}+(-2x+2)\mathrm{e}^x$
	Use limits correctly throughout	dep*M1		Tolerate (their value for $x=1$) (-0)
	$\mathrm{e}^{(1)}-2$ ISW Exact answer only	A1	6	Allow 0.718 → M1
				6
3	Volume = $(k)\int_0^{\pi}\sin^2 x\,(\mathrm{d}x)$	B1		where $k=\pi, 2\pi$ or 1; limits necessary
	Suitable method for integrating $\sin^2 x$	*M1		eg $\int +/-1+/-\cos 2x\,(\mathrm{d}x)$ or single integ by parts & connect to $\int \sin^2 x\,(\mathrm{d}x)$
	$\int \sin^2 x\,(\mathrm{d}x)=\frac{1}{2}\int 1-\cos 2x\,(\mathrm{d}x)$	A1		or $-\sin x\cos x+\int \cos^2 x(\mathrm{d}x)$
	$\int \cos 2x\,(\mathrm{d}x)=\frac{1}{2}\sin 2x$	A1		or $-\sin x\cos x+\int 1-\sin^2 x\,(\mathrm{d}x)$
	Use limits correctly	dep*M1		
	Volume = $\frac{1}{2}\pi^2$ WWW Exact answer	A1	6	**<u>Beware</u>**: wrong working leading to $\frac{1}{2}\pi^2$
				6
4	(i) $\left(1+\frac{x}{2}\right)^{-2}=1+(-2)\left(\frac{x}{2}\right)+\frac{-2.-3}{2}\left(\frac{x}{2}\right)^2+\frac{-2.-3.-4}{3!}\left(\frac{x}{2}\right)^3$	M1		Clear indication of method of ≥ 3 terms
	$=1-x$	B1		First two terms, not dependent on M1
	$+\frac{3}{4}x^2-\frac{1}{2}x^3$	A1		For both third and fourth terms
	$(2+x)^{-2}=\frac{1}{4}\left(\text{their exp of }(1+ax)^{-2}\right)$ mult out	√B1		Correct: $\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3$
	$\lvert x\rvert<2$ or $-2<x<2$ (but not $\lvert\frac{1}{2}x\rvert<1$)	B1	5	
	(ii) If (i) is $a+bx+cx^2+dx^3$ evaluate $b+d$	M1		
	$-\frac{3}{8}\ (x^3)$	√A1	2	Follow-through from $b+d$
				7

5(i) $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		
$=\frac{-4\sin 2t}{-\sin t}$	A1		Accept $\frac{4\sin 2t}{\sin t}$ WWW
$= 8\cos t$	A1		
≤ 8 **AG**	A1	**4**	with brief explanation eg $\cos t \le 1$
(ii) Use $\cos 2t = 2\cos^2 t +/-1$ or $1-2\cos^2 t$	M1		<u>If starting with</u> $y = 4x^2+1$, then
Use correct version $\cos 2t = 2\cos^2 t - 1$	A1		Subst $x=\cos t, y = 3+2\cos 2t$ M1
Produce WWW $y = 4x^2+1$ **AG**	A1	**3**	<u>Either</u> substitute <u>a</u> formula for cos *2t* M1
			Obtain 0=0 or $4\cos^2 t+1 = 4\cos^2 t+1$ A1 <u>Or</u> Manip to give formula for cos *2t* M1 Obtain corr formula & say it's correct A1
(iii) U-shaped parabola abve *x*-axis, sym abt *y*-axis	B1		Any labelling must be correct
Portion between $(-1,5)$ and $(1,5)$	B1	**2**	either $x=\pm1$ or $y=5$ must be marked
N.B. If (ii) answered or quoted before (i) attempted,	allow in part		(i) B2 for $\frac{dy}{dx}=8x$ +B1,B1 if earned. **9**
6 (i) $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$	B1		
Using $d(uv)=u\,dv+v\,du$ for the $(3)xy$ term	M1		
$\frac{d}{dx}(x^2+3xy+4y^2)=2x+3x\frac{dy}{dx}+3y+8y\frac{dy}{dx}$	A1		
Solve for $\frac{dy}{dx}$ & subst (x, y) = (2,3)	M1		or v.v. Subst now or at normal eqn stage; (M1 dep on either/both B1 M1 earned)
$\frac{dy}{dx} = -\frac{13}{30}$	A1		Implied if grad normal = $\frac{30}{13}$
Grad normal = $\frac{30}{13}$ follow-through	√B1		This f.t. mark awarded only if numerical
Find equ <u>any</u> line thro (2,3) with <u>any</u> num grad	M1		
$30x-13y-21=0$ AEF	A1	**8**	No fractions in final answer **8**
7 (i) Leading term in quotient = $2x$	B1		
<u>Suff evidence</u> of division or identity process	M1		
Quotient = $2x+3$	A1		Stated or in relevant position in division
Remainder = *x*	A1	**4**	Accept $\frac{x}{x^2+4}$ as remainder
(ii) their quotient + $\frac{\text{their remainder}}{x^2+4}$	√B1	**1**	$2x+3+\frac{x}{x^2+4}$
(iii) <u>Working with their expression in part (ii)</u>			
their *Ax* + *B* integrated as $\frac{1}{2}Ax^2+Bx$	√B1		
their $\frac{Cx}{x^2+4}$ integrated as $k\ln(x^2+4)$	M1		Ignore any integration of $\frac{D}{x^2+4}$
$k = \frac{1}{2}C$	√A1		
Limits used correctly throughout	M1		
$14+\frac{1}{2}\ln\frac{13}{5}$	A1	**5**	logs need not be combined.
			10

8	(i) Sep variables eg $\int \frac{1}{6-h}(dh) = \int \frac{1}{20}(dt)$	*M1		s.o.i. <u>Or</u> $\frac{dt}{dh} = \frac{20}{6-h} \rightarrow$ M1
	LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c) $\rightarrow$ A1+A1
	RHS = $\frac{1}{20}t$ (+c)	A1		
	Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
	Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
	Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	6	Must see $\ln 5 - \ln(6-h)$
	(ii) When h = 2, t = $20\ln\frac{5}{4}$ =4.46(2871)	B1	1	Accept 4.5, $4\frac{1}{2}$
	(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage
	h = 2.97(2.9673467…)	A1	2	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
	[In (ii),(iii) accept non-decimal (exact) answers but -1 once.**]** Accept truncated values in (ii),(iii).			
	(iv) Any indication of (approximately) 6 (m)	B1	1	
				10
9	(i) Use -6**i** + 8**j** -2**k** and **i** + 3**j** + 2**k** only	M1		
	Correct method for scalar product	M1		of <u>any</u> two vectors ($-6+24-4=14$)
	Correct method for magnitude	M1		of <u>any</u> vector ($\sqrt{36+64+4} = \sqrt{104}$ or $\sqrt{1+9+4} = \sqrt{14}$)
	68 or 68.5 (68.47546); 1.2(0) (1.1951222) rad	A1	4	
	[N.B. 61 (60.562) will probably have been generated by 5**i** – **j** -2**k** and 3**i** – 8**j**]			
	(ii) Indication that relevant vectors are parallel	M1		-6**i** + 8**j** -2**k** & 3**i** + c**j** + **k** with some indic of method of attack eg -6**i** + 8**j** -2**k** =λ(3**i** + c**j** + **k**)
	c **=** -4	A1	2	$c = -4$ WW $\rightarrow$ B2
	(iii) Produce 2/3 equations containing t,u (& c)	M1		eg $3+t = 2+3u, -8+3t = 1+cu$ and $2t = 3+u$
	Solve the 2 equations not containing 'c'	M1		
	t = 2, u = 1	A1		
	Subst their (t,u) into equation containing c	M1		
	c = -3	A1	5	
	<u>Alternative method for final 4 marks</u>			
	Solve two equations, one with 'c', for t and u in terms of c, and substitute into third equation	(M2)		
	$c = -3$	(A2)		**11**

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Q		Answer	Marks		Guidance
1		Method for finding magnitude of any vector	M1		Expect $\sqrt{14}$ and $\sqrt{6}$
		Method for finding scalar prod of any 2 vectors	M1		Expect $1.2+(-2).1+3.1=3$
		Using $\cos\theta = \dfrac{\mathbf{i}-2\mathbf{j}+3\mathbf{k}\,.\,2\mathbf{i}+\mathbf{j}+\mathbf{k}}{\lvert\mathbf{i}-2\mathbf{j}+3\mathbf{k}\rvert\lvert 2\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert}$	M1		Correct vectors only. Expect $\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}$
		70.9 (70.89, 70.893) WWW; 1.24 (1.237)	A1	**4**	Condone answer to nearest degree (71)
2	**(i)**	Correct format $\dfrac{A}{x+1}+\dfrac{B}{x+2}$	M1		stated or implied by answer
		$-\dfrac{1}{x+1}$ or $A=-1$	A1		
		$+\dfrac{2}{x+2}$ or $B=2$	A1	**3**	
	(ii)	$\int \dfrac{1}{x+1}\,dx = \ln(x+1)$ or $\ln\lvert x+1\rvert$			
		or $\int \dfrac{1}{x+2}\,dx = \ln(x+2)$ or $\ln\lvert x+2\rvert$	B1		
		$A\ln\lvert x+1\rvert + B\ln\lvert x+2\rvert + c$ ISW	√A1	**2**	Expect $-\ln\lvert x+1\rvert + 2\ln\lvert x+2\rvert + c$
3		<u>Method 1 (Long division)</u>			
		Clear correct division method at beginning	M1		x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4)=0$]
		Correct method up to & including x term in quot	M1		[At subtraction stage, cf $(x^3)=0$]
		<u>Method 2 (Identity)</u>			
		Writing $(x^2+2x-1)(x^2+bx+2)+cx+7$	M1		Probably equated to $x^4-2x^3-7x^2+7x+a$
		Attempt to compare cfs of x^3 or x^2 or x or const	M1		
		Then:			
		$b=-4$	A1		
		$c=-1$	A1		
		$a=5$	A1	**5**	
4		$\dfrac{d}{dx}(x^2y) = x^2\dfrac{dy}{dx}+2xy$	B1		s.o.i.;
		$\dfrac{d}{dx}(y^3) = 3y^2\dfrac{dy}{dx}$	B1		
		Substitute (*x*,*y*) = (1,1) and solve for $\dfrac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
			M1		dep on either/both B1 earned]
		$\dfrac{dy}{dx} = -\dfrac{11}{7}$ WWW	A1		Implied if grad normal = $\dfrac{7}{11}$
		Gradient normal = $-\dfrac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
		$7x-11y+4=0$ AEF	A1	**6**	No fractions in final answer.

5	**(i)** Use 3**i** – 4**j** + 2**k** and 2**i** – **j** – 5**k** only	M1	
	Use correct method for scalar prod of <u>any</u> 2 vectors	M1	(indep) May be as part of $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$
	Obtain 6 + 4 – 10, state = 0 & deduce perp **AG**	A1 **3**	
	(ii) Produce 3 equations in s and t	*M1	of the type $5+3s=2+2t$, $-2-4s=-2-t$ and $-2+2s=7-5t$
	Solve 2 of the equations for s and t	dep*M1	*<u>Or</u> Eliminate s (or t) from 2 pairs dep*M1*
	Obtain $(s,t) = \left(\frac{3}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{22},\frac{18}{11}\right)$ or $\left(\frac{3}{19},\frac{33}{19}\right)$	A1	*(5t=12,11t=18,19t=33) <u>or</u> (5s=3,22s= 9,19s=3) A1,A1*
	Substitute their values in 3rd equation	dep*M1	*State/show inconsistency <u>& state non-parallel</u>*
	State/show inconsistency <u>& state non-parallel</u> ∴ skew	A1 **5**	*∴ skew WWW* A1
6	**(i)** $1-4ax+...$	B1	
	$\frac{-4.-5}{1.2}(ax)^2$ or $\frac{-4.-5}{1.2}a^2x^2$ or $\frac{-4.-5}{1.2}ax^2$	M1	Do not accept $\begin{pmatrix}-4\\2\end{pmatrix}$ unless 10 also appears
	$...+10a^2x^2$	A1 **3**	
	(ii) f.t. (their cf x) + b(their const cf) = 1	√B1	Expect $b-4a=1$
	f.t. (their cf x^2) + b(their cf x) $=-2$	√B1	Expect $10a^2-4ab=-2$
	Attempt to eliminate 'b' and produce equation in 'a'	M1	Or eliminate 'a' and produce equation in 'b'
	Produce $6a^2+4a=2$ AEF	A1	Or $6b^2+4b=42$ AEF
	$a=\frac{1}{3}$ and $b=\frac{7}{3}$ only	A1 **5**	Made clear to be only (final) answer
7	**(i)** Perform an operation to produce an equation connecting A and B (or possibly in A or in B)	M1	Probably substituting value of θ, or comparing coefficients of sin x, and/or cos x
	$A=2$	A1	
	$B=-2$	A1 **3**	WW scores 3
	(ii) Write $4\sin\theta$ as $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)$ and re-write integrand as $A+\frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}$	M1	A and B need not be numerical – but, if they are, they should be the values found in (i).
	$\int A\,d\theta = A\theta$	√B1	general or numerical
	$\int \frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}d\theta = B\ln(\sin\theta+\cos\theta)$	√A2	general or numerical
	Produce $\frac{1}{4}A\pi + B\ln\sqrt{2}$ f.t. with their A,B	√A1 **5**	Expect $\frac{1}{2}\pi-\ln 2$ (Numerical answer only)
8	**(i)** $\frac{dx}{dt}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1	k non-numerical; i.e. 1 side correct
	$\frac{dx}{dt}=-kx^{\frac{1}{2}}$ or $\frac{dx}{dt}=kx^{\frac{1}{2}}$	A1 **2**	i.e. both sides correct
	(ii) Separate variables or invert, + attempt to integrate *	M1	Based <u>only</u> on above eqns or $\frac{dx}{dt}=x^{\frac{1}{2}}$, $-x^{\frac{1}{2}}$
	Correct result for their equation after integration	A1	Other than omission of 'c'
	Subst $(t,x)=(0,2)$ into eqn containing k &/or c dep*	M1	or substitute (5,1)
	Subst $(t,x)=(5,1)$ into eqn containing k <u>&</u> c dep*	M1	or substitute (0,2)
	Subst $x=0.5$ into eqn with their k & c subst dep*	M1	
	$t=8.5(8.5355339)$	A1 **6**	[1 d.p. requested in question]

9	**(i)** Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{\frac{dy}{dp}}{\frac{dx}{dp}}$	M1		Or conv to cartes form & att to find $\frac{dy}{dx}$ at P
	$= \frac{2t}{3t^2}$ or $\frac{2p}{3p^2}$	A1		
	Find eqn tgt thro (p^3, p^2) or (t^3, t^2) ,their gradient	M1		Using $y - y_1 = m(x - x_1)$ or $y = mx + c$
	$3py - 2x = p^3$ **AG**	A1	**4**	Do not accept t here
	(ii) Substitute $(-10,7)$ into given equation *	M1		to produce a cubic equation in p
	Satis attempt to find at least 1 root/factor dep*	M1		Inspection/factor theorem/rem theorem/t&i
	Any one root	A1		-1 or -4 or 5
	All 3 roots	A1		$-1, -4$ and 5
	$(-1,1)$, $(-64,16)$ and $(125,25)$	A1	**5**	All 3 sets; no f.t.
10	**(i)** $(1-x^2)^{\frac{3}{2}} \to \cos^3\theta$	B1		May be implied by $\int \sec^2\theta \, d\theta$
	$dx \to \cos\theta \, d\theta$	B1		
	$\frac{1}{(1-x^2)^{\frac{3}{2}}} dx \to \sec^2\theta \, (d\theta)$ or $\frac{1}{\cos^2\theta}(d\theta)$	B1		
	$\int \sec^2\theta \, (d\theta) = \tan\theta$	B1		
	Attempt change of limits (expect 0 & $\frac{1}{6}\pi / 30$)	M1		Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$
	$\frac{1}{\sqrt{3}}$ AEF	A1	**6**	Obtained with no mention of 30 anywhere
	(ii) Use parts with $u = \ln x$, $\frac{dv}{dx} = \frac{1}{x^2}$	*M1		obtaining a result $f(x) + / - \int g(x)(dx)$
	$-\frac{1}{x}\ln x + \int \frac{1}{x^2}(dx)$ AEF	A1		Correct first stage result
	$-\frac{1}{x}\ln x - \frac{1}{x}$	A1		Correct overall result
	Limits used correctly	dep*M1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1	**5**	
	<u>If substitution attempted in part (ii)</u>			
	$\ln x = t$	B1		
	Reduces to $\int t \, e^{-t} \, dt$	B1		
	Parts with $u = t, dv = e^{-t}$	M1		
	$-te^{-t} - e^{-t}$	A1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1		

4724 Core Mathematics 4

1	(a)	$2x^2-7x-4=(2x+1)(x-4)$ or $3x^2+x-2=(3x-2)(x+1)$	B1	
		$\frac{2x+1}{3x-2}$ as final answer; this answer only	B1	Do not ISW
			2	
	(b)	For correct leading term x in quotient	B1	Identity method
		For evidence of correct division process	M1	M1: $x^3+2x^2-6x-5=Q(x^2+4x+1)+R$
		Quotient = $x-2$	A1	M1: $Q=ax+b$ or $x+b, R=cx+d$ & ≥ 2 ops [N.B. If $Q=x+b$, this $\Rightarrow$ 1 of the 2 ops]
		Remainder = $x-3$	A1	A2: $a=1, b=-2, c=1, d=-3$ SR: B1 for two
			4	
2		Parts with correct split of $u=\ln x, \frac{dv}{dx}=x^4$	*M1	obtaining result $f(x)+/-\int g(x)\,dx$
		$\frac{x^5}{5}\ln x-\int\frac{x^5}{5}.\frac{1}{x}(dx)$	A1	
		$\frac{x^5}{5}\ln x-\frac{x^5}{25}$	A1	
		Correct method with the limits	dep*M1	Decimals acceptable here
		$\frac{4e^5}{25}+\frac{1}{25}$ ISW (Not '+c')	A1	Accept equiv fracts; like terms amalgamated
			5	
3	(i)	$\frac{d}{dx}(x^2y)=x^2\frac{dy}{dx}+2xy$ or $\frac{d}{dx}(xy^2)=2xy\frac{dy}{dx}+y^2$	*B1	
		Attempt to solve their differentiated equation for $\frac{dy}{dx}$	dep*M1	
		$\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$ only	A1	WWW **AG** Must have intermediate line &… …could imply "=0" on 1st line
			3	
	(ii)(a)	Attempt to solve **only** $y^2-2xy=0$ & derive $y=2x$	B1	**AG** Any effort at solving $x^2-2xy=0\rightarrow$ B0
		Clear indication why y = 0 is not acceptable	B1	Substituting $y=2x\rightarrow$ B0, B0
			2	
	(b)	Attempt to solve y = 2x simult with $x^2y-xy^2=2$	M1	
		Produce $-2x^3=2$ or $y^3=-8$	A1	AEF
		(–1, –2) or $x=-1, y=-2$ **only**	A1	
			3	

