



GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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4754A

Mark Scheme

June 2013

Annotations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

4754A

Mark Scheme

June 2013

Question	Answer	Marks	Guidance
1 (i)	$\frac{x}{(1+x)(1-2x)} = \frac{A}{1+x} + \frac{B}{1-2x}$ $\Rightarrow x = A(1-2x) + B(1+x)$ $x = \frac{1}{2} \Rightarrow \frac{1}{2} = B(1 + \frac{1}{2}) \Rightarrow B = \frac{1}{3}$ $x = -1 \Rightarrow -1 = 3A \Rightarrow A = -\frac{1}{3}$	M1 A1 A1 [3]	expressing in partial fractions of correct form (at any stage) and attempting to use cover up, substitution or equating coefficients Condone a single sign error for M1 only. www cao www cao (accept $A/(1+x) + B/(1-2x)$, $A = -1/3$, $B = 1/3$ as sufficient for full marks without needing to reassemble fractions with numerical numerators)

4754A

Mark Scheme

June 2013

Question	Answer	Marks	Guidance
1 (ii)	$\frac{x}{(1+x)(1-2x)} = \frac{-1/3}{1+x} + \frac{1/3}{1-2x}$ $= \frac{1}{3} [(1-2x)^{-1} - (1+x)^{-1}]$ $= \frac{1}{3} [1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \dots - (1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \dots)]$ $= \frac{1}{3} [1 + 2x + 4x^2 + \dots - (1 - x + x^2 + \dots)]$ $= \frac{1}{3} (3x + 3x^2 + \dots) = x + x^2 + \dots \text{ so } a = 1 \text{ and } b = 1$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>correct binomial coefficients throughout for first three terms of either $(1-2x)^{-1}$ or $(1+x)^{-1}$ oe ie 1, (-1), (-1)(-2)/2, not nCr form. Or correct simplified coefficients seen.</p> <p>$1 + 2x + 4x^2$</p> <p>$1 - x + x^2$ (or 1/3/-1/3 of each expression, ft their A/B)</p> <p>If $k(1-x+x^2)$ (A1) not clearly stated separately, condone absence of inner brackets (ie $1+2x+4x^2-1-x+x^2$) only if subsequently it is clear that brackets were assumed, otherwise A1A0.</p> <p>[ie $-1-x+x^2$ is A0 unless it is followed by the correct answer]</p> <p>Ignore any subsequent incorrect terms</p> <p>or from expansion of $x(1-2x)^{-1}(1+x)^{-1}$</p> <p>www cao</p>
	<p>OR</p> $x(1-x-2x^2) = x(1-(x+2x^2))$ $= x(1+x+2x^2 + (-1)(-2)(x+2x^2)^2/2 + \dots)$ $= x(1+x+2x^2+x^2+\dots)$ $= x+x^2+\dots \text{ so } a = 1 \text{ and } b = 1$	<p>M1</p> <p>A2</p> <p>A1</p>	<p>correct binomial coefficients throughout for $(1-(x+2x^2))$ oe (ie 1,-1), at least as far as necessary terms (1+x) (NB third term of expansion unnecessary and can be ignored)</p> <p>$x(1+x)$ www</p> <p>www cao</p>
	Valid for $-\frac{1}{2} < x < \frac{1}{2}$ or $ x < \frac{1}{2}$	<p>B1</p> <p>[5]</p>	<p>independent of expansion. Must combine as one overall range. condone \leq s (although incorrect) or a combination. Condone also, say $-\frac{1}{2} < x < \frac{1}{2}$ but not $x < \frac{1}{2}$ or $-1 < 2x < 1$ or $-\frac{1}{2} > x > \frac{1}{2}$</p>

