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ADVANCED GCE **4754/01A**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

WEDNESDAY 21 MAY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

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2

Section A (36 marks)

1 Express $\dfrac{x}{x^2 - 4} + \dfrac{2}{x + 2}$ as a single fraction, simplifying your answer. [3]

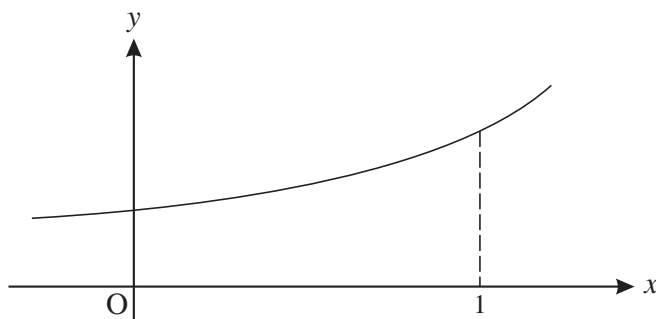**2** Fig. 2 shows the curve $y = \sqrt{1 + e^{2x}}$.

Fig. 2

The region bounded by the curve, the x-axis, the y-axis and the line $x = 1$ is rotated through $360°$ about the x-axis.

Show that the volume of the solid of revolution produced is $\frac{1}{2}\pi(1 + e^2)$. [4]

3 Solve the equation $\cos 2\theta = \sin \theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving your answers in terms of π. [7]

4 Given that $x = 2 \sec \theta$ and $y = 3 \tan \theta$, show that $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$. [3]

5 A curve has parametric equations $x = 1 + u^2$, $y = 2u^3$.

  (i) Find $\dfrac{dy}{dx}$ in terms of u. [3]

  (ii) Hence find the gradient of the curve at the point with coordinates $(5, 16)$. [2]

6 **(i)** Find the first three non-zero terms of the binomial series expansion of $\dfrac{1}{\sqrt{1 + 4x^2}}$, and state the set of values of x for which the expansion is valid. [5]

  (ii) Hence find the first three non-zero terms of the series expansion of $\dfrac{1 - x^2}{\sqrt{1 + 4x^2}}$. [3]

7 Express $\sqrt{3} \sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y = \sqrt{3} \sin x - \cos x$ for which $0 < x < 2\pi$. [6]

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Section B (36 marks)

8   The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.
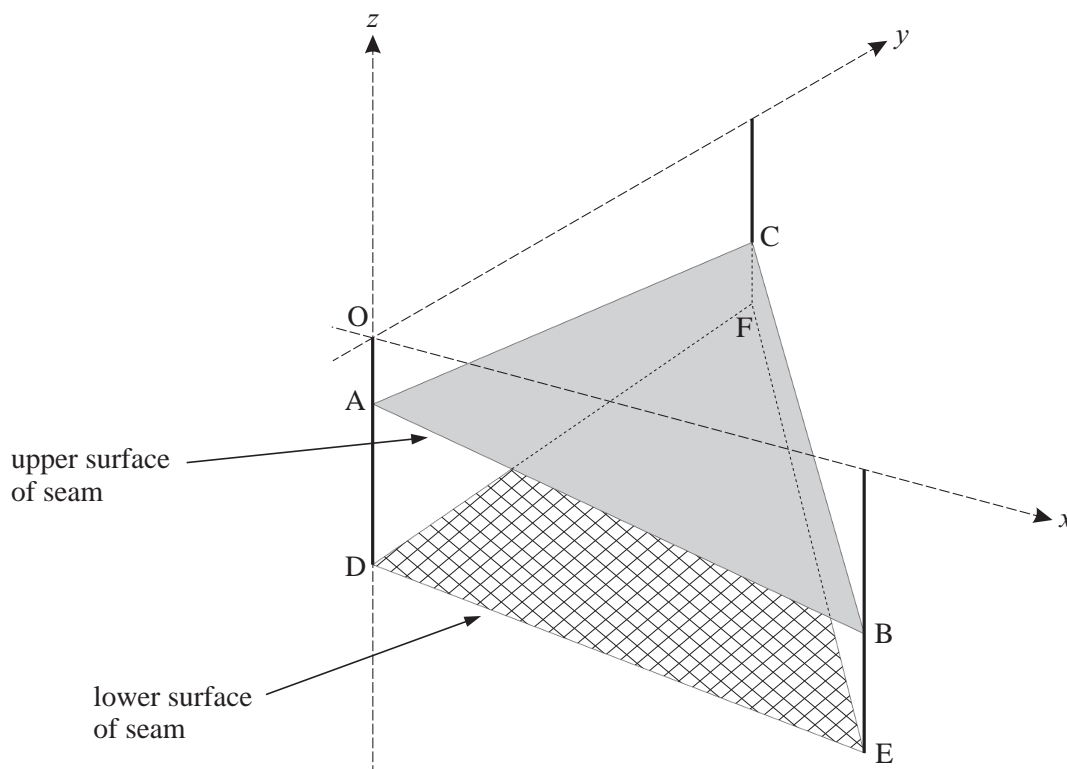

Fig. 8

Relative to axes Ox (due east), Oy (due north) and Oz (vertically upwards), the coordinates of the points are as follows.