4 (i)	For (either point) + t(difference between vectors) **r** = (3**i** + 2**j** + 3**k** or **i** + 3**j** + 4**k**)+ t(–2**i** + **j** + **k** or 2**i** – **j** – **k**)	**M1** **A1** 2	't' can be 's', 'λ' etc. '**r**' must be '**r**' but need not be bold Check other formats, e.g. $ta+(1-t)b$
(ii)	State/imply that their **r** and their –2**i** + **j** + **k** are perpendicular	***M1**	N.B.This *M1 is dep on M1 being earned in (i)
	Consider scalar product = 0	dep***M1**	
	Obtain $t=-\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$	**A1**	
	Subst their t into their equation of AB	**M1**	
	Obtain $\frac{1}{6}$(16**i** + 13**j** + 19**k**) AEF	**A1** 5	Accept decimals if clear
5 (i)	$(1-x)^{\frac{1}{2}}=1-\frac{1}{2}x-\frac{1}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq-\frac{1}{8}$ or 0
	$(1+x)^{-\frac{1}{2}}=1-\frac{1}{2}x+\frac{3}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq\frac{3}{8}$ or 0
	Product = $1-x+\frac{1}{2}x^2$ ignoring x^3 etc	**B1** 5	**AG**; with (at least) 1 intermediate step (cf x^2)
(ii)	$\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	**B1**	
	$\frac{37}{49}$ or $1-\frac{2}{7}+\frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	**B1**	
	$\frac{\sqrt{5}}{3}\approx\frac{37}{49}\Rightarrow\sqrt{5}\approx\frac{111}{49}$	**B1** 3	**AG**
6 (i)	Produce at least 2 of the 3 relevant equations in t and s	**M1**	1 + 2t = 12 + s, 3t = –4s, –5 + 4t = 5 – 2s
	Solve for t and s	**M1**	
	(t, s) = (4, –3) AEF	***A1**	
	Subst (4, –3) into suitable equation(s) & show consistency	dep***A1** 4	Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for
(ii)	Method for finding magnitude of any vector	***M1**	Expect $\sqrt{29}$ and $\sqrt{21}$
	Method for finding scalar product of any 2 vectors	***M1**	Expect -18
	Using $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors	dep***M1**	Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$
	137 (136.8359) or 43.2(43.164…)	**A1** 4	2.39 (2.388236..) or 0.753(0.75335…) rads

7	(i)	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	M1	
		Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$	A1	
		Show manipulation to –cosec *x* cot *x* (or vice-versa)	A1	WWW **AG** with $\geq$ 1 line intermed working
			3	
	(ii)	Separate variables, $\int(-)\frac{1}{\sin x\tan x}\,dx = \int\cot t\,dt$	M1	or $\int\frac{1}{\sin x\tan x}\,dx = \int(-)\cot t\,dt$
		<u>Style: For the M1 to be awarded, d*x* and d*t* must appear on correct sides or there must be $\int$ sign on both sides</u>		
		$\int -\operatorname{cosec} x\cot x\,dx = \operatorname{cosec} x$ (+c)	A1	or $\int \operatorname{cosec} x\cot x\,dx = -\operatorname{cosec} x$
		$\int\cot t\,dt = \ln\sin t$ or $\ln\lvert\sin t\rvert$ (+c)	B1	or $\int -\cot t\,dt = -\ln\sin t$ or $-\ln\lvert\sin t\rvert$
		Subst $(t,x)=\left(\frac{1}{2}\pi,\frac{1}{6}\pi\right)$ into their equation containing 'c'	M1	and attempt to find 'c'
		$\operatorname{cosec} x = \ln\sin t + 2$ or $\ln\lvert\sin t\rvert + 2$	A1	WWW ISW; cosec $\frac{\pi}{6}$ to be changed to 2
			5	
8	(i)	$A(t + 1) + B = 2t$ $A = 2$ B = –2	M1 A1 A1	<u>Beware</u>: correct values for *A* and/or *B* can be ... … obtained from a wrong identity <u>Alt method:</u> subst suitable values into given… …expressions
			3	
	(ii)	Attempt to connect d*x* and d*t* d*x* = *t* d*t* s.o.i. AEF	M1 A1	But not just d*x* = d*t*. As **AG**, look carefully.
		$x+\sqrt{2x-1}\to\frac{t^2+1}{2}+t=\frac{(t+1)^2}{2}$ s.o.i.	B1	Any wrong working invalidates
		$\int\frac{2t}{(t+1)^2}\,dt$	A1	**AG** WWW The 'd*t*' must be present
			4	
	(iii)	$\int\frac{1}{t+1}\,dt=\ln(t+1)$	B1	Or parts $u=2t, dv=(t+1)^{-2}$ or subst *u* = *t* + 1
		$\int\frac{1}{(t+1)^2}\,dt=-\frac{1}{t+1}$	B1	
		Attempt to change limits (expect 1 & 3) and use f(*t*)	M1	<u>or</u> re-substitute and use 1 and 5 on g(*x*)
		$\ln 4-\frac{1}{2}$	A1	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
			4	

9	(i)	$A: \theta = \frac{1}{2}\pi$ (accept 90°)	B1	
		$B: \theta = 2\pi$ (accept 360°)	B2	SR If B0 awarded for point *B*, allow B1 SR for any angle s.t. $\sin\theta = 0$
			3	
	(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1	or $\frac{dy}{d\theta} . \frac{d\theta}{dx}$ Must be used, not just quoted
		$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$	B1	
		$2 + 2\cos 2\theta = 4\cos^2\theta$ with $\geq$ 1 line intermed work	*B1	
		$\frac{dy}{dx} = \frac{4\cos\theta}{2 + 2\cos 2\theta}$ s.o.i.	A1	This & previous line are interchangeable
		$= \sec\theta$	dep***A1**	WWW **AG**
			5	
	(iii)	Equating $\sec\theta$ to 2 and producing at least one value of θ	M1	degrees or radians
		$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$	A1	'Exact' form required
		$(y =) -2\sqrt{3}$	A1	'Exact' form required
			3	

4724 Core Mathematics 4

1		Attempt to factorise numerator and denominator	M1	$\frac{A}{f(x)}+\frac{B}{g(x)}$;fg= $6x^2-24x$
		Any (part) factorisation of both num and denom	A1	Corres identity/cover-up
		Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{\frac{5}{6}}{x}$	A1	
			3	
2		Use parts with $u = x, \mathrm{d}v = \sec^2 x$	M1	result $f(x) +/- \int g(x)\,\mathrm{d}x$
		Obtain correct result $x \tan x - \int \tan x\,\mathrm{d}x$	A1	
		$\int \tan x\,\mathrm{d}x = k \ln \sec x$ or $k \ln \cos x$, where $k = 1$ or -1	B1	or $k \ln\lvert\sec x\rvert$ or $k \ln\lvert\cos x\rvert$
		Final answer = $x \tan x - \ln\lvert\sec x\rvert + c$ or $x \tan x + \ln\lvert\cos x\rvert + c$	A1	
			4	
3	**(i)**	$1 + \frac{1}{2}.2x + \frac{\frac{1}{2}.-\frac{1}{2}}{2}(4x^2 \text{ or } 2x^2) + \frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}(8x^3 \text{ or } 2x^3)$	M1	
		$= 1 + x$	B1	
		$\ldots -\frac{1}{2}x^2 + \frac{1}{2}x^3$ (AE fract coeffs)	A1 (**3**)	For both terms
	(ii)	$(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3$	B1	or $(1+x)^3 = 1 + 3x + 3x^2 + x^3$
		Either attempt at their **(i)** multiplied by $(1+x)^{-3}$	M1	or **(i)** long div by $(1+x)^3$
		$1 - 2x \ldots.$ √ $1 + (a-3)x$	A1	f.t. **(i)** = $1 + ax + bx^2 + cx^3$
		$\ldots + \frac{5}{2}x^2 \ldots.$ √ $(-3a + b + 6)x^2$	A1	
		$\ldots -2x^3$ √ $(6a - 3b + c - 10)x^3$	A1 (**5**)	(AE fract.coeffs)
	(iii)	$-\frac{1}{2} < x < \frac{1}{2}$, or $\lvert x\rvert < \frac{1}{2}$	B1 (**1**)	
			9	

4		Attempt to expand $(1+\sin x)^2$ and integrate it	*M1	Minimum of $1+\sin^2 x$
		Attempt to change $\sin^2 x$ into $f(\cos 2x)$	M1	
		Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	A1	dep M1 + M1
		Use $\int \cos 2x \, dx = \frac{1}{2}\sin 2x$	A1	dep M1 + M1
		Use limits correctly on an attempt at integration	dep* M1	Tolerate g $(\frac{1}{4}\pi)$ − 0
		$\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F	A1	WW 1.51… → M1 A0
			6	

5	**(i)**	Attempt to connect du and dx, find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	But not e.g. du = dx
		Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{du}{dx} = \frac{1}{2}x^{-½}$
		Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (**3**)	WWW
	(ii)	Attempt partial fractions	M1	
		$\frac{2}{u} - \frac{2}{1+u}$	A1	
		√ $A \ln u + B \ln (1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
		Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
		$\ln \frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (**5**)	Not involving ln 1
			8	

6	**(i)**	Solve 0 = $t-3$ & subst into $x=t^2-6t+4$	M1	
		Obtain $x=-5$	A1 (**2**)	(–5,0) need not be quoted
		N.B. If **(ii)** completed first, subst $y=0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		
	(ii)	Attempt to eliminate t	M1	
		Simplify to $x=y^2-5$ ISW	A1 (**2**)	
	(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
		Obtain $\frac{dy}{dx}=\frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-) $\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
		If t = 2, $x=-4$ and $y=-1$	B1	Awarded anywhere in **(iii)**
		Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
		$x+2y+6=0$ AEF(without fractions) ISW	A1 (**5**)	
			9	
7	**(i)**	Attempt direction vector between the 2 given points	M1	
		State eqn of line using format (**r**) = (either end) + s(dir vec)	M1	's' can be 't'
		Produce 2/3 eqns containing t and s	M1	2 different parameters
		Solve giving $t=3$, $s=-2$ or 2 or -1 or 1	A1	
		Show consistency	B1	
		Point of intersection = $(5,9,-1)$	A1 (**6**)	
	(ii)	Correct method for scalar product of 'any' 2 vectors	M1	Vectors from this question
		Correct method for magnitude of 'any' vector	M1	Vector from this question
		Use $\cos\theta=\frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the correct 2 vectors $\begin{pmatrix}1\\4\\-2\end{pmatrix}$ & $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Vects may be mults of dvs
		62.2 (62.188157…) 1.09 (1.0853881)	A1 (**4**)	
			10	

8	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y^3\right)=3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
		Consider $\frac{\mathrm{d}}{\mathrm{d}x}(xy)$ as a product	M1	
		$= \quad x\frac{\mathrm{d}y}{\mathrm{d}x}+y$	A1	Tolerate omission of '6'
		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6y-3x^2}{3y^2-6x}$ ISW AEF	A1 (**4**)	
	(ii)	$x^3=2^4$ or 16 and $y^3=2^5$ or 32	*B1	
		Satisfactory conclusion	dep* B1	**AG**
		Substitute $\left(2^{\frac{4}{3}},2^{\frac{5}{3}}\right)$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	or the numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$
		Show or use calc to demo that num = 0, ignore denom **AG**	A1 (**4**)	
	(iii)	Substitute (a,a) into eqn of curve	M1	& attempt to state 'a = …'
		$a=3$ only with clear ref to $a\neq 0$	A1	
		Substitute $(3,3)$ or (their a, their a) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	
		-1 only WWW	A1 (**4**)	from (their a,their a)
			12	
9	**(i)**	$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\ldots$	B1	
		$k(160-\theta)$	B1 (**2**)	The 2 @ 'B1' are indep
	(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int\frac{1}{160-\theta}\,\mathrm{d}\theta=\int k,\frac{1}{k},1\,\mathrm{d}t$
		Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
		RHS = kt or $\frac{1}{k}t$ or t (+ c)	A1	
		Subst. $t=0,\theta=20$ into equation containing 'c'	dep* M1	
		Subst $t=5,\theta=65$ into equation containing 'c' & 'k'	dep* M1	
		$c=-\ln 140$ (-4.94) ISW	A1	
		$k=\frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
		Using their 'c' & 'k', subst t = 10 & evaluate θ	dep*M1	
		$\theta=96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (**9**)	
			11	

4724 Core Mathematics 4

1 Long Division For leading term $3x^2$ in quotient B1

Suff evid of div process (ax^2 , mult back, attempt sub) M1

(Quotient) = $3x^2 - 4x - 5$ A1

(Remainder) = $-x + 2$ A1

Identity $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$ *M1

$Q = ax^2 + bx + c, R = dx + e$ & attempt ≥ 3 ops. dep*M1 — If $a = 3$,this $\Rightarrow$ 1 operation

$a = 3, b = -4, c = -5$ A1 — dep*M1; Q = $ax^2 + bx + c$

$d = -1, e = 2$ A1

Inspection Use 'Identity' method; if $R = e$, check cf(x) correct before awarding 2nd M1

4

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2 Indefinite Integral Attempt to connect $\mathrm{d}x$ & $\mathrm{d}\theta$ *M1 — Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$; not $\mathrm{d}x = \mathrm{d}\theta$

Reduce to $\int 1 - \tan^2\theta \,(\mathrm{d}\theta)$ A1 — A0 if $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \sec^2\theta$; but allow all following A marks

Use $\tan^2\theta = (1,-1) + (\sec^2\theta, -\sec^2\theta)$ dep*M1

Produce $\int 2 - \sec^2\theta \,(\mathrm{d}\theta)$ A1

Correct √ integration of function of type $d + e\sec^2\theta$ √A1 — including $d = 0$

EITHER Attempt limits change (allow degrees here) M1 — (This is 'limits' aspect; the integ need not be accurate)

OR Attempt integ, re-subst & use original ($\sqrt{3}$,1)

$\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required A1

7

Q	Part	Answer	Marks	Guidance
3	(i)	$\left(1+\frac{x}{a}\right)^{-2}=1+(-2)\frac{x}{a}+\frac{-2.-3}{2}\left(\frac{x}{a}\right)^2+\ldots$	M1	Check 3rd term; accept $\frac{x^2}{a}$
		$=1-\dfrac{2x}{a}+\ldots$ or $1+\left(-\dfrac{2x}{a}\right)$	B1	or $1-2xa^{-1}$ (Ind of M1)
		$\ldots+\frac{3x^2}{a^2}+\ldots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$)	A1	Accept $\frac{6}{2}$ for 3
		$(a+x)^{-2}=\frac{1}{a^2}\left\{\text{their expansion of }\left(1+\frac{x}{a}\right)^{-2}\right\}$ mult out	√A1 **4**	$\frac{1}{a^2}-\frac{2x}{a^3}+\frac{3x^2}{a^4}$ **;** accept eg a^{-2}
	(ii)	Mult out $(1-x)(\text{their exp})$ to produce all terms/cfs(x^2)	M1	Ignore other terms
		Produce $\frac{3}{a^2}+\frac{2}{a}(=0)$ or $\frac{3}{a^4}+\frac{2}{a^3}$ $(=0)$ or AEF	A1	Accept x^2 if in both terms
		$a=-\frac{3}{2}$ www seen anywhere in (i) or (ii)	A1 **3**	Disregard any ref to $a=0$
			7	
4	(i)	Differentiate as a product, $u\,\mathrm{d}v+v\,\mathrm{d}u$	M1	or as 2 separate products
		$\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin 2x)=2\cos 2x$ <u>or</u> $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cos 2x)=-2\sin 2x$	B1	
		$\mathrm{e}^x(2\cos 2x+4\sin 2x)+\mathrm{e}^x(\sin 2x-2\cos 2x)$	A1	terms may be in diff order
		Simplify to $5\,\mathrm{e}^x\sin 2x$ www	A1 **4**	Accept $10\mathrm{e}^x\sin x\cos x$
	(ii)	Provided result (i) is of form $k\,\mathrm{e}^x\sin 2x$, k const		
		$\int \mathrm{e}^x\sin 2x\,\mathrm{d}x=\dfrac{1}{k}\mathrm{e}^x(\sin 2x-2\cos 2x)$	B1	
		$[\,\mathrm{e}^x(\sin 2x-2\cos 2x)]_0^{\frac{1}{4}\pi}=\mathrm{e}^{\frac{1}{4}\pi}+2$	B1	
		$\frac{1}{5}\left(\mathrm{e}^{\frac{1}{4}\pi}+2\right)$	B1 **3**	Exact form to be seen
		SR Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.		
			7	

5	**(i)**	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used	M1	
		$= \frac{4t + 3t^2}{2 + 2t}$	A1	
		Attempt to find t from one/both equations	M1	<u>or</u> diff (ii) cartesian eqn $\rightarrow$ M1
		State/imply $t = -3$ is only solution of both equations	A1	subst $(3,-9)$, solve for $\frac{dy}{dx} \rightarrow$ M1
		Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$	A1 **5**	grad of curve = $-\frac{15}{4} \rightarrow$ A1
		[**SR** If $t = 1$ is given as solution & not disqualified, award A0 + √A1 for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;		
		If $t = 1$ is given/used as only solution, award A0 + √A1 for grad = $\frac{7}{4}$]		
	(ii)	$\frac{y}{x} = t$	B1	
		Substitute into either parametric eqn	M1	
		Final answer $x^3 = 2xy + y^2$	A2 **4**	
		[**SR** Any correct unsimplified form (involving fractions or common factors) $\rightarrow$ A1]		
			9	
6	**(i)**	$4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$	M1	
		$A = 5$	A1	'cover-up' rule, award B1
		$B = -5$	A1	
		$C = -6$	A1 **4**	'cover-up' rule, award B1
		Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1		
	(ii)	$\int \frac{A}{x-5} dx = A \ln(5-x)$ or $A \ln\lvert 5-x\rvert$ or $A \ln\lvert x-5\rvert$	√B1	but <u>not</u> $A \ln(x-5)$
		$\int \frac{B}{x-3} dx = B \ln(3-x)$ or $B \ln\lvert 3-x\rvert$ or $B \ln\lvert x-3\rvert$	√B1	but <u>not</u> $B \ln(x-3)$
		<u>If candidate is awarded B0,B0</u>, then award **SR** √ B1 for $A \ln(x-5)$ **and** $B \ln(x-3)$		
		$\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$	√B1	
		$5 \ln \frac{3}{4} + 5 \ln 2$ aef, isw √ $A \ln \frac{3}{4} - B \ln 2$	√ B1	Allow if **SR** B1 awarded
		-3 √ $\frac{1}{2}C$	√B1 **5**	
		[Mark at earliest correct stage & isw; no ln 1]	**9**	