4754A

Mark Scheme

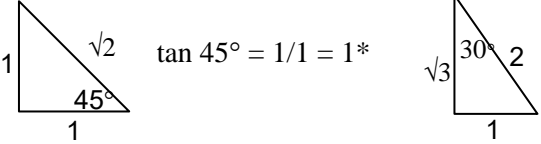
June 2013

Question	Answer	Marks	Guidance
2	$\operatorname{cosec} x + 5 \cot x = 3 \sin x$ $\Rightarrow \frac{1}{\sin x} + \frac{5 \cos x}{\sin x} = 3 \sin x$ $\Rightarrow 1 + 5 \cos x = 3 \sin^2 x = 3(1 - \cos^2 x)$ $\Rightarrow 3 \cos^2 x + 5 \cos x - 2 = 0 *$ $\Rightarrow (3 \cos x - 1)(\cos x + 2) = 0$ $\Rightarrow \cos x = 1/3,$ $x = 70.5^\circ,$ 289.5°	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>using $\operatorname{cosec} x = 1/\sin x$ and $\cot x = \cos x / \sin x$</p> <p>$\cos^2 x + \sin^2 x = 1$ used (both M marks must be part of same solution in order to score both marks)</p> <p>AG (Accept working backwards, with same stages needed)</p> <p>use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and -2 when multiplied out) or comp square oe</p> <p>$\cos x = 1/3$ www</p> <p>for 70.5° or first correct solution, condone over-specification (ie 70.5° or better eg $70.53^\circ, 70.5288^\circ$ etc),</p> <p>for 289.5° or second correct solution (condone over-specification) and no others in the range</p> <p>Ignore solutions outside the range</p> <p>SCA1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48</p> <p>SC Award A1A0 for 1.2,5.1 radians (or better)</p> <p>Do not award SC marks if there are extra solutions in the range</p>

4754A

Mark Scheme

June 2013

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3	 <p> $\tan 45^\circ = 1/1 = 1^*$ $\tan 30^\circ = 1/\sqrt{3}^*$ </p> <p> $\tan 75^\circ = \tan (45^\circ + 30^\circ)$ $= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$ $= \frac{1 + \sqrt{3}}{-1 + \sqrt{3}}$ $= \frac{(1 + \sqrt{3})^2}{3 - 1}$ $= \frac{(3 + 2\sqrt{3} + 1)}{3 - 1} = 2 + \sqrt{3}^*$ </p> <p> (oe eg $\frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{(3 + \sqrt{3})^2}{9 - 3}$) </p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>For both B marks AG so need to be convinced and need triangles but further explanation need not be on their diagram. Any given lengths must be consistent.</p> <p>Need $\sqrt{2}$ or indication that triangle is isosceles oe</p> <p>Need all three sides oe</p> <p>use of correct compound angle formula with $45^\circ, 30^\circ$ soi</p> <p>substitution in terms of $\sqrt{3}$ in any correct form</p> <p>eliminating fractions within a fraction (or rationalising, whichever comes first) provided compound angle formula is used as $\tan(A+B) = \frac{\tan(A+B)}{1 \pm \tan A \tan B}$.</p> <p>rationalising denominator (or eliminating fractions whichever comes second)</p> <p>correct only, AG so need to see working</p>

4754A

Mark Scheme

June 2013

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4 (i)	$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \dots$ $\dots + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>need \mathbf{r} (or another letter) = or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for first B1</p> <p>NB answer is not unique eg $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$</p> <p>Accept i/j/k form and condone row vectors.</p>
4 (ii)	$x + 3y + 2z = 4$ $\Rightarrow -2\lambda + 3(1 + \lambda) + 2(3 + 2\lambda) = 4$ $\Rightarrow 5\lambda = -5, \lambda = -1$ <p>so point of intersection is (2, 0, 1)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>substituting their line in plane equation (condone a slip if intention clear)</p> <p>www cao NB λ is not unique as depends on choice of line in (i)</p> <p>www cao</p>
4 (iii)	<p>Angle between $-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is θ where</p> $\cos \theta = \frac{-2 \times 1 + 1 \times 3 + 2 \times 2}{\sqrt{9} \sqrt{14}} = \frac{5}{3\sqrt{14}}$ $\Rightarrow \theta = 63.5^\circ$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Angle between $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and their direction from (i) ft condone a single sign slip if intention clear</p> <p>correct formula (including cosine), with substitution, for these vectors condone a single numerical or sign slip if intention is clear</p> <p>www cao (63.5 in degrees (or better) or 1.109 radians or better)</p>

