OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\dfrac{x}{x^2-1}+\dfrac{2}{x+1}$ as a single fraction, simplifying your answer. **[3]**

2 Fig. 2 shows the curve $y=\sqrt{1+x^2}$.

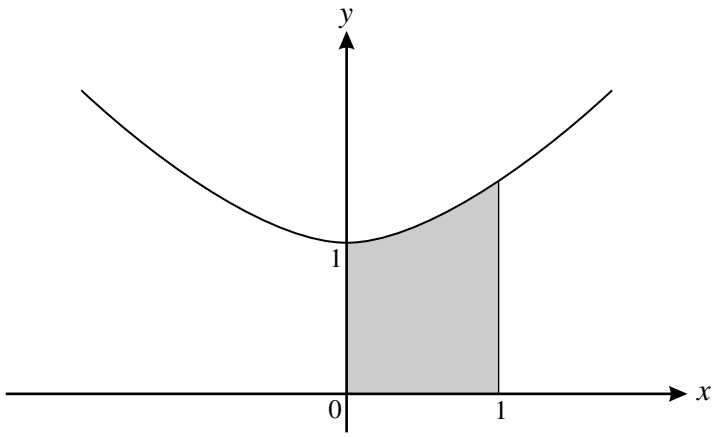

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

3 The parametric equations of a curve are

$$x=\cos 2\theta, \quad y=\sin\theta\cos\theta \quad \text{for } 0\leqslant\theta<\pi.$$

Show that the cartesian equation of the curve is $x^2+4y^2=1$.

Sketch the curve. **[5]**

4 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[5]**

5 **(i)** Express $\dfrac{3}{(y-2)(y+1)}$ in partial fractions. **[3]**

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2(y-2)(y+1),$$

show that $\dfrac{y-2}{y+1} = A\mathrm{e}^{x^3}$, where A is a constant. **[5]**

6 Solve the equation $\tan(\theta + 45^\circ) = 1 - 2\tan\theta$, for $0^\circ \leqslant \theta \leqslant 90^\circ$. **[7]**

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates (−200, 100, 0) and B has coordinates (100, 200, 100), where units are metres.

(i) Verify that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. **[3]**

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. **[6]**

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. **[4]**

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. **[5]**

[Question 8 is printed overleaf.]

8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

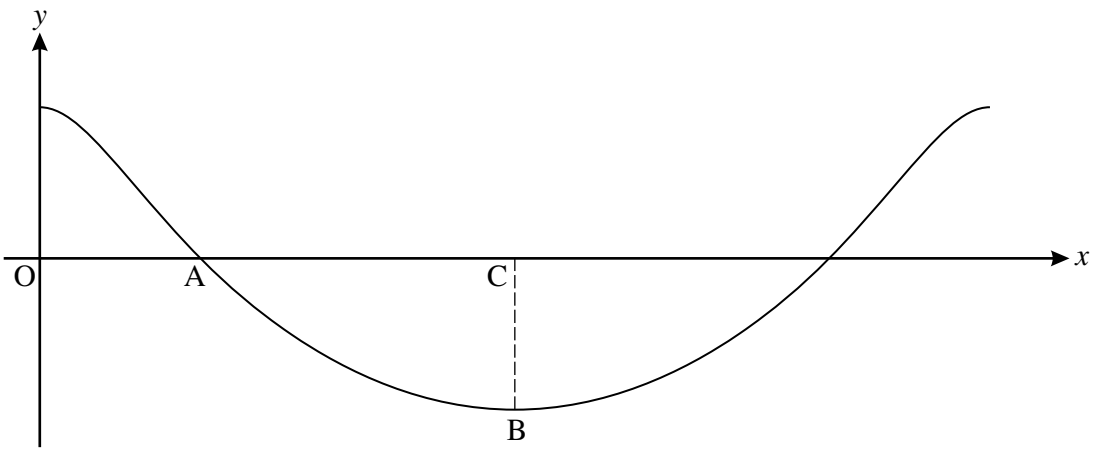

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$ **[2]**

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the Question Paper

OCR Supplied Materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Rough paper
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: Up to 1 hour

* 4 7 5 4 B *

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- This paper should be attached to the candidate's paper A script before sending to the examiner.

Examiner's Use Only:	
1	
2	
3	
4	
5	
Total	

For Examiner's Use

1 The train journey from Swansea to London is 307 km and that by road is 300 km. Carry out the calculations performed on the First Great Western website to estimate how much lower the carbon dioxide emissions are when travelling by rail rather than road. **[2]**

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2 The equation of the curve in Fig. 3 is

$$y = \frac{1}{10^4}\left(x^3 - 100x^2 - 10\,000x + 2\,100\,100\right).$$

Calculate the speed at which the car has its lowest carbon dioxide emissions and the value of its emissions at that speed. **[6]**

[An answer obtained from the graph will be given no marks.]

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3 **(i)** In line 109 the carbon dioxide emissions for a particular train journey from Exeter to London are estimated to be 3.7 tonnes. Obtain this figure. **[2]**

(ii) The text then goes on to state that the emissions per extra passenger on this journey are less than $\frac{1}{2}$ kg. Justify this figure. **[1]**

(i) .....

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(ii) .....

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For Examiner's Use

4 The daily number of trains, n, on a line in another country may be modelled by the function defined below, where P is the annual number of passengers.

$n = 10$ for $0 \leqslant P < 10^6$
$n = 11$ for $10^6 \leqslant P < 1.5 \times 10^6$
$n = 12$ for $1.5 \times 10^6 \leqslant P < 2 \times 10^6$
$n = 13$ for $2 \times 10^6 \leqslant P < 2.5 \times 10^6$
$n = 14$ for $2.5 \times 10^6 \leqslant P < 3 \times 10^6$
… and so on …

(i) Sketch the graph of n against P. **[2]**

(ii) Describe, in words, the relationship between the daily number of trains and the annual number of passengers. **[2]**

(i)

(ii) .....

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[Question 5 is printed overleaf.]

For Examiner's Use

5 The FGW website gives the conversion factor for miles to kilometres to 7 significant figures.

> "We got the distance between the two stations by road from theaa.com. We then converted this distance to kilometres by multiplying it by 1.609344."

Suppose this conversion factor is applied to a distance of exactly 100 miles.

State which one of the following best expresses the level of accuracy for the distance in metric units, justifying your answer.

A : to the nearest millimetre
B : to the nearest 10 centimetres
C : to the nearest metre **[3]**

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