7	**(i)**	Attempt scalar prod {**u**.(4**i** + **k**) or **u**.(4**i** + 3**j** + 2**k**)}= 0	M1	where **u** is the given vector
		Obtain $\frac{12}{13}+c=0$ <u>or</u> $\frac{12}{13}+3b+2c=0$	A1	
		$c=-\frac{12}{13}$	A1	
		$b=\frac{4}{13}$	A1	cao No ft
		Evaluate $\left(\frac{3}{13}\right)^2+(\text{their } b)^2+(\text{their } c)^2$	M1	Ignore non-mention of $\sqrt{\ }$
		Obtain $\frac{9}{169}+\frac{144}{169}+\frac{16}{169}=1$ AG	A1 **6**	Ignore non-mention of $\sqrt{\ }$
	(ii)	Use $\cos\theta=\frac{\boldsymbol{x}.\boldsymbol{y}}{\lvert\boldsymbol{x}\rvert\lvert\boldsymbol{y}\rvert}$	M1	
		Correct method for finding scalar product	M1	
		36° (35.837653…) Accept 0.625 (rad)	A1 **3**	From $\frac{18}{\sqrt{17}\sqrt{29}}$
		SR If 4**i**+**k** = (4,1,0) in (i) & (ii), mark as scheme but allow final A1 for 31°(31.160968) or 0.544		
			9	
8	**(i)**	$\frac{d}{dx}\left(y^2\right)=2y\frac{dy}{dx}$	B1	
		$\frac{d}{dx}(uv)=u\ dv+v\ du$ used on $(-7)xy$	M1	
		$\frac{d}{dx}\left(14x^2-7xy+y^2\right)=28x-7x\frac{dy}{dx}-7y+2y\frac{dy}{dx}$	A1	(= 0)
		$2y\frac{dy}{dx}-7x\frac{dy}{dx}=7y-28x\rightarrow\frac{dy}{dx}=\frac{28x-7y}{7x-2y}$ www AG	A1 **4**	As AG, intermed step nec
	(ii)	Subst x = 1 into eqn curve & solve quadratic eqn in y	M1	$(y=3$ or $4)$
		Subst x = 1 and (one of) their y-value(s) into given $\frac{dy}{dx}$	M1	$\left(\frac{dy}{dx}=7 \text{ or } 0\right)$
		Find eqn of tgt, with their $\frac{dy}{dx}$, going through (1, their y)	*M1	using (one of) y value(s)
		Produce either $y=7x-4$ or $y=4$	A1	
		Solve simultaneously their two equations	dep*M1	provided they have two
		Produce $x=\frac{8}{7}$	A1 **6**	
			10	

9	**(i)**	$\frac{20}{k_1}$ (seconds)	B1 **1**	
	(ii)	$\frac{d\theta}{dt} = -k_2(\theta - 20)$	B1 **1**	
	(iii)	Separate variables or invert each side	M1	Correct eqn or very similar
		Correct int of each side (+ c)	A1,A1	for each integration
		Subst $\theta = 60$ when $t = 0$ into eqn containing 'c'	M1	or $\theta = 60$ when t = their **(i)**
		c (or $-c$) = ln 40 or $\frac{1}{k_2}\ln 40$ or $\frac{1}{k_2}\ln 40k_2$	A1	Check carefully their 'c'
		Subst their value of c and $\theta = 40$ back into equation	M1	Use scheme on LHS
		$t = \frac{1}{k_2}\ln 2$	A1	Ignore scheme on LHS
		Total time = $\frac{1}{k_2}\ln 2 + \text{their (i)}$ (seconds)	√A1 **8**	

SR If the negative sign is omitted in part **(ii)**, allow all marks in **(iii)** with ln 2 replaced by $\ln\frac{1}{2}$.

SR If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

10

4724 Core Mathematics 4

1		Long division method			
		Correct leading term x^2 in quotient	B1		
		Evidence of correct div process	M1		Sufficient to convince
		(Quotient =) x^2+6x-4	A1		
		(Remainder =) $11x+9$	A1		
		Identity method			
		$x^4+11x^3+28x^2+3x+1=Q(x^2+5x+2)+R$	M1		
		$Q=ax^2+bx+c$ or x^2+bx+c ; $R=dx+e$ & ≥ 3 ops	M1		N.B. $a=1 \Rightarrow$ 1 of the 3 ops
		$a=1, b=6, c=-4, d=11, e=9$ (for all 5)	A2		S.R. B1 for 3 of these
			4		
2	(i)	Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA})$, $(\overrightarrow{BC}$ or $\overrightarrow{CB})$, $(\overrightarrow{AC}$ or $\overrightarrow{CA})$	M1		irrespect of label; any notation
		Use correct method to find scal prod of any 2 vectors	M1		or use corr meth for modulus
		Use $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{\lvert AB\rvert\lvert BC\rvert}=0$	M1		or use $\lvert\overrightarrow{AB}\rvert^2+\lvert\overrightarrow{BC}\rvert^2=\lvert\overrightarrow{AC}\rvert^2$
		Obtain $p=1$ (dep 3 @ M1)	A1	4	
	(ii)	Use equal ratios of appropriate vectors	M1		or scalar product method
		Obtain $p=-8$	A1	2	
			6		
3		Use $\cos 2x = a\cos^2 x+b\ /\ \pm\ \cos^2 x-\sin^2 x\ /\ 1-2\sin^2 x$	*M1		
		Obtain $\lambda+\mu\sec^2 x$	dep*M1		using 'reasonable' Pythag attempt
		$\int \lambda+\mu\sec^2 x\,dx=\lambda x+\mu\tan x$	A1		(λ or μ may be 0 here/prev line)
		Obtain correct result $2x-\tan x$	A1		no follow-through
		$\frac{1}{6}\pi-\sqrt{3}+1$ ISW	A1		exact answer required
			5		
4		Attempt to connect du and dt or find $\frac{du}{dt}$ or $\frac{dt}{du}$	M1		not du = dt but no accuracy
		$du=\frac{1}{t}dt$ or $\frac{du}{dt}=\frac{1}{t}$ or $dt=e^{u-2}du$ or $\frac{dt}{du}=e^{u-2}$	A1		
		Indef int $\rightarrow \int \frac{1}{u^2}(du)$	A1		no t or dt in evidence
		$=-\frac{1}{u}$	A1		
		Attempt to change limits if working with $f(u)$	M1		or re-subst & use 1 and e
		$\frac{1}{6}$ ISW	A1		ln e must be changed to 1, ln 1 to 0
			6		

5	**(i)**	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + ...$	B1		
		$\ldots \; -\frac{1}{9}x^2$	B1	**2**	$-\frac{2}{18}x^2$ acceptable
	(ii) (a)	$(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$	B1		not $16^{\frac{1}{3}}\left(\frac{1}{2}+x\right)^{\frac{1}{3}}$
		$(1+2x)^{\frac{1}{3}}$ = their **(i)** expansion with $2x$ replacing x	M1		not dep on prev B1
		$= 1+\frac{2}{3}x-\frac{4}{9}x^2 + \ldots$	√A1		$-\frac{8}{18}x^2$ acceptable
		Required expansion = 2 (expansion just found)	√B1	**4**	accept equiv fractions
		<u>N.B</u>. If not based on part **(i)**, award M1 for $8^{1/3} + \frac{1}{3}.8^{-2/3}(16x) + \frac{\frac{1}{3}.-\frac{2}{3}}{1.2}8^{-5/3}(16x)^2$, allowing $16x^2$ for $(16x)^2$, with 3 @ A1 for $2...+\frac{4}{3}x...-\frac{8}{9}x^2$, accepting equivalent fractions & ISW			
	(ii) (b)	$-\frac{1}{2} < x < \frac{1}{2}$ or $\|x\| < \frac{1}{2}$	B1	**1**	no equality
			7		
6		$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		quoted/implied
		$\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW	B1		
		$\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW	B1		
		Stating/implying $\frac{3t^2 - \frac{3}{t}}{9-\frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$	A1		WWW, totally correct at this stage
		$t = 3$ as final ans with clear log indication of invalidity of -3; ignore (non) mention of $t = 0$	A2		**<u>S.R.</u>** A1 if $t = \pm 3$ or $t = -3$ or ($t = 3$ **<u>&</u>** wrong/no indication)
			6		
7		Treat $\frac{d}{dx}(x^2y)$ as a product	M1		
		$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$	B1		
		$3x^2 + 2x^2\frac{dy}{dx} + 4xy = 3y^2\frac{dy}{dx}$	A1		Ignore $\frac{dy}{dx} =$ if not used
		Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa	M1		
		$\frac{dy}{dx} = -4$ WWW	A1		
		grad normal = $-\frac{1}{\text{their } \frac{dy}{dx}}$	√A1		stated or used
		Find eqn of line, through (2, 1), with either gradient	M1		using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their } \frac{dy}{dx}}$
		$x - 4y + 2 = 0$	A1		AEF with integral coefficients
			8		

Q	Part	Answer	Marks	Total	Guidance
8	**(i)**	$-\sin x\,e^{\cos x}$	B1	**1**	
	(ii)	$\int \sin x\,e^{\cos x}dx = -e^{\cos x}$	B1		anywhere in part **(ii)**
		Parts with split $u = \cos x, dv = \sin x\,e^{\cos x}$	M1		result $f(x) +/- \int g(x)\,dx$
		Indef Integ, 1st stage $-\cos x\,e^{\cos x} - \int \sin x\,e^{\cos x}dx$	A1		accept $\ldots -\int -e^{\cos x}.-\sin x\,dx$
		Second stage = $-\cos x\,e^{\cos x} + e^{\cos x}$	*A1		
		Final answer = 1	dep*A2	**6**	
				7	
9	**(i)**	P is $\begin{pmatrix}3\\1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$	B1		
		direction vector of ℓ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and of $\overrightarrow{OP}$ is their P	√B1		
		Use $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and their OP	M1		
		$\theta = 35.3$ or better (0.615… rad)	A1	**4**	
	(ii)	Use $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}=0$	M1		
		$1(3+t)-1(1-t)+2(1+2t)=0$	A1		
		$t=-\frac{2}{3}$	A1		
		Subst. into $\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}$ to produce $\begin{pmatrix}7/3\\5/3\\-1/3\end{pmatrix}$ ISW	A1	**4**	
	(iii)	Use $\sqrt{x^2+y^2+z^2}$ where $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is part **(ii)** answer	M1		
		Obtain $\sqrt{\frac{75}{9}}$ AEF, 2.89 or better (2.8867513….)	A1	**2**	
				10	

10 (i)	$\dfrac{\frac{1}{3}}{3-x}$ ……. $-\dfrac{\frac{1}{3}}{6-x}$	B1+1 **2**	
(ii)	**(a)** Separate variables $\int \frac{1}{(3-x)(6-x)}\,dx = \int k\,dt$	M1	or invert both sides
	<u>Style</u>: For the M1, d*x* & d*t* must appear on correct sides <u>or</u> there must be $\int$ sign on both sides		
	Change $\frac{1}{(3-x)(6-x)}$ into partial fractions from **(i)**	√B1	
	$\int \frac{A}{3-x}\,dx = \left(-A \text{ or } -\frac{1}{A}\right)\ln(3-x)$	B1	or $\int \frac{B}{6-x}\,dx = \left(-B \text{ or } -\frac{1}{B}\right)\ln(6-x)$
	$-\frac{1}{3}\ln(3-x)+\frac{1}{3}\ln(6-x)=kt\ (+c)$	√A1	f.t. from wrong multiples in **(i)**
	Subst (x = 0, t = 0) & (x = 1, t = 1) into eqn with ‘c’	M1	and solve for ‘k’
	Use $\ln a + \ln b = \ln ab$ or $\ln a - \ln b = \ln \frac{a}{b}$	M1	
	Obtain $k = \frac{1}{3}\ln\frac{5}{4}$ with sufficient working & WWW	A1 **7**	**AG**
	(b) Substitute $k = \frac{1}{3}\ln\frac{5}{4}$, $t = 2$ & their value of ‘c’	*M1	
	Reduce to an eqn of form $\frac{6-x}{3-x} = \lambda$	dep*M1	where λ is a const
	Obtain $x = \frac{27}{17}$ or 1.6 or better (1.5882353…)	A2 **4**	S.R. A1√ for $x = \frac{3\lambda-6}{\lambda-1}$
		13	

1 First 2 terms in expansion = $1-5x$ — B1 (simp to this, now or later)

3^{rd} term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$ — M1 $-\dfrac{8}{3}$ can be $-\dfrac{5}{3}-1$

$(3x)^2$ can be $9x^2$ or $3x^2$

$= +\ 20x^2$ — A1

4^{th} term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$ — M1 $-\dfrac{11}{3}$ can be $-\dfrac{5}{3}-2$

$(3x)^3$ can be $27x^3$ or $3x^3$

$= -\dfrac{220}{3}x^3$ ISW — A1 Accept $-\dfrac{440}{6}x^3$ ISW

N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

5

2 Attempt quotient rule — M1

[Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x)+/-\sin x+/-\cos x+/-\cos x$]

Numerator = $(1-\sin x).-\sin x-\cos x.-\cos x$ — A1 terms in any order

{ Product symbols must be clear or implied by further work }

Reduce correct numerator to $1-\sin x$ — B1 or $-\sin x+\sin^2 x+\cos^2 x$

Simplify to $\dfrac{1}{1-\sin x}$ ISW — A1 Accept $-\dfrac{1}{\sin x-1}$

4

3 $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$ — M1 For correct format

$A(x-1)(x-2)+B(x-2)+C(x-1)^2\equiv x^2$ — M1

$A=-3$ — A1

$B=-1$ — A1 (B1 if cover-up rule used)

$C=4$ — A1 (B1 if cover-up rule used)

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.

NB2: Having obtained B & C by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

5

<u>These special cases using different formats are the only other ones to be considered</u> <u>Max</u>

$\dfrac{A}{x-1}+\dfrac{Bx+C}{(x-1)^2}+\dfrac{D}{x-2}$; M1 M1; A0 for any values of A, B & C, A1 or B1 for $D=4$ — 3

$\dfrac{Ax+B}{(x-1)^2}+\dfrac{C}{x-2}$; M0 M1; A1 for $A=-3$ <u>and</u> $B=2$, A1 or B1 for $C=4$ — 3

4		Att by diff to connect dx & du or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not dx=du)	M1	no accuracy; not 'by parts'
		dx = 2u du or $\frac{du}{dx}=\frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF	A1	
		Indefinite integral $\rightarrow \int 2(u^2-2)^2 (\frac{u}{u})(du)$	A1	May be implied later
		{If relevant, cancel u/u and} attempt to square out	M1	
		{dep $\int k\text{I}(du)$ where $k=2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ }		
		Att to change limits if working with f(u) after integration	M1	or re-subst into integral attempt and use −1 & 7
		Indef integ = $\frac{2}{5}u^5 +/- \frac{8}{3}u^3 + 8u$ or $\frac{1}{10}u^5 +/- \frac{2}{3}u^3 + 2u$	A1	or $\frac{1}{5}u^5 +/- \frac{4}{3}u^3 + 4u$
		$\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c'	A1	
			7	
5		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1	Implied by e.g., $4x\frac{dy}{dx}+y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
		Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx}=0$	M1	
		Produce only $2x+4y=0$ (though AEF acceptable)	*A1	without any error seen
		Eliminate x or y from curve eqn & eqn(s) just produced	M1	
		Produce either $x^2=36$ or $y^2=9$	dep* A1	Disregard other solutions
		$(\pm 6, \mp 3)$ AEF, as the only answer ISW	dep* A1	Sign aspect must be clear
			7	
6	**(i)**	State/imply scalar product of any two vectors = 0	M1	
		Scalar product of correct two vectors = 4 + 2a −6	A1	($4+2a-6=0 \rightarrow$ M1A1)
		$a=1$	A1 **3**	
	(ii)	**(a)** Attempt to produce at least two relevant equations	M1	e.g. $2t=3+2s$ …..
		Solve two not containing 'a' for s and t	M1	
		Obtain at least one of $s=-\frac{1}{2}$, $t=1$	A1	
		Substitute in third equation & produce $a=-2$	A1 **4**	
		(b) Method for finding magnitude of any vector	M1	possibly involving 'a'
		Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\boldsymbol{b}\rvert}$ for the pair of direction vectors	M1	possibly involving 'a'
		107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516) c.a.o.	A1 **3**	1.87, 1.88 (1.87707) or 1.26
			10	

7	**(i)**	Differentiate x as a quotient, $\frac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\frac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$	M1	or product clearly defined
		$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$	A1	WWW $\rightarrow$ 2
		$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$	B1	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1	quoted/implied and used
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks)	*A1	ignore ref $t=-1, t=-3$
		State <u>squares</u> +ve or $(t+1)^2$ & $(\mathrm{t}+3)^2$ + ve $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}$ +ve	dep*A1 **6**	or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore ≥ 0
	(ii)	Attempt to obtain t from either the x or y equation	M1	No accuracy required
		$t = \frac{2-x}{x-1}$ AEF <u>or</u> $t = \frac{2}{y} - 3$ AEF	A1	
		Substitute in the equation not yet used in this part	M1	or equate the 2 values of t
		Use correct meth to eliminate ('double-decker') fractions	M1	
		Obtain <u>$2x + y = 2xy + 2$</u> ISW AEF	A1 **5**	but not involving fractions **[11]**
8	**(i)**	<u>Long division method</u>		<u>Identity method</u>
		Evidence of division process as far as 1st stage incl sub	M1	$\equiv Q(x-1) + R$
		(Quotient =) $x - 4$	A1	$Q = x - 4$
		(Remainder =) 2 ISW	A1 **3**	$R = 2$; N.B. might be B1
	(ii)	**(a)** Separate variables; $\int \frac{1}{y-5}\,\mathrm{d}y = \int \frac{x^2 - 5x + 6}{x-1}\,\mathrm{d}x$	M1	'$\int$' may be implied later
		Change $\frac{x^2-5x+6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$)	M1	
		$\ln(y-5)$ =√ (integration of their previous result) (+c)ISW	√A1 **3**	f.t. if using Quot + $\frac{\text{Rem}}{x-1}$
	(ii)	**(b)** Substitute y = 7, x = 8 into their eqn containing 'c'	M1	& attempt 'c' (–3.2 , $\ln\frac{2}{49}$)
		Substitute x = 6 and their value of 'c'	M1	& attempt to find y
		<u>y = 5.00 (5.002529)</u> Also $5 + \frac{50}{49}e^{-6}$	A2 **4**	Accept 5, 5.0,
		Beware: <u>any</u> wrong working anywhere $\rightarrow$ A0 even if answer is one of the acceptable ones.		
			[10]	