4754A

Mark Scheme

June 2013

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5	$\begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ $\Rightarrow 3\lambda - \mu = 10$ $2\lambda + \mu = 5 \Rightarrow 5\lambda = 15, \lambda = 3$ $\Rightarrow 9 - \mu = 10, \mu = -1$ $-5 = -\lambda + 2\mu, \quad -5 = -3 + 2 \times -1 \text{ true}$ <p>coplanar</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>[6]</p>	<p>required form, can be so from two or more correct equations</p> <p>forming at least two equations and attempting to solve oe</p> <p>www</p> <p>www</p> <p>verifying third equation, do not give BOD</p> <p>accept a statement such as $\begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix} = 3 \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + -1 \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ as</p> <p>verification</p> <p>Must clearly show that the solutions satisfy all the equations.</p> <p>oe independent of all above marks</p>

4754A

Mark Scheme

June 2013

Question	Answer	Marks	Guidance
6 (i)	$v dv/dx + 4x = 0$ $\int v dv = -\int 4x dx$ $\frac{1}{2} v^2 = -2x^2 + c$ When $x = 1, v = 4$, so $c = 10$ so $v^2 = 20 - 4x^2$ *	M1 A1 B1 A1 [4]	separating variables and intending to integrate oe condone absence of c . [Not immediate $v^2 = -4x^2 (+c)$] finding c , must be convinced as AG, need to see at least the statement given here oe (condone change of c) AG following finding c convincingly Alternatively, SC $v^2=20-4x^2$, by differentiation, $2v dv/dx = -8x$ $v dv/dx + 4x = 0$ scores B2 if, in addition, they check the initial conditions a further B1 is scored (ie $16=20-4$). Total possible 3/4.
6 (ii)	$x = \cos 2t + 2\sin 2t$ when $t = 0, x = \cos 0 + 2 \sin 0 = 1$ * $v = dx/dt = -2\sin 2t + 4 \cos 2t$ $v = 4 \cos 0 - 2\sin 0 = 4$ *	B1 M1 A1 A1 [4]	AG need some justification differentiating, accept $\pm 2, \pm 4$ as coefficients but not $\pm 1, \pm 2$ and not $\pm 1/2, \pm 1$ from integrating cao www AG

Question	Answer	Marks	Guidance
6 (iii)	$\cos 2t + 2 \sin 2t = R \cos(2t - \alpha) = R(\cos 2t \cos \alpha + \sin 2t \sin \alpha)$ $R = \sqrt{5}$ $R \cos \alpha = 1, R \sin \alpha = 2$ $\tan \alpha = 2,$ $\alpha = 1.107$ $x = \sqrt{5} \cos(2t - 1.107)$ $v = -2\sqrt{5} \sin(2t - 1.107)$	 B1 M1 M1 A1 A1	SEE APPENDIX 1 for further guidance or 2.24 or better (not \pm unless negative rejected) correct pairs soi correct method cao radians only, 1.11 or better (or multiples of π that round to 1.11) differentiating or otherwise, ft their numerical R, α (not degrees) required form SC B1 for $v = \sqrt{20} \cos(2t + 0.464)$ oe
	EITHER $v^2 = 20 \sin^2(2t - \alpha)$ $20 - 4x^2 = 20 - 20 \cos^2(2t - \alpha)$ $= 20(1 - \cos^2(2t - \alpha)) = 20 \sin^2(2t - \alpha)$ so $v^2 = 20 - 4x^2$	 M1 A1	squaring their v (if of required form with same α as x), and x , and attempting to show $v^2 = 20 - 4x^2$ ft their R, α (incl. degrees) [α may not be specified]. cao www (condone the use of over-rounded α (radians) or degrees)
	OR multiplying out $v^2 = (-2 \sin 2t + 4 \cos 2t)^2$ $= 4 \sin^2 2t - 16 \sin 2t \cos 2t + 16 \cos^2 2t$ and $4x^2 = 4(\cos^2 2t + 4 \sin 2t \cos 2t + 4 \sin^2 2t)$ $= 4 \cos^2 2t + 16 \sin 2t \cos 2t + 16 \sin^2 2t$ (need middle term) and attempting to show that $v^2 + 4x^2 = 4(\sin^2 2t + \cos^2 2t) + 16(\cos^2 2t + \sin^2 2t)$ $= 4 + 16 = 20$ (or $20 - 4x^2 = v^2$) oe so $v^2 = 20 - 4x^2$	M1 A1 [7]	differentiating to find v (condone coefficient errors), squaring v and x and multiplying out (need attempt at middle terms) and attempting to show $v^2 = 20 - 4x^2$ cao www

