Oxford Cambridge and RSA

Friday 24 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence show that the equation $\cos\theta - 3\sin\theta = 4$ has no solution. **[6]**

2 Given that $\left(1+\frac{x}{p}\right)^q = 1 - x + \frac{3}{4}x^2 + \ldots$, find p and q, and state the set of values of x for which the expansion is valid. **[7]**

3 Fig. 3 shows the curve $y = x^4$ and the line $y = 4$.

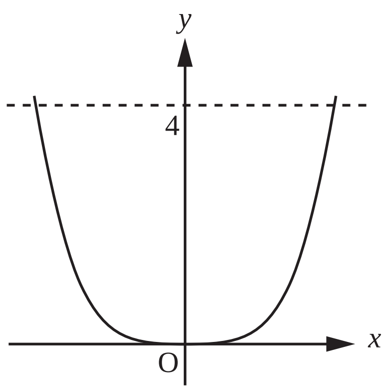

Fig. 3

The finite region enclosed by the curve and the line is rotated through 180° about the y-axis. Find the exact volume of revolution generated. **[4]**

4 Solve the equation $2\sin 2\theta = 1 + \cos 2\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

5 In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all θ.

AB = x and AE = $2x$.

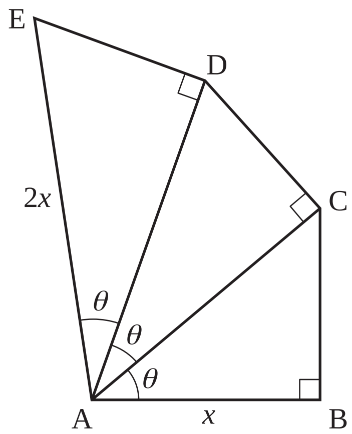

Fig. 5

(i) Show that $\sec^3\theta = 2$. **[3]**

(ii) Hence show the ratio of lengths ED to CB is $2^{\frac{2}{3}} : 1$. **[4]**

6 P is a general point on the curve with parametric equations $x = 2t$, $y = \frac{2}{t}$. This is shown in Fig. 6. The tangent at P intersects the x- and y-axes at the points Q and R respectively.

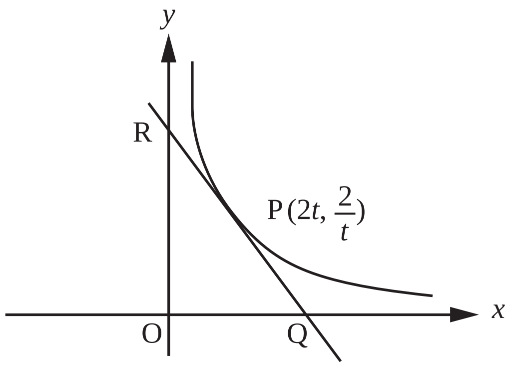

Fig. 6

Show that the area of the triangle OQR, where O is the origin, is independent of t. **[7]**

Section B (36 marks)

7 Fig. 7 shows a cuboid OABCDEFG with coordinates as shown. The point P has coordinates (4, 2, 0).

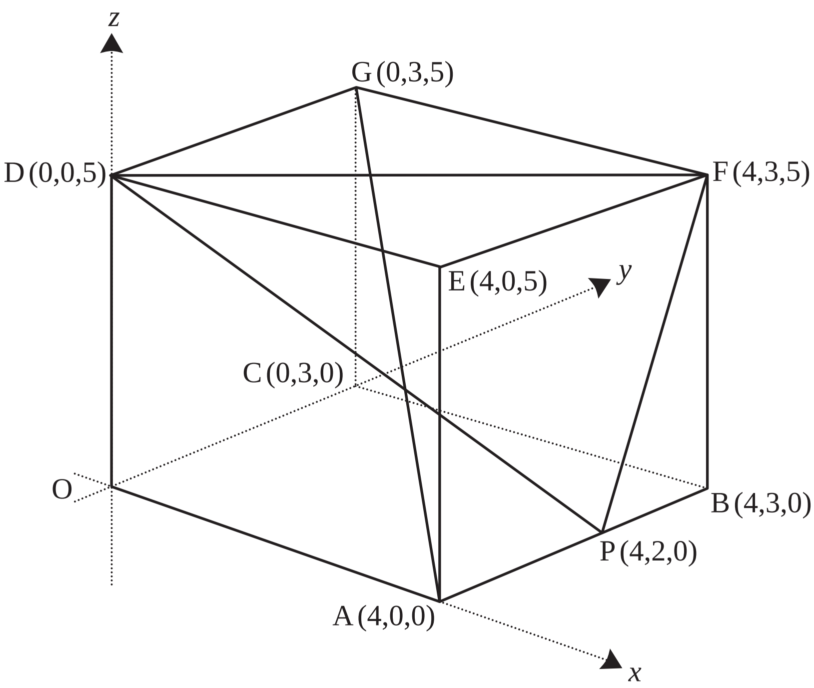

Fig. 7

(i) Find the length of the diagonal AG. **[2]**

(ii) Show that the vector $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal to the plane DPF. Hence find the cartesian equation of this plane. **[6]**

The diagonal AG intersects the plane DPF at Q.

(iii) Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ : QG. **[6]**

(iv) Find the acute angle between the line AG and the plane DPF. **[4]**

8 **(i)** Show that $\dfrac{1}{2+x}+\dfrac{1}{2-x}=\dfrac{4}{(2+x)(2-x)}$. **[1]**

In a chemical reaction, the time t minutes taken for a mass x mg of a substance to be produced is modelled by the equation

$$t=\ln\left(\frac{2+x}{2-x}\right).$$

(ii) Show that when $t = 0$, $x = 0$. **[2]**

(iii) Show that the rate of change of x is proportional to the product of $(2 + x)$ and $(2 - x)$, and find the constant of proportionality. **[4]**

(iv) Show that $x=\dfrac{2(1-e^{-t})}{1+e^{-t}}$.

Hence determine the long-term mass of the substance predicted by this model. **[4]**

In another chemical reaction, the mass x mg at time t minutes is modelled by the differential equation

$$\frac{dx}{dt}=k(2+x)(2-x)e^{-t},$$

where k is a positive constant, and $x = 0$ when $t = 0$.

(v) Show by integration that, for this reaction, $\ln\left(\dfrac{2+x}{2-x}\right)=4k(1-e^{-t})$. **[5]**

(vi) Given that the long-term mass of this substance is 1.85 mg, find the value of k. **[2]**

END OF QUESTION PAPER

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