9 (i)	Attempt to multiply out $(x+\cos 2x)^2$	M1	Min of 2 correct terms
	<u>Finding $\int 2x\cos 2x\,dx$</u>		
	Use $u=2x, dv=\cos 2x$	M1	1st stage $f(x) +/- \int g(x)\,dx$
	1st stage $x\sin 2x-\int \sin 2x\,dx$	A1	
	$\therefore \int 2x\cos 2x\,dx = x\sin 2x+\frac{1}{2}\cos 2x$	A1	
	<u>Finding $\int \cos^2 2x\,dx$</u>		
	Change to $k\int +/-1+/-\cos 4x\,dx$	M1	where $k=\frac{1}{2}$, 2 or 1
	Correct version $\frac{1}{2}\int 1+\cos 4x\,dx$	A1	
	$\int \cos 4x\,dx=\frac{1}{4}\sin 4x$	B1	seen anywhere in this part
	Result = $\frac{1}{2}x+\frac{1}{8}\sin 4x$	A1	
	(i) ans $=\frac{1}{3}x^3+x\sin 2x+\frac{1}{2}\cos 2x+\frac{1}{2}x+\frac{1}{8}\sin 4x$ (+ c)	A1 **9**	Fully correct
(ii)	V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (dx)$	M1	
	Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer	M1	
	(**i**) correct value = $\frac{1}{24}\pi^3-\frac{1}{2}+\frac{1}{4}\pi-\frac{1}{2}$	A1	
	Final answer = $\pi\left(\frac{1}{24}\pi^3+\frac{1}{4}\pi-1\right)$	A1 **4**	**c.a.o.** No follow-through
		13	

<u>Alternative methods</u>

2 If $y=\dfrac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x)=\cos x$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x+(1-\sin x)\frac{dy}{dx}=-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y=\dfrac{\cos x}{1-\sin x}$ is changed into $y=\cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{dy}{dx}\right)=\cos^2 x(1-\sin x)^{-2}+(1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

6(ii)(a) If candidates use some long drawn-out method to find '*a*' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A2 for $a=-2$

7(ii) Marks for obtaining this Cartesian equation are not available in part (i).

If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:

Method 1 where candidates differentiate implicitly

M1 for attempt at implicit differentiation

A1 for $\frac{dy}{dx}=\frac{2y-2}{1-2x}$ AEF

M1 for substituting parametric values of x and y

A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$

A1 for finish as in original method

Method 2 where candidates manipulate the Cartesian equation to find x = or y=

M1 for attempt to re-arrange so that either $y = f(x)$ or $x = g(y)$

A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF

M1 for differentiating as a quotient

A2 for obtaining $\frac{dy}{dx}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$

A1 for finish as in original method

8(ii)(b) If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A2 for 5, 5.0, 5.00 (5.002529) with caveat as in main scheme dep M2

1	**(i)**	First two terms are $1-\frac{1}{2}x........$	B1	
		Third term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}[(-x)^2 \text{ or } x^2 \text{ or } -x^2]$	M1	
		$= -\frac{1}{8}x^2$	A1 **3**	$-\frac{1}{8}x^2$ without work $\to$ M1 A1
	(ii)	Attempt to replace x by $2y-4y^2$ or $2y+4y^2$	M1	or write as $1-(2y-4y^2 \text{ or } 2y+4y^2)$
		First two terms are $1-y$	B1	
		Third term = $+\frac{3}{2}y^2$ or √ $(4b+2)y^2$	A1√ **3**	where b = cf (x^2) in part (i)
			6	
2	**(i)**	$A(x-2)+B=7-2x$	M1	or $A(x-2)^2+B(x-2)=(7-2x)(x-2)$
		$A=-2$	A1	
		$B=3$	A1 **3**	
	(ii)	$\int \frac{A}{x-2}\,dx = \left(A \text{ or } \frac{1}{A}\right)\ln(x-2)$	B1	Accept $\ln\lvert x-2\rvert$, $\ln\lvert 2-x\rvert$, $\ln(2-x)$
		$\int \frac{B}{(x-2)^2}\,dx = -\left(B \text{ or } \frac{1}{B}\right).\frac{1}{x-2}$	B1	Negative sign <u>is</u> required
		Correct f.t. of A & B; $A\ln(x-2)-\frac{B}{x-2}$	B1√	Still accept lns as before
		Using limits = $-2\ln 3+2\ln 2+\frac{1}{2}$ ISW	B1 **4**	<u>No</u> indication of ln(negative)
			7	
3	**(i)**	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x = \frac{1}{\cos x}$
		Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv - v\,du}{v^2}$ & wrong trig signs
		Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
		Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. <u>Not</u> just = $\sec x.\tan x$
	(ii)	Use $\cos 2x = +/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm(\cos^2 x-\sin^2 x)$
		Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
		Evidence that $\frac{\tan x}{\cos x}=\sec x\tan x$ or $\int \frac{\tan x}{\cos x}\,dx=\sec x$	B1	irrespective of any const multiples
		$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for x except final line
			8	

4	**(i)**	Attempt to use $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dt}.\frac{dt}{dx}$	M1	Not just quote formula
		$\frac{4}{2t}$ or $\frac{2}{t}$	A1 **2**	
	(ii)	Subst $t=4$ into their (i), invert & change sign	M1	
		Subst $t=4$ into (x,y) & use num grad for tgt/normal	M1	
		$y=-2x+52$ AEF CAO (no f.t.)	A1 **3**	Only the eqn of normal accepted
	(iii)	Attempt to eliminate t from the 2 given equations	M1	
		$x=2+\frac{y^2}{16}$ or $y^2=16(x-2)$ AEF ISW	A1 **2**	Mark at earliest acceptable form.
				7
5	**(i)**	Attempt to connect dx and du	M1	Including $\frac{du}{dx}=$ or $du=...dx$; not dx = du
		$5-x=4-u^2$	B1	perhaps in conjunction with next line
		Show $\int\frac{4-u^2}{2+u}.2u\,du$ reduced to $\int 4u-2u^2\,du$ **AG**	A1	In a fully satisfactory & acceptable manner
		Clear explanation of why limits change	B1	e.g. when x = 2, u = 1 <u>and</u> when x = 5, u = 2
		$\frac{4}{3}$	B1 **5**	not dependent on any of first 4 marks
	(ii)(a)	$5-x$	*B1 **1**	Accept $4-x-1=5-x$ (this is not **AG**)
	(b)	Show reduction to $2-\sqrt{x-1}$	dep*B1	
		$\int\sqrt{x-1}\,dx=\frac{2}{3}(x-1)^{\frac{3}{2}}$	B1	Indep of other marks, seen anywhere in (b)
		$\left(10-\frac{2}{3}.8\right)-\left(4-\frac{2}{3}\right)=\frac{4}{3}$ or $4\frac{2}{3}-3\frac{1}{3}=\frac{4}{3}$	B1 **3**	Working must be shown
				9
6	**(i)**	Work with correct pair of direction vectors	M1	
		Demonstrate correct <u>method</u> for finding scalar product	M1	Of <u>any</u> two 3x3 vectors rel to question
		Demonstrate correct <u>method</u> for finding modulus	M1	Of <u>any</u> vector relevant to question
		24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)	A1 **4**	Mark earliest value, allow trunc/rounding
	(ii)	Attempt to set up 3 equations	M1	Of type $3+2s=5, 3s=3+t, -2-4s=2-2t$
		Find correct values of $(s,t)=(1,0)$ or $(1,4)$ or $(5,12)$	A1	Or 2 diff values of s (or of t)
		Substitute their (s,t) into equation not used	M1	and make a relevant deduction
		<u>Correctly</u> demonstrate failure	A1 **4**	dep on all 3 prev marks
	(iii)	Subst their (s,t) from first 2 eqns into new 3rd eqn	M1	New 3rd eqn of type $a-4s=2-2t$
		$a=6$	A1 **2**	
				10

7		Attempt parts with $u = x^2+5x+7$, $\mathrm{d}v = \sin x$	M1		as far as $\mathrm{f}(x) +/- \int \mathrm{g}(x)\,\mathrm{d}x$
		1st stage = $-(x^2+5x+7)\cos x + \int (2x+5)\cos x\,\mathrm{d}x$	A1		signs need not be amalgamated at this stage
		$\int (2x+5)\cos x\,\mathrm{d}x = (2x+5)\sin x - \int 2\sin x\,\mathrm{d}x$	B1		indep of previous A1 being awarded
		$= (2x+5)\sin x + 2\cos x$	B1		
		I = $-(x^2+5x+7)\cos x + (2x+5)\sin x + 2\cos x$	A1		WWW
		(Substitute $x=\pi$) $-$(Substitute $x=0$)	M1		An attempt at subst $x = 0$ must be seen
		$\pi^2+5\pi+10$ WWW **AG**	A1	**7**	
					7
8	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}(y^2) = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
		$\frac{\mathrm{d}}{\mathrm{d}x}(-5xy) = (-)(5)x\frac{\mathrm{d}y}{\mathrm{d}x} + (-)(5)y$	M1		i.e. reasonably clear use of product rule
		LHS completely correct $4x - 5x\frac{\mathrm{d}y}{\mathrm{d}x} - 5y + 2y\frac{\mathrm{d}y}{\mathrm{d}x} (= 0)$	A1		Accept "$\frac{\mathrm{d}y}{\mathrm{d}x} =$" provided it is not used
		Substitute $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{8}$ or solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$ & then equate to $\frac{3}{8}$	M1		Accuracy not required for "solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$"
		Produce $x = 2y$ WWW **AG** **(**Converse acceptable)	A1	**5**	Expect $17x = 34y$ and/or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y-4x}{2y-5x}$
	(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1		
		Produce either $x^2 = 36$ or $y^2 = 9$	A1		
		AEF of $(\pm 6, \pm 3)$	A1	**3**	ISW Any correct format acceptable
					8
9	**(i)**	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,\mathrm{d}x = \int q\,\mathrm{d}t$	M1		Or invert as $\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
		$\int \frac{1}{(x-8)^{1/3}}\,\mathrm{d}x = k(x-8)^{2/3}$	A1		k const
		All correct (+ c)	A1		
		For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
		Correct corresponding value of c from correct eqn	A1		
		Subst their c & $x = 35$ back into eqn	M1		
		$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	**7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
	(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
		Substitute $x = 8$ back into equation containing numeric 'c'	M1		
		$t = 6$	A1	**3**	
					10

1 When an acceptable answer has been obtained, ignore subsequent working (ISW) unless stated otherwise.

2 Ignore working which has no relevance to question as set; e.g. in Qu.1, ignore all terms in x^3 etc.

3 The ‘M’ marks are awarded if it is clear that candidate is attempting to do what he/she should be doing.

4 If an ans is given (**AG**), working must be checked minutely as answer shown will nearly always be ‘correct’. More reasoning/explanation is generally required than when the answer is not given.

Comments or Alternative methods

Question 1(ii)

Beware: there are often double mistakes leading to the correct terms – errors invalidate marks.

Question 2(ii)

For the first 2 marks, we’re really testing $\int \frac{1}{x-2}\,\mathrm{d}x$ and $\int \frac{1}{(x-2)^2}\,\mathrm{d}x$; this is why we accept $\frac{1}{A}$ and/or $-\frac{1}{B}$.

For the 1st & 3rd marks, accept $\ln(2-x)$ as these are the indef integ stages. At final, definite, stage, it will be penalised..

‘Exact value’ is required; so 0.0945…. without equivalent log version $\to B0$ $2\ln 2 - 3\ln 3$ need not be simplified.

Question 4

Allow marks for part (iii) to be awarded at any stage of question.

So, if the Cartesian equation is worked out first of all, then award marks in part (i) as follow:

if cart. eqn is found in the form $x = \mathrm{f}(y)$, award M1 for finding $\frac{\mathrm{d}x}{\mathrm{d}y}$, inverting & subst $y = 4t$ (in either order)

if cart. eqn is found in the form $y = \mathrm{g}(x)$, award M1 for finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ and substituting $x = 2 + t^2$

and, finally, A1 as in main scheme.

Question 5(i)

The problem here will centre on how the candidate manipulates the equation $u = \sqrt{x-1}$ to get x in terms of u. He/she could get $x = u^2 + 1$ (correct) or, perhaps, $x = u^2 - 1$ or $x = 1 - u^2$ (incorrect) or some other incorrect version.

The 1st, 4th & 5th marks in part (i) are unaffected by the correctness or otherwise of this manipulation. However, any error seen must destroy the 2nd and 3rd marks – but candidates can still score 3 of the 5 marks.

For the A1, there must be some evidence of reduction to the given answer; the one main case that we are not accepting is where $\frac{8u - 2u^3}{2+u}$ is said to be $4u - 2u^2$ without any supporting evidence; long division will suffice; or if $8u - 2u^3$ is said to be $(2+u)(4u - 2u^2)$, then we will accept (as multiplication can easily be checked in the head whereas division is not reckoned to be). Note that ‘2’ into ‘$8u$’ gives ‘$4u$’ and ‘u’ into ‘$-2u^3$’ gives ‘$-2u^2$’.

Question 5(ii)(a)

This is just a ‘1’ mark part so we give 1 or 0 purely dependent on the answer and we ignore any sloppy working.

A candidate writing $4 - x - 1 = 3 - x$ will be awarded 0 marks; however, another candidate writing $4 - x - 1 = 5 - x$ will be awarded the B1 mark. This is not an **AG** so the candidate does not know the required answer.

Question 6(i)

For demonstrating correct method for finding scalar product, I expect to see at least $2/3$ of the working correct.

Likewise for modulus: examine either vector, $\sqrt{2^2+3^2-4^2}$ will score M1 { $\frac{2}{3}$ correct, prob $\sqrt{29}$ will follow anyway}

Question 6(ii)

Occasionally candidates do not follow a 'sensible' method. However, the first M1 is always standard. The remaining 3 marks must be awarded for convincing arguments and/for accurate results.

Question 7

This is a question where signs are crucial and where the given answer may be obtained even with errors in the working; also the fact that the answer is **AG** means that many candidates will state it on the final line.

Using the standard method, 3 marks out of the 7 are fixed (the 2 @ M1 and the final A1) but the other 4 marks depend on the capability of the candidate to integrate sin x and cos x.

If he/she uses cos x for the integral of sin x, candidate should get $-(\text{our version of 1st main stage})$, so that's A0 but he/she still has to integrate $(2x+5)\cos x$ for the 2nd stage. Admittedly he/she may then make a further mistake when integrating cos x but the 2 @ B1 are available. These 2 marks are an independent pair and only depend on the integral of $(2x+5)\cos x$ being attempted. Whether it's the integral of $(2x+5)\cos x$ or of $-(2x+5)\cos x$ is immaterial. This gives a maximum of 4 out of 7 if sin x is incorrectly integrated.

Even though I have bracketed the 3 terms as (x^2+5x+7), we can expect some candidates to multiply out as 3 separate integrals., $\int x^2 \sin x \, dx$ and $\int 5x \sin x \, dx$ and $\int 7 \sin x \, dx$

Their equivalent 1st stages are:

$-x^2 \cos x + \int 2x \cos x \, dx$; $-5x \cos x + \int 5 \cos x \, dx$; $-7 \cos x$ **M1 + A1**

Their equivalent 2nd stages are:

$2x \sin x + 2 \cos x$ **B1** $5 \sin x$ **B1**

To obtain the corresponding marks, all components must be correct.