4754A

Mark Scheme

June 2013

Question		Answer	Marks	Guidance
6	(iv)	$x = \sqrt{5}\cos(2t - \alpha)$ or otherwise $x \text{ max} = \sqrt{5}$ when $\cos(2t - \alpha) = 1$, $2t - 1.107 = 0$, $2t = 1.107$ $t = 0.55$	B1 M1 A1 [3]	ft their R oe (say by differentiation) ft their α in radians or degrees for method only cao (or answers that round to 0.554)
7	(i)	$u = 10, x = 5 \ln 10 = 11.5$ so $OA = 5 \ln 10$ when $u = 1$, $y = 1 + 1 = 2$ so $OB = 2$ When $u = 10, y = 10 + 1/10 = 10.1$ So $AC = 10.1$	M1 A1 M1 A1 A1 [5]	Using $u = 10$ to find OA accept 11.5 or better Using $u = 1$ to find OB or $u = 10$ to find AC In the case where values are given in coordinates instead of OA=,OB=,AC=, then give A0 on the first occasion this happens but allow subsequent As. Where coordinates are followed by length eg B(0, 2), length=2 then allow A1.

4754A

Mark Scheme

June 2013

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7 (ii)	$\frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{1-1/u^2}{5/u}$ $\left[= \frac{u^2-1}{5u} \right]$	M1 A1	their dy/du /dx/du Award A1 if any correct form is seen at any stage including unsimplified (can isw)
	EITHER When $u = 10$, $dy/dx = 99/50 = 1.98$ $\tan(90 - \theta) = 1.98 \Rightarrow \theta = 90 - 63.2$ $ = 26.8^\circ$	M1 M1 A2	substituting u =10 in their expression or by geometry , say using a triangle and the gradient of the line 26.8°, or 0.468 radians (or better) cao SC M1M0A1A0 for 63.2° (or better) or 1.103 radians(or better)
	OR When $u = 10$, $dy/dx = 99/50 = 1.98$ $\tan(90 - \theta) = 99/50 \Rightarrow \tan\theta = 50/99$ $ \theta = 26.8^\circ$	M1 M1 A2 [6]	allow use of their expression for M marks 26.8°, or 0.468 radians (or better) cao
7 (iii)	$x = 5 \ln u \Rightarrow x/5 = \ln u, u = e^{x/5}$ $\Rightarrow y = u + 1/u = e^{x/5} + e^{-x/5}$	M1 A1 [2]	Need some working Need some working as AG