$$\text{A: } (0, 0, -15) \qquad \text{B: } (100, 0, -30) \qquad \text{C: } (0, 100, -25)$$
$$\text{D: } (0, 0, -40) \qquad \text{E: } (100, 0, -50) \qquad \text{F: } (0, 100, -35)$$

  (i) Verify that the cartesian equation of the plane ABC is $3x + 2y + 20z + 300 = 0$. [3]

 (ii) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$. Show that the vector $2\mathbf{i} - \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]

(iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R (15, 34, 0) in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

 (iv) Write down a vector equation of the line RS.

   Calculate the coordinates of S. [5]

                                         [Turn over

9 A skydiver drops from a helicopter. Before she opens her parachute, her speed $v\,\text{m s}^{-1}$ after time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10\mathrm{e}^{-\frac{1}{2}t}.$$

When $t = 0$, $v = 0$.

  (i) Find v in terms of t.   [4]

  (ii) According to this model, what is the speed of the skydiver in the long term?   [2]

She opens her parachute when her speed is $10\,\text{m s}^{-1}$. Her speed t seconds after this is $w\,\text{m s}^{-1}$, and is modelled by the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\tfrac{1}{2}(w - 4)(w + 5).$$

(iii) Express $\dfrac{1}{(w - 4)(w + 5)}$ in partial fractions.   [4]

(iv) Using this result, show that $\dfrac{w - 4}{w + 5} = 0.4\mathrm{e}^{-4.5t}$.   [6]

  (v) According to this model, what is the speed of the skydiver in the long term?   [2]

ADVANCED GCE **4754/01B**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper B: Comprehension

WEDNESDAY 21 MAY 2008 Afternoon

Time: Up to 1 hour

Additional materials: Rough paper
MEI Examination Formulae and Tables (MF2)

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- The total number of marks for this paper is 18.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

FOR EXAMINER'S USE	
1	
2	
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TOTAL	

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1 Complete these Latin square puzzles.

   (i)

2	1	3
3		

[1]

   (ii)

2	3	
	2	

[1]

2 In line 51, the text says that the Latin square

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

could not be the solution to a Sudoku puzzle.

Explain this briefly. [2]

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3 On lines 114 and 115 the text says "It turns out that there are 16 different ways of filling in the remaining cells while keeping to the Sudoku rules. One of these ways is shown in Fig. 10."

Complete the grid below with a solution different from that given in Fig. 10. [2]

1	2	3	4

4 Lines 154 and 155 of the article read "There are three other embedded Latin squares in Fig. 14; one of them is illustrated in Fig. 16."

Indicate one of the other two embedded Latin squares on this copy of Fig. 14. [2]

4	2	3	1
		2	4
		4	2
2	4	1	3

5 The number of 9×9 Sudokus is given in line 121.

Without doing any calculations, explain why you would expect 9! to be a factor of this number. [2]

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6 In the table below, *M* represents the maximum number of givens for which a Sudoku puzzle may have no unique solution (Investigation 3 in the article). *s* is the side length of the Sudoku grid and *b* is the side length of its blocks.

Block side length, b	Sudoku, $s \times s$	M
1	1×1	–
2	4×4	12
3	9×9	
4	16×16	
5		

   (i) Complete the table. [2]

 (ii) Give a formula for *M* in terms of *b*. [2]

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7   A man is setting a Sudoku puzzle and starts with this solution.

1	2	3	4	5	6	7	8	9
4	5	6	8	9	7	3	1	2
7	8	9	3	1	2	5	6	4
2	3	1	5	6	4	8	9	7
5	6	4	9	7	8	1	2	3
8	9	7	1	2	3	6	4	5
3	1	2	6	4	5	9	7	8
6	4	5	7	8	9	2	3	1
9	7	8	2	3	1	4	5	6

He then removes some of the numbers to give the puzzles in parts **(i)** and **(ii)**. In each case explain briefly, and without trying to solve the puzzle, why it does not have a unique solution.

[2, 2]

(i)

1	2		4		6			9
4			8	9			1	
	8						6	
2		1			4			7
	6	4		7	8	1	2	
8	9			2			4	
	1		6	4		9	7	
6	4		7		9			1
9		8	2		1	4		6

(ii)

1	2	3	4	5	6	7	8	9
4	5	6	8	9	7	3	1	2
7	8	9				5	6	4
2	3	1	5	6	4	8	9	7
5	6	4	9	7	8	1	2	3
8	9	7				6	4	5
3	1	2	6	4	5	9	7	8
6	4	5	7	8	9	2	3	1
9	7	8				4	5	6

(i) ....

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(ii) ....

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