Q			Mark	Total	Guidance
1		Attempt to factorise **both** numerator & denominator	M1		completely or partially
		Num = e.g. $(x^2-1)(x^2-9)$ or $(x^2-2x-3)(x^2+2x-3)$	B1		or $(x-3)(x+3)(x-1)(x+1)$
		Denominator = e.g. $(x^2-2x-3)(x+5)(x+3)$	B1		or $(x-3)(x+1)(x+5)(x+3)$
		$\frac{x-1}{x+5}$ or $1-\frac{6}{x+5}$ WWW	A1	**4**	ISW but not if any further 'cancellation'
		Alternative start, attempting long division			
		Expand denom as quartic & attempt to divide $\frac{\text{numerator}}{\text{denominator}}$	M1		but not divide $\frac{\text{denominator}}{\text{numerator}}$
		Obtain quotient = 1 & remainder = $-6x^3-6x^2+54x+54$	B1		
		Final B1 A1 available as before			
					4
2		$2^2+(-3)^2+\left(\sqrt{12}\right)^2$ soi e.g. 25 or 5	M1		Allow $2^2-3^2+\sqrt{12}^2$
		5	A1		May be implied by 5 or 1/5 in final answer
		$\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{5}\\-\frac{3}{5}\\\frac{\sqrt{12}}{5}\end{pmatrix}$ AEF	√A1	**3**	FT their '5'. Accept $-\frac{1}{5}\begin{pmatrix}\\\\\end{pmatrix}$ or $\frac{1}{\pm 5}\begin{pmatrix}\\\\\end{pmatrix}$
					3
3	**(i)**	The words quotient and remainder need not be explicit			
		Long division For leading term $3x$ in quotient	B1		
		Suff evidence of div process ($3x$, mult back, attempt sub)	M1		
		(Quotient) = $3x-1$	A1		
		(Remainder) = x **AG**	A1	**4**	No wrong working, partic on penult line
		Identity $3x^3-x^2+10x-3=Q(x^2+3)+R$	*M1		
		$Q=ax+b, R=cx+d$ & attempt at least 2 operations	dep*M1		If $a=3$, this $\Rightarrow$ 1 operation
		$a=3, b=-1$	A1		
		$c=1, d=0$	A1		No wrong working anywhere
		Inspection $3x^3-x^2+10x-3=(x^2+3)(3x-1)+x$	B2		or state quotient $=3x-1$
		Clear demonstration of LHS = RHS	B2		
	(ii)	Change integrand to 'their (i) quotient' + $\frac{x}{x^2+3}$	M1		
		Correct FT integration of 'their (i) quotient'	√A1		
		$\int \frac{x}{x^2+3}\,dx=\frac{1}{2}\ln\left(x^2+3\right)$	A1		
		Exact value of integral = $\frac{1}{2}+\frac{1}{2}\ln 4-\frac{1}{2}\ln 3$ AEF ISW	A1	**4**	Answer as decimal value (only) $\rightarrow$ A0
					8

4		<u>Indefinite integral</u> Attempt to connect $\mathrm{d}x$ and $\mathrm{d}\theta$	M1	Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}=, \frac{\mathrm{d}\theta}{\mathrm{d}x}=$, $\mathrm{d}x = \ldots\mathrm{d}\theta$; not $\mathrm{d}x=\mathrm{d}\theta$
		Denominator $\left(1-9x^2\right)^{3/2}$ becomes $\cos^3\theta$	B1	
		Reduce original integral to $\frac{1}{3}\int \frac{1}{\cos^2\theta}\,\mathrm{d}\theta$	A1	May be implied, seen only as $\frac{1}{3}\int \sec^2\theta\,\mathrm{d}\theta$
		Change $\int \frac{1}{\cos^2\theta}\,\mathrm{d}\theta$ to $\tan\theta$	B1	Ignore $\frac{1}{3}$ at this stage
		Use <u>appropriate</u> limits for θ (allow degrees) or x	M1	Integration need not be accurate
		$\frac{\sqrt{3}}{9}$ AEF, exact answer required, ISW	A1 **6**	
				6
5	**(i)**	Attempt to set up 3 equations	M1	of type $4+3s=1, 6+2s=t, 4+s=-t$
		$(s,t)=(-1,4)$ or $(-1,-3)$ or $(-\frac{10}{3},-\frac{2}{3})$	*A1	or $s=-1$ & $-\frac{10}{3}$ <u>or</u> $t=$ two of $(4,-3,-\frac{2}{3})$
		Show clear contradiction e.g. $3\neq-4,\ 4\neq-3,\ -6\neq1$	dep*A1 **3**	Allow ✓ unsimpl contradictions. No ISW.
		<u>SC</u> If $s=\frac{-10}{3}$ found from 2nd & 3rd eqns and contradiction shown in 1st eqn, all 3 marks may be awarded.		
	(ii)	Work with $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\-1\end{pmatrix}$	M1	
		Clear method for scalar product of any 2 vectors	M1	
		Clear method for modulus of any vector	M1	
		$79.1^{(\circ)}$ or better (79.1066..) 1.38 (rad) (1.38067..) ISW	A1 **4**	(From $\frac{1}{\sqrt{14}.\sqrt{2}}$)
	(iii)	Use $\begin{pmatrix}4+3s\\6+2s\\4+s\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix}=0$	M1	
		Obtain $s=-2$	A1	from $12+9s+12+4s+4+s=0$
		A is $\begin{pmatrix}-2\\2\\2\end{pmatrix}$ or $-2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ final answer	<u>B</u>1 **3**	Accept $(-2, 2, 2)$
				10

6 $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}ax$ …….. $+\frac{\frac{1}{2}\cdot\frac{-1}{2}}{2}(ax)^2$ B1, B1 N.B. third term = $-\frac{1}{8}a^2x^2$

Change $(4-x)^{-\frac{1}{2}}$ into $k\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$, where k is likely to be $\frac{1}{2}$ /2/4 / –2 , & work out expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\frac{1}{8}x$ …. $+\frac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\left(\frac{(-)x}{4}\right)^2$ B1, B1 N.B. third term = $\frac{3}{128}x^2$

<u>OR</u> Change $\{4-x\}^{\frac{1}{2}}$ into $l\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$, where l is likely to be $\frac{1}{2}$ / 2 / 4 /– 2,& work out expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1-\frac{1}{8}x-\frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k=\frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l=2$ (<u>with no further marks available</u>)

Multiply $(1+ax)^{\frac{1}{2}}$ by $(4-x)^{-\frac{1}{2}}$ or $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) <u>identified as</u>

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 **8** A1 for one correct term + A1 for other two

<u>SC</u> B1 for $\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$; B1 for $\left(1-\frac{x}{4}\right)^{-1}=1+\frac{x}{4}+\frac{x^2}{16}$; M1 for multiplying $(1+ax)$ by their $(4-x)^{-1}$.

If result is $p+qx+rx^2$, then to find $\left(p+qx+rx^2\right)^{\frac{1}{2}}$ award B1 for $p^{\frac{1}{2}}(......)$,

B1 correct 1st & 2nd terms of expansion, B1 correct 3rd term; A1,A1 as before, for correct answers.

8

7 Attempt to sep variables in format $\int py^2\,(\mathrm{d}y)=\int\frac{q}{x+2}(\mathrm{d}x)$ M1 where constants p and/or q may be wrong

Either y^3 & $\ln(x+2)$ <u>or</u> $\frac{1}{3}y^3$ & $\frac{1}{3}\ln(x+2)$ A1+A1 Accept $\frac{1}{3}\ln(3x+6)$ for $\frac{1}{3}\ln(x+2)$ & $|\ |$ for ( )

<u>If indefinite integrals are being used (most likely scenario)</u>

Substitute $x=1, y=2$ into an eqn <u>containing '+const'</u> M1

Sub <u>y</u> = 1.5 and their value of 'const' & solve for <u>*x or q*</u> M1

x or q = –1.97 <u>only</u> A2

[SC x or q = –1.970 or –1.971 or –1.9705 or –1.9706 A1] **7**

<u>If definite integrals are used (less likely scenario)</u>

Use $\int_{1.5}^{2}...\mathrm{d}y=\int_{q}^{1}...\mathrm{d}x$ where 2 corresponds with 1..... M2 & 1.5 corresp with q (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use $\int_{1.5}^{2}...\mathrm{d}y=\int_{1}^{q}...\mathrm{d}x$ where 2 corresponds with q….. M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 <u>only</u>

7

8	**Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded**		
(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$		
	<u>OR</u> sub $t = -2$ into para eqs, obtain $(-1,-3)$ & state $y = 3x$		
	<u>OR</u> other similar methods producing (or verifying) $t = -2$	B1	
	Value of t at other point is 2	B1 **2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	
	$= -(t+1)^2$	A1	or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1	
	Gradient normal = 1 cao	A1	
	Subst $t = -2$ into the parametric eqns.	M1	to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal <u>WWW</u>	A1 **6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1	
	Produce $t = 0$ as final answer cao	A1 **2**	This is dep on final A1 in (ii)
	N.B. If $y = x - 2$ is found fortuitously in (ii) (& $\therefore$ given A0 in (ii)), you must award A0 here in (iii).		
(iv)	Attempt to eliminate t from the parametric equations	M1	
	Produce <u>any</u> correct equation	A1	e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1 **3**	Must be seen in (iv)
	{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}		
		13	

9 (i)	Treat $x \ln x$ as a product	M1	If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
	Obtain $x.\frac{1}{x} + \ln x$	A1	$x \ln x - \int 1\, dx = x \ln x - x$
	Show $x.\frac{1}{x} + \ln x - 1 = \ln x$ WWW **AG**	A1 **3**	And state given result
(ii)(a)	Part (a) is mainly based on the indef integral $\int (\ln x)^2\, dx$		
	[A candidate stating e.g. $\int (\ln x)^2 dx = \int 2 \ln x\, dx$ or $= \int (\ln x - x)^2 dx$ **is awarded 0 for (ii)(a)]**		
	Correct use of $\int \ln x\, dx = x \ln x - x$ anywhere in this part	B1	Quoted from (i) or derived
	Use integ by parts on $\int (\ln x)^2\, dx$ with $u = \ln x, dv = \ln x$	M1	or $u = (\ln x)^2, dv = 1$
	[For 'integration by parts, candidates must get to a 1st stage with format $f(x) +/- \int g(x) dx$]		
	1st stage $= \ln x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx$ soi	A1	$x(\ln x)^2 - \int x.\frac{2}{x} \ln x\, dx$
	2nd stage $= x(\ln x)^2 - 2x \ln x + 2x$ AEF (unsimplified)	A1	
	$\therefore$ Value of definite integral between 1 & e $= e - 2$ cao	A1	Use limits on 2nd stage & produce cao
	Volume $= \pi(e - 2)$ ISW	A1 **6**	Answer as decimal value (only) $\rightarrow$ A0
	Alternative method when subst. $u = \ln x$ used		
	Attempt to connect dx and du	M1	
	Becomes $\int u^2 e^u\, du$	A1	
	First stage $u^2 e^u - \int 2u\, e^u du$	A1	
	Third stage $(u^2 - 2u + 2)e^u$	A1	
	Final A1 A1 available as before		
(b)	Indication that reqd vol = vol cylinder – vol inner solid	M1	
	Clear demonstration of either vol of cylinder being πe^2 (including reason for height $= \ln e$) or rotation of $x = e$ about the y-axis (including upper limit of $y = \ln e$)	A1	Could appear as $\pi \int_0^1 e^2\, dy$
	$(\pi)\int x^2\, dy = (\pi)\int e^{2y} dy$	B1	
	$\frac{\pi(e^2 + 1)}{2}$ or 13.2 or 13.18 or better	B1 **4**	May be from graphical calculator
		13	

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is.. e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3}\cos\theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol √ is sometimes used to indicate 'follow-through' in this scheme.

Question			Answer	Marks	Guidance	
1			f(x) = $\left(x^2+1\right)\left(x^2+4x+2\right)+\left(x-1\right)$	M1	written or clearly intended	(Alt)Long div with 3 stages/equate quots/equate rems
			$x^4+4x^3+...$	B1		
			$+ \ldots 3x^2+5x+1$	A1		
				[3]		
2	(i)		**a** = $\begin{pmatrix}4\\2\\7\end{pmatrix}$ or $\begin{pmatrix}5\\-4\\-1\end{pmatrix}$	B1	Accept any notation	
			b = Difference between the two points	M1		
			Provided final answer is of form <u>**r** = **a** + *t***b**</u> $\begin{pmatrix}1\\-6\\-8\end{pmatrix}$ or $\begin{pmatrix}-1\\6\\8\end{pmatrix}$	A1		
				[3]		
2	(ii)		Method for magnitude of <u>any</u> vector	M1	Accept e.g. $\sqrt{1^2-6^2-8^2}$	
			Method for scalar product of <u>any</u> 2 vectors	M1		
			Using $\cos\theta=\frac{\mathbf{c.d}}{\lvert\mathbf{c}\rvert\lvert\mathbf{d}\rvert}$ for their **b** and $\begin{pmatrix}1\\2\\3\end{pmatrix}$	M1		
			21.4 or better (21.444513); 0.374 or better (0.374277)	A1		
				[4]		

Question			Answer	Marks	Guidance	
3	(i)		Treat $(x+3)(y+4)$ or xy as a product	M1	attempting $u.\mathrm{d}v + v.\mathrm{d}u$	
			$\frac{\mathrm{d}}{\mathrm{d}x}(x+3)(y+4) = (x+3)\frac{\mathrm{d}y}{\mathrm{d}x} + (y+4)$ or $\frac{\mathrm{d}}{\mathrm{d}x}(xy) = x\frac{\mathrm{d}y}{\mathrm{d}x} + y$	A1		
			$\frac{\mathrm{d}}{\mathrm{d}x}(y^2) = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
			$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x-y-4}{x-2y+3}$	B1 **[4]**	AEF including $-\frac{a}{b}, \frac{-a}{b}, \frac{a}{-b}$	
3	(ii)		State or imply that denominator is zero	B1	Provided denom is $x-2y+3$ or $-x+2y-3$	
			Tangents are parallel to y-axis	B1 **[2]**	Accept vertical or of the form $x = k$	
3	(iii)		Substitute (6,0) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ $(= \frac{8}{9})$	M1		
			$8x-9y=48$ FT $fx-gy=6f$	A1 FT **[2]**	FT their numerical $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{f}{g}$ www in this part	
4	(i)		First two terms in expansion = $1-x$	B1	(simplify to this, now or later)	
			Third term shown as $\frac{\frac{1}{4}.-\frac{3}{4}}{2}(-4x)^2$	M1	$-\frac{3}{4}$ can be $\frac{1}{4}-1$; $(-4x)^2$ can be $-4x^2$ or $-16x^2$	
			$= -\frac{3}{2}x^2$	A1		
			Fourth term shown as $\frac{\frac{1}{4}.-\frac{3}{4}.-\frac{7}{4}}{2.3}(-4x)^3$	M1	Similar allowances as for first M1	
			$= -\frac{7}{2}x^3$	A1 **[5]**	[Complete expansion is $1-x-\frac{3}{2}x^2-\frac{7}{2}x^3...$]	

Question			Answer	Marks	Guidance	
4	(ii)		$\left(1+bx^2\right)^7$ shown (implied) as $1+7bx^2+...$	B1		
			Clear indic that terms involving x and x^2 must cancel	M1		
			$a=-1$	A1 FT	If **(i)** = $1+\lambda x+\mu x^2$, $a=\lambda$	
			$b=-\frac{3}{14}$	A1 FT	If **(i)** = $1+\lambda x+\mu x^2$, $b=\frac{1}{7}\mu$	
				[4]	FT from wrong **(i)** only, not wrong $\left(1+bx^2\right)^7$	
5			Attempt to connect du and dx or find $\frac{\mathrm{d}u}{\mathrm{d}x}$	M1	no accuracy ; not $\mathrm{d}u=\mathrm{d}x$	
			$\mathrm{d}u=-\sin x\ \mathrm{d}x$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x$	A1		
			Indefinite integral becomes $-\int\left(1-u^2\right)u^2\ (\mathrm{d}u)$	A1 FT	FT only from $\frac{\mathrm{d}u}{\mathrm{d}x}=\sin x$	
			$-\int\left(1-u^2\right)u^2\ (\mathrm{d}u)=-\frac{1}{3}u^3+\frac{1}{5}u^5$	B1	Award also for $\int\left(1-u^2\right)u^2\ \mathrm{d}u=\frac{1}{3}u^3-\frac{1}{5}u^5$	
			Use new limits if f(u) or original limits if resubstitution	M1	no accuracy	
			$\frac{47}{480}$ AE Fraction	A1	ISW **www** If A0, answer of 0.0979… $\rightarrow$ M1	
				[6]		

Question			Answer	Marks	Guidance	
6			State or imply that graphs cross at $x=\frac{1}{4}\pi$	B1	(Limits on integrals may clarify)	Be lenient here
			$\pi\int y^2\mathrm{d}x$ used with either $y = \sin x$ or $y = \cos x$	*M1	The ‘ π ’ element(s) may not appear until later in the working.	
			$\pi\int_0^{\frac{1}{4}\pi}\sin^2 x\ (\mathrm{d}x)+\pi\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}\cos^2 x\ (\mathrm{d}x)$ or $2\pi\int_0^{\frac{1}{4}\pi}\sin^2 x\ (\mathrm{d}x)$	A1		
			Changing $\sin^2 x$ or $\cos^2 x$ into f(cos 2x)	**dep*M1**		
			$\sin^2 x=\frac{1}{2}(1-\cos 2x)$ or $\cos^2 x=\frac{1}{2}(1+\cos 2x)$	A1		
			$\int\cos 2x\ (\mathrm{d}x)=\frac{1}{2}\sin 2x$ anywhere in this part	B1		
			$\frac{1}{4}\pi^2-\frac{1}{2}\pi$	A1 **[7]**	ISW	
7	(i)		Use $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+t\\-t\\2\end{pmatrix}$	B1		
			$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\-1\\0\end{pmatrix}=0$	M1		
			$\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\\2\end{pmatrix}$ or $\frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+2\mathbf{k}$	A1 **[3]**		

Question			Answer	Marks	Guidance	
7	(ii)		$(1+t)^2+t^2+4=3^2$ or $\sqrt{(1+t)^2+t^2+4}=3$	M1	FT from their **(i)** *P*	
			$t=1$ or -2	A1	SR If A0A0 award A1A0 for either value of *t* leading to its correct answer.	
			$\begin{pmatrix}2\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\2\end{pmatrix}$	A1		
				[3]		
8	(i)		$\frac{dy}{dx}=\frac{\text{attempt at }\frac{dy}{d\theta}}{\text{attempt at }\frac{dx}{d\theta}}$ but not $\frac{4-3\sin^2\theta}{2\sin\theta}$	M1		Alternative Change to Cartesian form, differentiate and resubstitute Correct differentiation of correct equation
			$4\cos\theta-3\sin^2\theta\cos\theta$ seen	B1	indep	
			$\left(\frac{dy}{dx}=\right)\frac{4\cos\theta-3\sin^2\theta\ \cos\theta}{2\sin\theta\cos\theta}=\frac{4-3\sin^2\theta}{2\sin\theta}$ **AG**	A1		
				[3]		
8	(ii)		Equating given $\frac{dy}{dx}$ to 2 & producing quadratic equation	M1		
			$\sin\theta=\frac{2}{3}$	A1	ignore any other given value	
			P is $\left(\frac{4}{9},\frac{64}{27}\right)$	A1	Accept 0.444… and 2.37… or better	
				[3]		
8	(iii)		Identify problem as solving $4-3\sin^2\theta=0$	M1		
			Show convincingly that $4-3\sin^2\theta=0$ has no solutions	A1	Consider magnitude of $\sin\theta$	
				[2]		
8	(iv)		Attempt to eliminate $\sin\theta$ from the 2 given equations	M1	e.g. $y=4\sqrt{x}-\left(\sqrt{x}\right)^3$	
			Produce $y^2=x(4-x)^2$ or $16x-8x^2+x^3$	A1	ISW	
				[2]		