4754A

Mark Scheme

June 2013

Question	Answer	Marks	Guidance
7 (iv)	$\text{Vol of rev} = \int_0^{5\ln 10} \pi y^2 dx = \int_0^{5\ln 10} \pi(e^{x/5} + e^{-x/5})^2 dx$ $= \int_0^{5\ln 10} \pi(e^{2x/5} + 2 + e^{-2x/5}) dx$ $= \pi \left[\frac{5}{2} e^{2x/5} + 2x - \frac{5}{2} e^{-2x/5} \right]_0^{5\ln 10}$ $= \pi(250 + 10\ln 10 - 0.025 - 0)$ $= 858$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>need $\pi (e^{x/5} + e^{-x/5})^2$ and dx soi. Condone wrong limits or omission of limits for M1.</p> <p>Allow M1 if y prematurely squared as eg $(e^{2x/5} + e^{-2x/5})$</p> <p>including correct limits at some stage (condone 11.5 for this mark)</p> <p>$[\frac{5}{2}e^{2x/5} + 2x - \frac{5}{2}e^{-2x/5}]$ allow if no π and/or no limits or incorrect limits</p> <p>substituting both limits (their OA and 0) in an expression of correct form ie $ae^{2x/5} + be^{-2x/5} + cx, \quad a,b,c \neq 0$ and subtracting in correct order (- 0 is sufficient for lower limit) Condone absence of π for M1</p> <p>A1 accept 273π and answers rounding to 273π or 858</p> <p>NB The integral can be evaluated using a change of variable to u. This involves changing dx to $(dx/du)x du$. For completely correct work from this method award full marks. Partially correct solutions must include the change in dx. If in doubt consult your TL.</p> <p>Remember to indicate second box has been seen even if it has not been used.</p>

APPENDIX 1

ADDITIONAL GUIDANCE for 6(iii)

$R\cos\alpha=1, R\sin\alpha=2, R=\sqrt{5}, \alpha=1.107$

1) Missing R

I.e $\cos\alpha=1, \sin\alpha=2$.

We reluctantly condone this provided that it is followed by working that suggests R was implied such as $\tan\alpha=2, \alpha=1.107$ M1M1A1. Other methods are possible.

We do not award M1 for $\cos\alpha=1, \sin\alpha=2$ if it is followed by $\alpha=\text{inv cos } 1$ as R is not implied.

B1 is still available.

2) Incorrect pairs

eg $R\sin\alpha=1, R\cos\alpha=2$ scores M0 but would obtain the second M1ft if it was followed by $\tan\alpha=1/2$. M0M1A0. B1 is possible.

3) Incorrect method

$R\sin\alpha=2, R\cos\alpha=1$ followed by $\tan\alpha=1/2$ scores M1M0A0. B1 is possible.

4) Incorrect pairs and incorrect method

$R\sin\alpha=1, R\cos\alpha=2, \tan\alpha=2$ is M0M0A0. B1 is possible. This is easily over-looked and is a double error leading to an apparently correct answer.

5) Incorrect signs (all could score B1)

(a) $R\cos\alpha=1, R\sin\alpha=-2, \tan\alpha=-2, M1, M1ft, A0$

(b) $R\cos\alpha=1, R\sin\alpha=-2, \tan\alpha=2, M1 M0ft, A0$ sign error

(c) $R\cos\alpha=1, R\sin\alpha=-2, R=\sqrt{5}, \sin\alpha=-2/\sqrt{5}, M1 M1 A0$ sign error

(d) $R\cos\alpha=1, R\sin\alpha=-2, \sin\alpha=2/\sqrt{5}, M1M0$ sign error A0

(e) $R\cos\alpha=1, R\sin\alpha=-2, \cos\alpha=1/\sqrt{5}, \alpha=1.107 M1, M1, A0$ sign error (even though not used)

6) Incorrect R

a) $R\cos\alpha=1, R\sin\alpha=2, R=5$ (say), $\cos\alpha=1/5$, scores B0 M1M1ftA0

b) $R\cos\alpha=1, R\sin\alpha=2, R=5$ (say), $\tan\alpha=2, \alpha=1.107$ scores M1M1B0A1 (allow)

7) Missing Working

a) $\tan\alpha=2, \alpha=1.107, R=\sqrt{5}$, scores M1M1A1B1 soi

b) $\tan\alpha=1/2, R=\sqrt{5}$ scores M1M0B1A0 (either correct pairs or correct method but not both)

c) $R\cos\alpha=1, R=\sqrt{5}, \alpha=1.107 M1M1B1A1$ soi

Other options are possible. Examiners should consult their Team Leaders if in doubt.

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4754B**: Applications of Advanced Mathematics: Paper B

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last

4754B**Mark Scheme****June 2013**

(complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

4754B

Mark Scheme

June 2013

Question	Answer	Marks	Guidance	
1	Point R marked correctly at (-60, 45) Point M marked correctly at (0, 20)	B1 B1 [2]	half way up the relevant square Generously applied if above half way If unclear, B0B0.	Need labelling with letters or co-ordinates oe (condone one not labelled if other is labelled (and no others marked)) SC B1 BOD both marked (and no others) and no labelling.
2	For 30° S, 10 hours daylight For 60° S, 5.5 hours daylight. Difference 4.5 hours	B1 B1 B1ft [3]	cao soi allow $5 < t < 6$ soi 10- t dependent on both previous B marks soi	10 only not 5 or 6 SC(1) allow B3 for $4 < \text{difference in length} < 5$ (without wrong working) SC(2) allow B2 if uses $t=5$ or $t=6$ eg, 10,6,4 www SC(3) If calculates (using the equation (4)) can obtain all three marks. Approximate values are 10.07, 5.51, 4.56. (May also work with 5 and $2.5 < t < 3$ if doubled at end eg 5 B1 dependent on later doubling 2.6 B1 dependent on later doubling (5-2.6) x 2 = 4.8 B1)
IT IS ESSENTIAL TO CHECK & ANNOTATE PAGES ATTACHED TO QUESTIONS 101, 102, 103				

4754B

Mark Scheme

June 2013

Question	Answer	Marks	Guidance
3	10° north is B 1° north is A 5° south is D 15° south is C	B2 [2]	All four answers correct SC B1 Any two answers correct
4	$\tan y = -\frac{1}{\tan \alpha} \cos(15t)$ $\alpha = 23.44^\circ, y = 60^\circ, t \text{ is to be found}$ $\cos(15t) = -0.7509\dots$ $15t = 138.6737\dots$ $t = 9.2449\dots$ Daylight hours are $2 \times 9.2449\dots = 18.4898\dots$ So 18.5 hours (to 3s.f.) OR Using $t=9$ $\alpha=23.44, t=9, y$ is to be found $\tan y = 1.6309\dots\dots$ $y=58.485^\circ$ so approx 60°	M1 A1 DM1 A1 M1 DM1 A2 [4]	substitute in formula and attempt to solve (as far as $15t=\text{invcos}\dots\dots$)oe accept 9.2 or better doubling, dependent on first M1 or approx 18 hours www (accept 18.4898,18.489,18.49,18.5 or 18.4 (from 2×9.2),18.48) any reasonable accuracy or stating error is approx 0.49 oe any reasonable accuracy

4754B

Mark Scheme

June 2013

Question		Answer	Marks	Guidance
5	(i)	$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times (n+10)\right)$ <p>On February 2nd, $n = 31 + 2 = 33$</p> $\alpha = -23.44 \times \cos\left(\frac{360}{365} \times 43\right)$ $a = -17.31$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>calculate $n = 31 + 2 = 33$ (days in Jan + Feb) soi</p> <p>substitution of their $n+10$ in equation (3) and attempt to evaluate</p> <p>or -17.306 or rounds to -17.3</p> <p>SC B1 condone 30+2</p> <p>Where $n=31,32,33,34$ only</p> <p>NB $n = 32+10$ gives -17.576 gaining B1M1A0</p>
5	(ii)	$\tan y = -\frac{1}{\tan \alpha} \cos(15t)$ $\tan 53 = -\frac{1}{\tan(-17.306)} \times \cos(15t)$ $t = \frac{1}{15} \arccos(-\tan 53 \times \tan(-17.306))$ $t = 4.3717$ <p>Sunset is at 12 hrs + 4 hours 22 minutes, and so 16:22 hrs</p>	<p>M1</p> <p>DM1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>use of their α in equation (4)</p> <p>making t the subject</p> <p>4.37 or better</p> <p>cao</p> <p>SC ft from -17.576 Obtains A1ft for 4.343 (or 4.34) And then A1ft for 16:21 (or 16:20)</p>

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