Question			Answer	Marks	Guidance	
9			Use $u = x^2 + 1,\ \mathrm{d}v = \mathrm{e}^{2x}$ or $u = x^2,\ \mathrm{d}v = \mathrm{e}^{2x}$	M1	1st stage = f(x)+/- $\int g(x)\ \mathrm{d}x$	
			1st stage= $\frac{1}{2}(x^2+1)\ \mathrm{e}^{2x} - \int x\ \mathrm{e}^{2x}\ \mathrm{d}x$ or $\frac{1}{2}x^2\mathrm{e}^{2x} - \int x\mathrm{e}^{2x}\mathrm{d}x$	A1		
			For $\int x\ \mathrm{e}^{2x}\ \mathrm{d}x$, use $u = x,\ \mathrm{d}v = \mathrm{e}^{2x}$	M1	ditto	
			$= \frac{1}{2}x\ \mathrm{e}^{2x} - \frac{1}{4}\ \mathrm{e}^{2x}$	A1	tolerate second sign error in $-\int x\mathrm{e}^{2x}\mathrm{d}x$	
			Complete final stage = $\frac{1}{2}(x^2+1)\ \mathrm{e}^{2x} - \frac{1}{4}(2x-1)\ \mathrm{e}^{2x}$	A1	soi; may be separate terms	
			Correct (method) use of limits seen anywhere	M1	Do not accept (…..) − 0	
			Final answer = $\frac{3}{4}\mathrm{e}^2 - \frac{3}{4}$	A1	ISW; if A0, answer of 4.79… $\rightarrow$ M1	
				[7]		
10	(i)		$\frac{1}{2}(y^2+1)^{-\frac{1}{2}}.2y$ or better	B1 **[1]**	Tolerate "$\frac{\mathrm{d}y}{\mathrm{d}x}$ = …" but, otherwise, no $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$	
10	(ii)		Separate variables; $\int \frac{y}{\sqrt{y^2+1}}\ \mathrm{d}y = \int \frac{x-1}{x}\ \mathrm{d}x$	*M1	$\int$ may be implied later	
			Change $\frac{x-1}{x}$ into $1-\frac{1}{x}$	M1		
			RHS = $x - \ln x$	A1		
			LHS = $\sqrt{y^2+1}$	B1	Quoted or derived	
			Subst $y = \sqrt{\mathrm{e}^2 - 2\mathrm{e}},\ x = \mathrm{e}$ into their eqn. with 'c'	Dep*M1		
			$\sqrt{y^2+1} = \sqrt{(\mathrm{e}-1)^2} = \mathrm{e} - 1$	A1	Ignore lack of/no ref to $1-\mathrm{e}$	
			$c = 0$	A1	Ignore any ref to $c = 2 - 2\mathrm{e}$	
			$\sqrt{y^2+1} = x - \ln x$	A1	ISW	
				[8]		

Question			Answer	Marks	Guidance
1	(i)		$x^2-3x+2=$ $(x-1)(x-2)$ or $(1-x)(2-x)$ oe	B1	
			Obtain $-\frac{1}{x-2}$ or $\frac{1}{2-x}$ or $\frac{-1}{x-2}$ or $\frac{1}{-(x-2)}$ ISW	B1	Not $\frac{-1}{-(2-x)}$ Accept WW
			If Partial Fractions are used, apply normal mark scheme.	**[2]**	
1	(ii)		Attempt single fraction or 2 fractions with same relevant denom	M1	e.g. $(x-1)(x-4)[(x-3)\text{ or }(x-3)^2]$
			Fully correct fraction(s) before any simplification	A1	
			Relevant numerator = $3x-9$ or $3x^2-18x+27$	B1	Can award if no denominator
			Final answer = $\frac{3}{(x-1)(x-4)}$ or $\frac{3}{x^2-5x+4}$ ISW	A1	
				[4]	
			S.R. If partial fractions are used on each fraction	(M1)	
			$-\frac{1}{x-1}+\frac{2}{x-3}$	(A1)	
			$\frac{2}{x-3}-\frac{1}{x-4}$	(A1)	
			$-\frac{1}{x-1}+\frac{1}{x-4}$ ISW	(A1)	
2			Write (or imply as) $\int 1.\ln(x+2)(\mathrm{d}x)$ (ln x+ ln 2 → M0)	M1	OR: t = ln(x+2) and attempt to connect dx and dt
			Correct 'by parts' 1st stage $x\ln(x+2)-\int\frac{x}{x+2}(\mathrm{d}x)$	A1	$\int te^t(dt)$
			Any suitable <u>starting idea</u> for integrating $\frac{x}{x+2}$	M1	Attempt by parts with u = t, $\frac{\mathrm{d}v}{\mathrm{d}t}=e^t$
			[e.g. change num to $x+2-2$ or use substitution $x+2=u$]		
			$\int\frac{x}{x+2}(\mathrm{d}x)==$ $x-2\ln(x+2)$ or $x+2-2\ln(x+2)$	A1	te^t-e^t
			Overall result = $x\ln(x+2)-x+2\ln(x+2)$ [(+c) or (-2+c)] ISW	A1	
				[5]	
			SR: Correct answer with no working	(B2)	

Question			Answer	Marks	Guidance
3	(i)		**The first 5 marks are awarded for expansions of either** $(1+4x)^{-\frac{1}{2}}$ **or** $(1+4x)^{\frac{1}{2}}$		
			Expansion of $(1+4x)^{-\frac{1}{2}}$; First 2 terms $= 1-2x$	B1	Or $(1+4x)^{\frac{1}{2}} = 1+2x$.......
			3rd term = $\frac{-\frac{1}{2}.(-\frac{1}{2}-1)}{2}.16x^2$ [Accept $4x^2$ for $16x^2$]	M1	3rd term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}.16x^2$ [ditto]
			$= +\ 6x^2$	A1	$= -2x^2$
			4th term $= \frac{-\frac{1}{2}.(-\frac{1}{2}-1).(-\frac{1}{2}-2)}{2.3}.64x^3$ [Accept $4x^3$ for $64x^3$]	M1	4th tm = $\frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{2.3}.64x^3$ [ditto]
			$= \ -20x^3$	A1	$= +\ 4x^3$
			$1-2x+7x^2-22x^3$; $1+ax+(b+1)x^2+(a+c)x^3$	A1 ft	ft only $(1+4x)^{-\frac{1}{2}} = 1+ax+bx^2+cx^3$ provided *a*, *b* and *c* attempted and at least one @ M1 obtained
				[6]	
3	(ii)		$\lvert x\rvert < \frac{1}{4}$; $-\frac{1}{4} < x < \frac{1}{4}$; $\{-\frac{1}{4} < x\ ,\ x < \frac{1}{4}\}$ no equality	B1	But not $\{-\frac{1}{4} < x$ **<u>OR</u>** $x < \frac{1}{4}\}$ If choice mark what appears to be the final answer.
				[1]	
4			+/– $\int e^{2y}\,(dy)$ and +/– $\int \tan x\,(dx)$ seen	M1	may be implied later
			$\int e^{2y}(dy) = \frac{1}{2}e^{2y}$	B1	
			$\int \tan x\,(dx) = \ln\lvert\sec x\rvert$ or $-\ln\lvert\cos x\rvert$	B1	Accept ln sec*x* or –ln cos*x*
			Subst $x = 0$, $y = 0$ into their equation containing f(x), g(y) and c	M1	S.R. Using def integrals: M1 $\int_0^x = \int_0^y$ followed by A2 or A0
			$c = \frac{1}{2}$ **WWW** (or poss $-\frac{1}{2}$ if c on LHS)	A1	
			$y = \frac{1}{2}\ln(1-2\ln\lvert\sec x\rvert)$ or $\frac{1}{2}\ln(1+2\ln\lvert\cos x\rvert)$ oe WWW	A1	Accept omission of modulus
				[6]	

Question			Answer	Marks	Guidance
5	(i)		Use $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$	M1	
			Obtain $\left(\cos\theta = \frac{6}{12}\right)$ $\theta = 60$ or $\frac{1}{3}\pi$ or 1.05 or better	A1	Better: 1.0471976 (rot)
				[2]	
5	(ii)		Indicate **a** – **b** is vector joining ends of **a** and **b** or equiv $\lvert\mathbf{a}-\mathbf{b}\rvert = \lvert\mathbf{a}\rvert - \lvert\mathbf{b}\rvert$,or anything similar, $\rightarrow$ M0	M1	
			Use cosine rule correctly on 3, 4 and included (i) angle	M1	Or any other correct method
			Obtain $\sqrt{13}$ or 3.61 or better (No ft from wrong θ)	A1	3.6055513 (rot)
				[3]	
6			Attempt diff to connect du and dx or find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	no accuracy, not just du = dx
			Correct e.g. $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or dx = $(2u-2)du$ AEF	*A1	
			Indefinite integral in terms of u = $\int \frac{2u-2}{u}(du)$	A1dep*	
			Provided of form $\int \frac{au+b}{u}(du)$, change to $\int a + \frac{b}{u}(du)$	M1	Or by parts
			Integrate to $au + b\ln\lvert u\rvert$ or $au + b\ln u$	A1 ft	
			Use correct variable for limits after attempt at integral of f(u)	M1	i.e. use new values of u (usually) or orig values of x (if resubst)
			Show as $8-2\ln 4-6+2\ln 3$ (oe) = $2+2\ln\frac{3}{4}$ **AG** WWW	A1	Some 'numerical' working must be shown before giving final ans
				[7]	

Question			Answer	Marks	Guidance
7			Satisfactory start method eg <u>attempt</u> square of $(1-\sin 3x)$	M1	Not e.g. $\frac{(1-\sin 3x)^3}{3}$.
			[N.B. The squaring process might include a term $\sin^2 9x$]		
			<u>The next **2** marks are awarded for integrating</u> $-2\sin 3x$		
			Obtain $\int -2\sin 3x\,dx = \frac{2}{3}\cos 3x$	*A1	
			Obtain $-\frac{2}{3}$ or $(...+0...)-(...+\frac{2}{3}...)$	A1dep*	
			<u>The next **3** marks are awarded for integrating</u> $\sin^2 3x$		<u>or for integrating</u> $\sin^2 ax$ where a = 6 or 9 **only**
			Use $\sin^2 3x = k(+/-1+/-\cos 6x)$	M1	$\sin^2 ax = k(+/-1+/-\cos 2ax)$
			Correct version = $\frac{1}{2}(1-\cos 6x)$	A1	Correct = $\frac{1}{2}(1-\cos 2ax)$
			$\int \cos 6x\,dx = \frac{1}{6}\sin 6x$, seen anywhere, indep	B1	or $\int \cos 2ax\,dx = \frac{1}{2a}\sin 2ax$
			Final answer = $\frac{1}{4}\pi + their -\frac{2}{3}$	A1	Check that the $\frac{1}{4}\pi$ is from $\left[\frac{3}{2}x - \frac{1}{12}\sin 6x\right]_0^{\frac{1}{6}\pi}$
				[7]	

Question			Answer	Marks	Guidance
8	(a)		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1	
			$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
			Substitute $(-1,-1)$ for (x, y) & attempt to solve for $\frac{dy}{dx}$	M1	or solve then substitute
			Obtain $\frac{dy}{dx} = -1$ WWW	A1	
				[4]	
8	(b)	(i)	Tangent parallel y-axis $\rightarrow \frac{dx}{dt} = 0$ or $\frac{dy}{dx} \rightarrow \infty$ or $\frac{dy}{dx} = \infty$	M1	Accept clear intention
			Obtain $t = 0$	A1	
			$(-1,0)$ with no other possibilities	A1	Accept $x = -1, y = 0$
				[3]	
8	(b)	(ii)	State or imply or use $\frac{dy}{dt} = \frac{dx}{dt}$	M1	
			Produce $3t^2 + 1 = 4t$ oe	A1	
			$t = \frac{1}{3}$ or 1	A1	
				[3]	

Question			Answer	Marks	Guidance
9	(i)		$\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$	B1	i.e. correct partial fractions
			$A(x-2)^2+B(x+1)(x-2)+C(x+1)=x^2-x-11$	M1	or equivalent identity or method
			$A=-1$	A1	B1 if cover up method used
			$B=2$	A1	
			$C=-3$	A1	B1 if cover up method used
				[5]	
			Special Cases The problems arise when we see how candidates deal with the denominator $(x-2)^2$: $\frac{A}{x+1}+\frac{Bx+C}{(x-2)^2}$; allow B1 for PF format, M1 for associated identity, B1 for $A=-1$ (max 3) $\frac{A}{x+1}+\frac{B}{x-2}+\frac{Cx+D}{(x-2)^2}$; allow B1 for PF format, M1 for assoc identity, B1 for $A=-1$ (max 3) $\frac{A}{x+1}+\frac{Bx}{(x-2)^2}$; allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$) $\frac{A}{x+1}+\frac{B}{(x-2)^2}$: allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$)		
9	(ii)		No marks are to be awarded for integrating a fraction with a zero numerator. Irrespective of the format used for the Partial Fractions in part (i), award marks as follow:		
			$\int \frac{\lambda}{x+1}\,dx=\left(\lambda \text{ or } \frac{1}{\lambda}\right)\ln(x+1)$ or….	B1	$\int \frac{\lambda}{x-2}\,dx=\left(\lambda \text{ or } \frac{1}{\lambda}\right)\ln(x-2)$
			$\int \frac{\mu}{(x-2)^2}\,dx=-\left(\mu \text{ or } \frac{1}{\mu}\right).\frac{1}{x-2}$	B1	
			$-\frac{3}{2}$	B1 ft	ft $\frac{C}{2}$
			$.....+\ln\frac{16}{5}$ ISW for either term	B1 ft	ft $.....+\ln\left\{\left(\frac{5}{4}\right)^A.2^B\right\}$
				[4]	

Question			Answer	Marks	Guidance
10			If MR, mark according to the scheme & follow-through from candidate's data. Award M, A & B marks (where possible) & apply penalty of 1 mark (by withholding one A mark in the question). E.g. in (i), product to be 'correct' & 'not perpendicular' to be stated.		
	(i)		α. Full justification that $t=-1$. May be 'by inspection'. [No equations not satisfied by $t=-1$ to be shown] ['unusual' attempts must be carefully checked; if convinced, award the B1 e.g. displacement vector between (-3**i** + 6**k**) and (-**i** + 2**j** + 7**k**) = ±(2**i** + 2**j** + **k**)]	B1	No other t = to be mentioned
			β. Consider scalar product $\begin{pmatrix}-3\\0\\6\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}$	M1	
			Show **– 6 + (0) + 6** = 0 and somewhere state perpendicularity oe	A1	
			[If $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ quoted, ignore accuracy of work involving $\lvert\mathbf{a}\rvert$ and $\lvert\mathbf{b}\rvert$]	**[3]**	
10	(ii)		Use **r** = v (-3**i** +6**k)** and ℓ_2	*M1	or $(-3\mathbf{i}+6\mathbf{k})$ + v$(-3\mathbf{i}+6\mathbf{k})$
			Attempt to produce at least two relevant equations	M1dep*	
			Solve two equations & produce $(v,s)=\left(\frac{1}{3},-3\right)$ soi	A1	$(v,s)=\left(-\frac{2}{3},-3\right)$
			Demonstrate clearly that these satisfy third equation	B1 **[4]**	Numerical proof required
10	(iii)		Method for finding $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{AB}\rvert$	M1	Method for finding $\overrightarrow{OB}$ or $\overrightarrow{BO}$ or $\overrightarrow{AB}$ or $\overrightarrow{BA}$
			$\lvert\overrightarrow{OB}\rvert=\sqrt{5}$ or $\lvert\overrightarrow{OA}\rvert=\sqrt{45}$ oe or $\lvert\overrightarrow{BA}\rvert=\sqrt{20}$ oe	A1	$\overrightarrow{OB}=\begin{pmatrix}-1\\0\\2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\0\\4\end{pmatrix}$
			Obtain 3:2 oe	A1 **[3]**	Answer 3:2 WW → B3

Question			Answer	Marks	Guidance	
1			$u = x$ and $\mathrm{d}v = \cos 3x$	M1	integration by parts as far as $\mathrm{f}(x) \pm \int \mathrm{g}(x)\,\mathrm{d}x$	Check if labelled $v, \mathrm{d}u$
			$x \times \frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,\mathrm{d}x$	A2	**A1** for $x \times k\sin 3x - \int k\sin 3x\,\mathrm{d}x$; $k \neq \frac{1}{3}$ or 0	k may be negative
			$\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x$ [+ c] cao www ISW	A1	Not ….. $\frac{1}{3}\left(\frac{1}{3}\cos 3x\right)$ or ….. $--\frac{1}{9}\cos 3x$	
				[4]		
2			The first 3 marks refer to the expansion…	……..	of $\left(1-\frac{16x}{9}\right)^{3/2}$ and to no other expansion	
			First 2 terms = $1 - \frac{8}{3}x$	B1	Allow any equiv fraction for the $-\frac{8}{3}$ and ISW	$\frac{3}{2}.-\frac{16}{9}$ is not an equiv fraction
			3^{rd} term = $\frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\left(-\frac{16x}{9}\right)^2$	M1	Allow clear evidence of intention,e.g. $\frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\frac{-16x^2}{9}$	
			$= \frac{32}{27}x^2$	A1	Allow any equiv fraction for the $\frac{32}{27}$ and ISW	
			Complete expansion $\approx 27 - 72x + 32x^2$	A1	cao No equivalents. Ignore any further terms	If expansion $(a+b)^n$ used, award B1,B1,B1 for $27, -72x, 32x^2$
			valid for $\frac{-9}{16} < x < \frac{9}{16}$ or $\lvert x \rvert < \frac{9}{16}$	B1	oe Beware, e.g. $x < \left\lvert \frac{9}{16} \right\rvert$	condone $\leq$ instead of $<$
				[5]		

Question			Answer	Marks	Guidance	
3			For attempt at product rule on xy^2 $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{2x-y^2}{2xy}$ or $\frac{1-x^{-2}}{2y}$ Stationary point $\rightarrow$ (their) $\frac{dy}{dx}=0$ soi $x^2=1$ <u>or</u> $y^2=2$ <u>or</u> $y^4=4$ $(1,\sqrt{2})$, $(1,-\sqrt{2})$	M1 B1 A1 M1 A1 A1,A1 **[7]**	or changing equation to $y^2=x+x^{-1}$ soi in the differentiating process Award <u>B</u>1 for $(\pm)\frac{1}{2}(x+x^{-1})^{-\frac{1}{2}}(1-x^{-2})$ Ignore any other values Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$	SR. Award **A1** only if extra co-ordinates presented with both correct answers
4	(i)		Produce (at least 2) relevant equations Eliminate either λ or μ from 2 of them and solve for the other (μ or λ) $\lambda=2$ <u>and</u> $\mu=-1$ cao Check that $(\lambda,\mu)=(2,-1)$ satisfies all eqns P is (5, 4, 6) cao www	M1 M1 A1 B1 A1 **[5]**	e.g. $1+2\lambda=6+\mu$, $2+\lambda=8+4\mu$, $3\lambda=1-5\mu$ soi by correct (λ,μ) or e.g. $\lambda=2$ from 2 different pairs <u>This must be convincing</u>. Check unusual arguments Allow any reasonable vector notation	Dep previous M1M1A1 earned
4	(ii)		Using $\begin{pmatrix}2\\1\\3\end{pmatrix}$ and $\begin{pmatrix}1\\4\\-5\end{pmatrix}$ Using $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ giving value $\frac{n}{\sqrt{a}\sqrt{b}}$ 68.2°... (not 111.8…)	M1 M1 A1 **[3]**	i.e. correct parts for direction vectors for any 2 meaningful vectors in this question using meaningful scalar product & modulus or 1.19 (radians)	Expect $\frac{-9}{\sqrt{14}\sqrt{42}}$

Question			Answer	Marks	Guidance	
5	(i)		their $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} \Big/ \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$	M1		
			$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2\sin\theta}{3\cos\theta}$	A1		
			their $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}$	M1		
			$\tan\theta = \dfrac{3}{4}$	A1	If $\tan\theta = \dfrac{3}{4}$ not seen, award this A1 only if coords are correct	
			$(3.8, -0.6)$ or $\left(\dfrac{19}{5}, -\dfrac{3}{5}\right)$ or $x = 3.8,\ y = -0.6$	A1		
				[5]		
5	(ii)		Manipulating equations into form $\sin\theta = \mathrm{f}(x)$ and $\cos\theta = \mathrm{g}(y)$ and then using $\sin^2\theta + \cos^2\theta = 1$	M1	<u>If part (ii) is attempted first, and then part (i), allow</u> B1 for obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4(x-2)}{9(y-1)}$ M1 for equating their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to $\dfrac{1}{2}$ A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn... A1 for $(3.8, -0.6))$	<u>the following marks in part (i):-</u> ….and their Cartesian equation
			$\dfrac{(x-2)^2}{9} + \dfrac{(1-y)^2}{4} = 1$ oe www ISW Accept e.g. $\left(\dfrac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	A1		
				[2]		

Question			Answer	Marks	Guidance	
6			Attempt diff to connect du & dx	M1	or find $\frac{du}{dx}$ or $\frac{dx}{du}$	
			Correct result e.g. $\frac{du}{dx} = 2$ or du = 2 dx	A1		
			Indef integ in terms of u = $\frac{1}{2}\int \frac{2u-3}{u^5}(\mathrm{d}u)$	A1	Must be completely in terms of u.	
			Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe	A1A1	or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$	Award B1,B1 for $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$ or for $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$ or for $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$ or for $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$
			Use correct variable & correct values for limits	M1	Provided minimal attempt at $\int f(u)\mathrm{d}u$ made	
			$= \frac{-23}{384}$ oe $(-0.059895\ .....)$	A1	Accept decimal answer only if minimum of first 3 marks scored	
			[ISW,e.g. changing to $\frac{23}{384}$]			
				[7]		

Question			Answer	Marks	Guidance	
7	(i)	I	$\frac{\cos x}{1+\sin x} - \frac{-\sin x}{\cos x}$ or $\frac{\cos x}{1+\sin x} + \frac{\sin x}{\cos x}$	B2	Each half (including 'middle' sign) scores B1	
			$\frac{+/-\cos^2 x + /-\sin x(1+\sin x)}{(1+\sin x)\cos x}$	M1	Combine, provided derivative was of form $^{f'(x)}/_{f(x)}$	Allow only variations num signs
			$\frac{1+\sin x}{\cos x(1+\sin x)} = \frac{1}{\cos x}$ www **AG**	A1	$\cos^2 x + \sin^2 x = 1$ in intermediate step required	
		II	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Change to ln(sec x + tan x)	B1	Not $\ln(\frac{1}{\cos x} + \tan x)$	
			Diff as $\frac{\text{attempt at } \frac{d}{dx}(\sec x + \tan x)}{\sec x + \tan x}$	M1		
			Reduce to $\sec x = \frac{1}{\cos x}$	A1		
		III	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Diff as $\frac{\text{attempt at quotient differentiation}}{\frac{1+\sin x}{\cos x}}$	M1		
			Fully correct differentiation	A1		
			Correct reduction to $\frac{1}{\cos x}$	A1 **[4]**		
7	(ii)		Indef integral = $\ln(1+\sin x) - \ln(\cos x)$ [Method I]	B1	or $\ln(\sec x + \tan x)$ [Method II]	
			Substitute limits & use log manipulation	M1	Use of ln A – ln B = $\ln\frac{A}{B}$ anywhere in question	
			Answer = $\ln(2 + \sqrt{3})$	B1 **[3]**	Accept ln 3.73 or $\ln\frac{2+\sqrt{3}}{1}$ …..but not $\ln\frac{1+\sqrt{3}/2}{1/2}$	Answer has not been given

Question			Answer	Marks	Guidance	
8	(i)		$AB = \sqrt{(+/-2)^2 + (+/-2^2 + (+/-4)^2)}$ $AD = \sqrt{(+/-2)^2 + (+/-4)^2 + (+/-2)^2}$	B1 B1 **[2]**	oe oe	If $AB^2 = AD^2 = 24$, then SR B1 $AB = AD$ to be stated for 2nd B1
8	(ii)		midpoint is (3, 5, 0) Clear method for finding direction vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{j} - \mathbf{k})$ oe or e.g. $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(-3\mathbf{j} + \mathbf{k})$ cao	B1 M1 A1 **[3]**	Accept any reasonable vector notation. Expect $3\mathbf{j} - \mathbf{k}$ or $-3\mathbf{j} + \mathbf{k}$ **"r ="** is essential. No f.t. for wrong mid-point.	
8	(iii)		substitution of $\lambda = +/-5$ or $\mu = +/-4$	M1 **[1]**	Based on correct answer to (ii)	
8	(iv)		Kite	B1 **[1]**		

Question			Answer	Marks	Guidance	
9	(i)		Separating variables $\int \frac{1}{\theta+20}\,\mathrm{d}\theta = \int -k\,\mathrm{d}t$	M1	or invert each side: $\frac{\mathrm{d}t}{\mathrm{d}\theta} = -\frac{1}{k(\theta+20)}$	Must see $\frac{1}{\theta+20}$; ignore posn 'k'
			ln(θ + 20) = -kt (+ c) or equivalent	A1	"Eqn A"	
			$\theta = Ae^{-kt} - 20$ oe (i.e. $\theta = e^{-kt+c} - 20$)	A1	"Eqn B"	
				[3]		
9	(ii)		$(-)3 = -k(40+20)$	M1	Using $t=0, \theta=40, \frac{\mathrm{d}\theta}{\mathrm{d}t} = (-)3$ in <u>given</u> equation	
			$k = \frac{1}{20}$ oe	*A1	Not $k = -\frac{1}{20}$	
			Subst $t=0, \theta=40$ & their k (where necessary) into their Eqn A or their Eqn B and solve for the arbitrary constant	M1		
			Subst $\theta = 0$ & their values of k and the arbitrary constant into their Eqn A or their Eqn B	M1		
			t = 21.9722 = 22 minutes cao www	dep*A1		
				[5]		
9	(iii)		k is larger	B1 **[1]**		

Question			Answer	Marks	Guidance	
10	(i)		Clear start to algebraic division (Quotient) = $x-1$ (Remainder) = $x+7$ Final answer: $x-1+\dfrac{x+7}{x^2-x-6}$	M1 A1 A1 A1 **[4]**	at least as far as x term in quot & subseq mult back final answer in correct form This must be shown in part (i) or, if not, then implied in part (ii) If no long division shown but only comparison of coefficients or otherwise, SR M0 B1 B1 B1	& attempt at subtraction Accept $A=1, B=-1, C=1, D=7$
10	(ii)		Convert their $\dfrac{Cx+D}{x^2-x-6}$ to Partial Fracts $\dfrac{x+7}{x^2-x-6}=\dfrac{2}{x-3}-\dfrac{1}{x+2}$ Their …. $\int Ax+B\,dx=\frac{1}{2}Ax^2+Bx$ or $\dfrac{(Ax+B)^2}{2A}$ $\int \dfrac{E}{x-3}+\dfrac{F}{x+2}\,dx=E\ln(x-3)+F\ln(x+2)$ Using limits in a correct manner $8+\ln\dfrac{27}{4}$ $\left(8+\ln\dfrac{54}{8}\right)$ isw	M1 A1A1 B1 ft B1 ft M1 A1 **[7]**	<u>Correct</u> fraction converted to <u>correct</u> PFs Tolerate some wrong signs provided intention clear Answer required in the form $a+\ln b$, so giving <u>only</u> a decimalised form is awarded A0	

Question			Answer	Marks	Guidance	
1			$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1	**<u>SC</u>** $\frac{(x-7)(x-2)}{(x+2)(x-1)^2} \equiv \frac{A}{x+2}+\frac{Bx+C}{(x-1)^2}$ [If no partial fractions seen anywhere, B0]	B1
			$(x-7)(x-2) \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$ [Allow careless minor error but not algebraic method error] or any equiv identity such as $\frac{(x-7)(x-2)}{(x-1)^2} \equiv A + \frac{B(x+2)}{(x-1)} + \frac{C(x+2)}{(x-1)^2}$ (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1	$(x-7)(x-2) \equiv A(x-1)^2 + (Bx+C)(x+2)$ [Allow careless minor error but not algebraic method error] or any equivalent identity (as in previous column) (or even the identity on the 1st line), in which values of x are substituted (or cfs compared)	M1
			$A=4, B=-3, C=2$ or $\frac{4}{x+2}-\frac{3}{x-1}+\frac{2}{(x-1)^2}$ ISW The 3 @ A1 are dep on the used identity being correct. <u>Cover-up:</u> A=4,C=2 score B1,B1; $B=-3$ needs M1, then A1	A1,1,1 **[5]**	$A = 4,\ B=-3, C=5$ or $\frac{4}{x+2}+\frac{-3x+5}{(x-1)^2}$	A1 This gives max 3/5 for easier case

Question			Answer	Marks	Guidance	
2			$u = \ln 3x$ and $\mathrm{d}v$ or $\frac{\mathrm{d}v}{\mathrm{d}x} = x^8$	M1	integ by parts as far as $\mathrm{f}(x) +/- \int \mathrm{g}(x)(\mathrm{d}x)$	If difficult to assess, x^8 must be integrated, so look for term in x^9
			$\frac{\mathrm{d}}{\mathrm{d}x}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$	B1	stated or clearly used	
			$\frac{x^9}{9}\ln 3x - \int \frac{x^9}{9}$ their $\frac{\mathrm{d}u}{\mathrm{d}x}(\mathrm{d}x)$ FT	√A1	i.e. correct understanding of 'by parts'…	..even if ln(3*x*) incorrectly differentiated
			Indication that $\int kx^8 \mathrm{d}x$ is required	M1	i.e. before integrating, product of terms must be taken	The product may already have been indicated on the previous line
			$\frac{x^9}{9}\ln 3x - \frac{x^9}{81}$ or $\frac{1}{9}x^9\left(\ln 3x - \frac{1}{9}\right)$ ISW (+c) <u>cao</u>	A1	$\frac{1}{9}\frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis	
				[5]		
			<u>If candidate manipulates ln(3*x*) first of all</u> $\ln(3x) = \ln 3 + \ln x$	B1		If, however, ln(3*x*) is said to be ln 3.ln *x*, then B0 followed by possible M1 A1 A1 in line with alternative solution on LHS, where the 'M' mark is for dealing with $\int x^8 \ln x \, \mathrm{d}x$ 'by parts' in the right order and the 2 @ A1 are for correct results.
			$u = \ln x$ and $\mathrm{d}v = x^8$	M1	In order to find $\int x^8 \ln x \, \mathrm{d}x$:	
			$\frac{x^9}{9}\ln x - \int \frac{x^9}{9}.\frac{1}{x}(\mathrm{d}x)$ or better	A1		
			$\frac{x^9}{9}\ln x - \frac{x^9}{81}$	A1		
			Their $\int x^8 \ln x \, \mathrm{d}x + \frac{x^9}{9}\ln 3$ (+ c) FT ISW	√A1		

Question			Answer	Marks	Guidance	
3			Set up the 3 relevant equations $1 + 2\lambda = \mu - 1 \quad -\lambda = 5 - \mu \quad 3 + 5\lambda = 2 - 5\mu$	M1	'M' mark so intention must be clear; minor error(s) only accepted	MR must be consistent; correct version anywhere $\Rightarrow$ not MR
			Attempt to find λ or μ from 2 of the equations & then find μ or λ from any of the 3 equations.	M1	Or find λ, say, from (i)(ii) & then from (ii)(iii) [values shown at next stage] – inconsistency dep*A1 also awarded here	
			$(\lambda, \mu) =$ (3,8) or $(-2\frac{3}{5}, 2\frac{2}{5})$ or $(-\frac{11}{15}, \frac{8}{15})$ or $(3, -3\frac{1}{5})$ or $(-\frac{11}{15}, 4\frac{4}{15})$ or $(-2\frac{3}{5}, -3\frac{1}{5})$ or $(\frac{1}{5}, 2\frac{2}{5})$ or $(-8\frac{1}{5}, 8)$ or $(-4\frac{7}{15}, \frac{8}{15})$	A1	Accept equivalent proper/improper fractional values or decimal equivalent values	These are all of the solutions obtainable using different combinations of the 3 equations; e.g. using just i & ii or using i & ii to find λ & iii to find μ
			Demonstrate inconsistency i.e. substitute the correct values into a correct equation (but not the immediate last one used)	M1	e.g. after (3,8), subst in iii & write $3 + 5 \times 3 \neq 2 - 5 \times 8$ or $3 + 5 \times 3 = 2 - 5 \times 8 \therefore$ do not intersect	
			State "skew"	A1	Dep on 3 @ M1 + A1	
			(a) Identify direction vectors; (b) state "not identical/same/equal/equiv/multiples" or eval cos(angle) & state $\neq 1(\text{or } -1)$; (c) state "not parallel"	B1	dvs must be identified: $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$ Accept any vector notation.	
				[6]		

Question			Answer	Marks	Guidance	
4			Use of $\sin 2x = +/- 2\sin x\cos x$ or $+/- \cos\left(\frac{\pi}{2} - 2x\right)$ *or* $\cos 2x = +/- 2\cos^2 x +/- 1$ etc	M1	Seen anywhere in the solution	
			$\left(\frac{dy}{dx} =\right)$ $-2\sin 2x(\text{or } -4\sin x\cos x)$; $+2\cos x$	B1,B1		
			their $\frac{dy}{dx} = 0$	*M1		
			$\left(\frac{\pi}{2}, 1\right)$; $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ and $\left(\frac{5\pi}{6}, \frac{3}{2}\right)$	dep* A1; A1 **[6]**	-1(once) for using degrees in an answer instead of radians. If B0 &/or B0 <u>because of sign error</u>, allow A1 to be awarded for $\left(\frac{\pi}{2}, 1\right)$	<u>SC</u> If A0 but all 3 x-values are correct, award SC A1 <u>SC</u> B2 for 3 ✓answers without working
5	**(i)**		$\frac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$	M1	Combine (or write as 2 separate fractions) using common denominator	Accept with/without brackets in num Accept $1-\tan x.1+\tan x$ in denom
			$= \frac{2\tan x}{1-\tan^2 x} = \tan 2x$ Answer Given	A1 **[2]**	$\frac{2\tan x}{1-\tan^2 x}$ essential stage N.B. If $\tan x$ changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles	A0 for omission of any necessary brackets

Question			Answer	Marks	Guidance	
5	(ii)		$\int \tan 2x \, dx = \lambda \ln(\sec 2x)$ or $\mu \ln(\cos 2x)$ [= F(x)]	M1		
			$\lambda = \frac{1}{2}$ or $\mu = -\frac{1}{2}$	A1		
			their F[$\frac{\pi}{6}$] – their F[$\frac{\pi}{12}$]	M1	dependent on attempt at integration……..	……i.e. not for $\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$
			$\frac{1}{2}\ln 2 - \frac{1}{2}\ln\frac{2}{\sqrt{3}}$ oe	A1	i.e. any correct but probably unsimplified numerical version	
			$\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{4}\ln 3$ or $\ln 3^{\frac{1}{4}}$ or $\frac{1}{2}\ln\frac{6}{2\sqrt{3}}$ oe ISW	+A1	i.e. any correct version in the form $a \ln b$	
				[5]		

Question			Answer	Marks	Guidance	
6			Find du in terms of dx (or vv) or $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$	M1	An attempt - not necessarily accurate	
			Substitute, changing given integral to $\int \frac{u-1}{u^2}(\mathrm{d}u)$	A1	No evidence of x at this A1 stage	
			Provided of form $\frac{au+b}{u^2}$, <u>either</u> split as $\frac{au}{u^2}+\frac{b}{u^2}\ldots$	M1	<u>or</u> use 'parts' with 'u' = $au+b$, 'dv' = $\frac{1}{u^2}$	
			Integrate as $\ln u+\frac{1}{u}$ or FT as $a\ln u-\frac{b}{u}$ [=F(u)]	√A1	or $-(au+b)\frac{1}{u}+a\ln u$ FT [=G(u)]	
			Re-substitute $u = 1 + \ln x$ in F(u)	M1	Re-substitute $u = 1 + \ln x$ in G(u)	
			$\ln(1+\ln x)+\frac{1}{1+\ln x}$ (+ c) ISW	A1	or $\ln(1 + \ln x)-\frac{\ln x}{1+\ln x}$ (+ c) ISW	
				[6]		
			<u>**In each part, mark the answers, ignoring the labels**</u>		<u>**To invoke MR, evidence must be clear**</u>	
7	(i)		$AB=\sqrt{91}$; $AC=\sqrt{27}$ or $3\sqrt{3}$ ISW	B1; B1	9.54 or 9.539392..; 5.2(0) or 5.1961524..	
			Attempting to use $\overrightarrow{AB}.\overrightarrow{AC}=AB.AC\cos\theta$	M1	or $BC^2=AB^2+AC^2-2AB.AC\cos\theta$	
			angle BAC = 171 (3 sf) or 2.99 (rad) (3 sf) ISW	A1	Final acute answer [8.68 or 0.152] /choice → A0	171 to 171.317 or 2.99
				[4]		
7	(ii)		6**i** + 4**j** – 2**k** or – 6**i** – 4**j** +2**k**	B1	seen, irrespective of any labelling	
			6×(–1) + 4×(–3) – 2×(–9) = 0 (∴ perpendicular) **AG**	B1	oe using (6,4,–2) or (–6,–4,2) and…	....(–1,–3,–9) or (1,3,9)
			6×1 + 4×1 – 2×5 = 0 (∴ perpendicular) **AG**	B1	oe using (6,4,–2) or (–6,–4,2) and…	… (1,1,5) or (–1,–1,–5)
				[3]		
7	(iii)		(AD =) $\sqrt{56}$ or $2\sqrt{14}$ or 7.48… soi	B1		
			area ABC= ½(their)AB × (their)AC × sin(their)BAC	M1	(✓= 3.74… but M mark, not A)	
			$9.3 \le V < 9.35$, $9\frac{1}{3}$ ISW	A1	Accept even if (i) angle given as 8.68…..	i.e. the acute version not accepted in (i)
				[3]		

Question			Answer	Marks	Guidance	
8	(i)		$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{\sqrt{r}}$ oe	B2	B1 for $\frac{\mathrm{d}r}{\mathrm{d}t} =$; B1 for $\frac{k}{\sqrt{r}}$	SR: B1 for $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \frac{1}{\sqrt{r}}$
			Sep variables of their diff eqn (or invert) & integrate each side, increasing powers by 1 (or $\frac{1}{r} \to \ln r$)	*M1	their d.e. must be $\frac{\mathrm{d}r}{\mathrm{d}t}$ (or $\frac{\mathrm{d}t}{\mathrm{d}r}$) = f($r$)	Ignore absence of '+c' after integration
			Subst $\frac{\mathrm{d}r}{\mathrm{d}t} = 1.08, r = 9$ into their diff eqn to find k	M1	their d.e. must include $\frac{\mathrm{d}r}{\mathrm{d}t}$ (or $\frac{\mathrm{d}t}{\mathrm{d}r}$), r & k	(✓k = 3.24 but M mark, not A)
			Substitute t = 5, r = 9 to find 'c'	dep*M1	Must involve '+c' here	
			Correct value of c (probably = 1.8 or -1.8)	A1	Check other values	
			$r = (4.86t + 2.7)^{\frac{2}{3}}$ ISW	A1	Answer required in form $r = \mathrm{f}(t)$	
				[7]		
8	(ii)		subst t = 0 into any version of (i) result to find finite r	M1		(✓$r \approx 1.938991$….but M mark, not A)
			Any V in range $30.5 \le V < 30.55$, but not fortuitously	A1	Accept 9.72π or $\frac{243}{25}\pi$	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{dy}{dt} = 2(+) - \frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW	B1, B1		
			$\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right), \frac{2t^3 - 2}{-t}, -t^2\left(2 - \frac{2}{t^3}\right)$ oe	B1 **[3]**	ISW. Must not involve (implied)'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2 - 2t^{-3}}{-t^2}$
9	(ii)		(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$	M1		
			$t = 1 \rightarrow$ (stationary point) = (0, 3)	A1	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$	
			Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$	M1		
			Hence (0, 3) is a minimum point www	A1 **[4]**	Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
9	(iii)		Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y	M1		
			$y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	A1 **[2]**		

Question			Answer	Marks	Guidance	
10	(i)		$(1-x)^{-3} = 1 + -3.-x + \frac{-3.-4}{2}(-x)^2 + \ldots$ oe; accept $3x$ for $-3.-x$ &/or $-x^2$ or $(x)^2$ for $(-x)^2$ multiplication by x to produce **AG** (**A**nswer **G**iven)	M1 A1 **[2]**	As result is given, this expansion must be shown and then simplified. It must not just be stated as $1+3x+6x^2+\ldots$	For alternative methods such as expanding $(1-x)^3$ and multiplying by $x+3x^2+6x^3$ <u>or</u> using long division, consult TL
10	(ii)		Clear indication that $x = 0.1$ is to be substituted (estimated value is) $0.1+3(0.1)^2+6(0.1)^3 = \underline{0.136}$	M1 A1 **[2]**	e.g. $0.1+3(0.1)^2+6(0.1)^3$ stated	Calculator value $\rightarrow$ M0 (0.13717… is calculator value of $\frac{100}{729}$)
10	(iii)		Sight of $1-x = x\left(\frac{1}{x}-1\right)$ or $1-x = -x\left(1-\frac{1}{x}\right)$ or $\left(\frac{1}{x}-1\right)^3 = -\left(1-\frac{1}{x}\right)^3$ or $(\frac{1}{x}-1)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or $\left(\frac{1}{x}-1\right)^{-3} = -\left(1-\frac{1}{x}\right)^{-3}$ or equivalent Complete satisfactory explanation (no reference to style) www $[1+(-3)(-\frac{1}{x})+\frac{(-3)(-4)}{2}\left(-\frac{1}{x}\right)^2+\ldots]$ $\rightarrow -\frac{1}{x^2}-\frac{3}{x^3}-\frac{6}{x^4}$	B1 B1 M1 A1 **[4]**	(Answer Given) Simplified expansion may be quoted – it may have come from result in part (i). Answer for this expansion is not **AG**.	

Question			Answer	Marks	Guidance	
10	(iv)		Must say "Not suitable" and one of following: Either: requires $\left\|\frac{1}{x}\right\| < 1$, which is not true if x = 0.1 Or: substitution of positive/small value of x in the expansion gives a negative/large value (which cannot be an approximation to 100/729).	B1 **[1]**	This B1 is dep on x = 0.1 used in (ii). Or "because $\frac{1}{x} > 1$" Or "it gives -63100"	Realistic reason If choice given, do not ignore incorrect comments, but ignore irrelevant/unhelpful ones

Question			Answer	Marks	Guidance	
1			$x(1-x^2)+(1+x)+2(1-x)$ oe	M1	condone one sign error	if M0B0, SC1 for any pair of terms correctly combined into a single fraction, may be unsimplified
			$1-x^2$ oe	B1	any correct denominator common to all three fractions	
			$\dfrac{3-x^3}{1-x^2}$ oe cao	A1	must be fully simplified; mark the final answer	eg $\dfrac{x(3-x^3)}{x(1-x^2)}$ oe may score a maximum of M1B1A0
				[3]		
2			**±** **(**(3 – 2)**i** + (− 3 − 8)**j** + (6 – 2)**k)** soi	B1	NB **i** – 11**j** + 4**k**	or
			their **±** **(i** – 11**j** + 4**k). ±**(5**i** + 5**j** + 8**k**) both diagonals used ; evaluation not essential	M1	if M0 SC2 for 84° (or 84.5°), or 52(.3°) or 39° or (38.5° or 43(.2°) or 46(.0°) found from scalar product or SC1 for the equivalent obtuse angle	B3 for correct use of Cosine Rule (using the midpoint of the diagonals of the parallelogram) $[\cos\theta]=\dfrac{34.5+28.5-72}{2\sqrt{34.5}\sqrt{28.5}}$ oe
			$\pm(1\times5+(-11)\times5+4\times8)$ $=\sqrt{1^2+11^2+4^2}\times\sqrt{5^2+5^2+8^2}\cos\theta$ oe	A1	must be fully correct	
			$\theta=\cos^{-1}\dfrac{\pm18}{\sqrt{138}\times\sqrt{114}}$	A1		
			81.7 to 82°	A1 **[5]**	1.4 to 1.43 rad	B2 for 81.7 to 82° unsupported or B3 + B2 possible for Cosine Rule

Question			Answer	Marks	Guidance	
3	(i)		$1+(-\frac{1}{2})(-2x)+(-\frac{1}{2})(\frac{-3}{2})\frac{(\pm 2x)^2}{2!}[+...]$	B1 B1	first two terms third term	allow recovery from omission of brackets do not allow $2x^2$ unless fully recovered in answer
			$1+x+\frac{3}{2}x^2$ oe	B1 **[3]**		
	(ii)		use of $(x+3)\times$ their$(1+x+\frac{3}{2}x^2)$	M1		
			coefficient is 5.5 oe	A1 **[2]**	or **B2** www in either part	may be embedded (eg $5.5x^2$ alone or in expansion)
4			$\int \frac{\cos 2x}{1+\sin 2x}\,(\mathrm{d}x)$	B1* B1*	$\cos 2x = 1 - 2\sin^2 x$ or $(1 +)\sin 2x = (1 +)\ 2\sin x\cos x$ seen numerator and denominator both correct in the integral soi	if B0B0M0A0, SC4 for $F[x] = ½\ln(1 + 2\sin x\cos x)$ or $½\ln(1 + \sin 2x)$ final mark may still be awarded
			$F[x]= k\ln(1 + \sin 2x)$ soi	M1dep*	or $k\ln(1 + u)$ or $k\ln(u)$ following their substitution www	
			$k = ½$	A1	correct k for their substitution	
			$½\ln(1 + \sin(^{\pi}/_{2})) - ½\ln(1 + 0)$ $= ½\ln 2$	A1 AG **[5]**	correct use of limits www	minimum working: $½\ln 2 - ½\ln 1$ or $½\ln(1 + 1)$ oe

Question			Answer	Marks	Guidance	
5	(i)		$1 - s = 2 + t$ $4 + 2s = 8 + 3t$ $1 + 2s = 2 + 5t$	B1	for all three equations NB third equation may appear later, or with values already substituted	
			value of either s or t obtained from valid method	M1		or M1 for one value (of s or t) found from one pair of equations
			correct pair of values	A1	eqns (i) and (ii): $s = 0.2$, $t = -1.2$ eqns (i) and (iii): $s = {}^{-4}/_{7}$, $t = {}^{-3}/_{7}$ eqns (ii) and (iii) $s = 4.25$, $t = 1.5$	A1 for substitution of this value (of s or t) in third equation and obtaining the other parameter (ie of t or s); NB (0.2, − 0.12) or $({}^{-4}/_{7}, {}^{-12}/_{7})$ or (4.25, − 5.25) if s found first and (– 2.5, –1.2) or $({}^{19}/_{14}, {}^{-3}/_{7})$ or (– 2.5, 1.5) if t found first
			eg $1 + 2\times0.2 \neq 2 + 5\times-1.2$ oe isw NB A0 for $1 + 2\times0.2 = 2 + 5\times-1.2$ unless clarified by suitable comment	A1	correct substitution of correct values in correct equation	**or** find same parameter from second pair of equations A1 for correct demonstration of inconsistency NB clear statement needed if two different values of same parameter found
				[4]		
5	(ii)		$2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k} = -2(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ oe	B1	allow equivalent in words, but scale factors must be correct	eg direction of A is $-\frac{1}{2}\times$direction of C
			eg line A goes through (1, 4, 1) but line C goes through (1, 15, 11), so they do not coincide so the lines are parallel eg demonstration of different y or z values on each line for (say) $x = 1$, so lines are parallel	B1		
				[2]		

Question			Answer	Marks	Guidance	
6			$3y^2\frac{dy}{dx}$	B1	or $2x\frac{dx}{dy}$	if B0B0 M0
			$2x-12\frac{dy}{dx}-8$	B1	$3y^2-8\frac{dx}{dy}-12$	SC2 for $\frac{dy}{dx}=$
			their $3y^2\frac{dy}{dx}-12\frac{dy}{dx}=8-2x$ soi must be two terms on each side and must follow from RHS = 0	M1	their $2x\frac{dx}{dy}-8\frac{dx}{dy}=-3y^2+12$ must be two terms on each side must follow from RHS = 0	$\frac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x+8+12\frac{dy}{dx})$ M1 may be earned for setting correct denominator equal to 0
			$\frac{dy}{dx}=\frac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\frac{dx}{dy}=0$ is substituted and there is no evidence for an incorrect expression for $\frac{dx}{dy}$	
			their $3y^2-12=0$	M1*		$x\neq 4$ not required
			$y=(\pm)\ 2$	A1	A0 if $\frac{dy}{dx}$ incorrect	
			substitution of their positive y value in original equation	M1dep*		ignore substitution of -2
			$x=10$, $x=-2$ and no others cao	A1	A0 if $\frac{dy}{dx}$ incorrect	condone omission of formal statement of coordinates (10, 2) and (–2, 2)
				[8]		

Question		Answer	Marks	Guidance	
7	(i)	$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of x = 2sin$t$
		$\frac{dy}{dx} = \text{their} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
		$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
		$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to 1 – 2sin$t$ www	A1	or equivalent intermediate step	
		(1, 1½)	B1	NB $t = \frac{\pi}{6}$	from 1 - 2sin$t$ = 0
			[5]		
7	(ii)	(y =) 1 – 2sin^2t + 2sin$t$	B1	may be awarded after correct substitution for x eg (y =) $1 - {}^{x^2}/_4 - \sin^2 t + 2\sin t$	or (y =) x + cos2t
		substitution of sin$t$ = ½x to eliminate t	M1		substitution of $t = \sin^{-1}({}^{x}/_{2})$ to eliminate t
		$y = 1 + x - ½x^2$ oe isw	A1	or B3 www	$y = x + \cos 2(\sin^{-1}({}^{x}/_{2}))$ oe isw
			[3]		

Question			Answer	Marks	Guidance	
7	(iii)		$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \le 2$ or $\lvert x \rvert \le 2$	B1	cao	
			sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	
			positive *y*-intercept and one distinguishing feature isw	A1		one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), *y* – intercept is (0, 1), *x*-intercept is $(1 - \sqrt{3}, 0)$
				[3]		
8	(i)		t^2 in quotient and $t^3 + 2t^2$ seen	B1	or $\dfrac{t(t^2-4)+4t}{(t+2)}$	or $\dfrac{(t+2)^3-6t^2-12t-8}{(t+2)}$
			$-2t$ in quotient and $-2t^2-(-2t^2-4t)=4t$ seen	B1	$\dfrac{t(t+2)(t-2)}{(t+2)}+\dfrac{4t}{t+2}$	$\dfrac{(t+2)^3}{(t+2)}-\dfrac{6((t+2)^2-4t-4)+12t+8}{(t+2)}$ oe
			completion to obtain correct quotient and remainder identified www	B1	$t(t-2)+\dfrac{4(t+2)-8}{t+2}$	$(t+2)^2-6(t+2)+\dfrac{12t+16}{t+2}$ oe $= t^2+4t+4-6t-12+\dfrac{12(t+2)-8}{t+2}$ oe both steps needed for final B1
				[3]		
8	(i)		alternatively $\dfrac{t^3}{t+2} \equiv At^2+Bt+C+\dfrac{D}{(t+2)}$	B1	or $t^3 \equiv (At^2+Bt+C)(t+2)+D$	or B1 for $\dfrac{t^2(t+2)-2t^2}{(t+2)}$
			equate coefficients to obtain correctly $A = 1$, $0 = 2A + B$ and $B = -2$ www	B1		B1 for $t^2+\dfrac{-2t(t+2)+4t}{(t+2)}$
			$0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www	B1		B1 for $t^2-2t+\dfrac{4(t+2)-8}{(t+2)}$
				[3]		

Question			Answer	Marks	Guidance	
8	(ii)		integration by parts with $u = \ln(t + 2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt)$	M1*	f(t) must include t^3 and g(t) must **not** include a logarithm	ignore spurious dx etc
			$2t^3 \ln(t+2) - \int \frac{2t^3}{t+2}(dt)$ cao	A1		alternatively, following $u = t + 2$
			result from part (i) seen in integrand; must follow award of at least first M1	M1*	no integration required for this mark	$\int 2(u^2 - 6u + 12 - \frac{8}{u})du$ oe
			$F[t] = 2t^3 \ln(t+2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16\ln(t+2)$	A1	$2t^3 \ln(t+2) - \frac{2t^3}{3} + 2t^2 - 8t + 16\ln(t+2)$	$\frac{2u^3}{3} - 6u^2 + 24u - 16\ln u$ and $2t^3 \ln(t+2)$
			their F[2] – F[1]	M1dep*	at least one of their terms correctly integrated	NB limits following substitution are $u = 4$ and $u = 3$
			$-6\frac{2}{3} - 18\ln 3 + 32\ln 4$ oe cao	A1 **[6]**		
9			$\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ may be seen in later work	B1	or $\frac{A}{1+2x} + \frac{Bx+C}{(1-x)^2}$ may be seen later in later work	if B0M0, SC1 for $\frac{1}{1+2x}$ seen
			$2 + x^2 \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$	M1	or $A(1-x)^2 + (Bx + C)(1 + 2x)$	allow only sign errors , not algebraic errors
			$A = 1$, $B = 0$ and $C = 1$ www	A1A1A1		
			$\int \left(\frac{1}{1+2x} + \frac{1}{(1-x)^2}\right) dx =$			
			$a\ln(1 + 2x) + b(1 - x)^{-1}$	M1*	a and b are non-zero constants	ignore extra terms
			$F(x) = \frac{1}{2}\ln(1 + 2x) + (1 - x)^{-1}$	A1		
			their $\frac{1}{2}\ln(\frac{3}{2}) + \frac{4}{3} - (\frac{1}{2}\ln 1 + 1)$	M1dep*		

Question			Answer	Marks	Guidance	
			$\frac{1}{2}\ln(\frac{3}{2})+\frac{4}{3}-0-1$	A1 **[9]**	and completion to given result www	NB $\frac{1}{2}\ln(\frac{3}{2})+\frac{1}{3}$
10	**(i)**		$\frac{dV}{dt}=\pm 0.01$	B1		
			by similar triangles, $\frac{h}{4.5}=\frac{r}{3}$	B1	may be implied by $r=\frac{2h}{3}$ oe	
			$\frac{dV}{dh}=\frac{4}{9}\pi h^2$ oe	B1		
			$\frac{dh}{dt}=\pm 0.01\times$ their $\frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either r or h or both
			$-0.01=(\frac{4}{9}\pi h^2)\times\frac{dh}{dt}$	A1 **[5]**	completion to given result www	$h^2\frac{dh}{dt}=\frac{-0.09}{4\pi}=\frac{-9}{400\pi}$
10	**(ii)**		$\int h^2 dh=\int\frac{-9}{400\pi}dt$ oe soi	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
			$\frac{h^3}{3}=\frac{-9}{400\pi}t(+c)$	A1		
			substitution of t = 0 and h = 4.5 in their expression following integration	M1	expression must include c and powers must be correct on each side	
			$h=\sqrt[3]{\frac{729}{8}-\frac{27t}{400\pi}}$ oe isw	A1 **[4]**	allow – 0.0215 or – 0.02148591…r.o.t to 4 sf or more and similarly 91.125	$91.125 = {}^{729}/_{8}$
10	**(iii)**		set h = 0 and solve to obtain positive t	**M1**	or $(t=)\frac{1}{3}\pi\times 3^2\times 4.5\div 0.01$ (= 1350π)	NB 1350π = 4241.150082…
			71 minutes cao	**A1** **[2]**		