

4754

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<p>1 $\sin x - \sqrt{3} \cos x = R \sin(x - \alpha)$ $= R(\sin x \cos \alpha - \cos x \sin \alpha)$ $\Rightarrow R \cos \alpha = 1, R \sin \alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$ $\tan \alpha = \sqrt{3}/1 = \sqrt{3} \Rightarrow \alpha = \pi/3$</p> <p>$\Rightarrow \sin x - \sqrt{3} \cos x = 2 \sin(x - \pi/3)$ x coordinate of P is when $x - \pi/3 = \pi/2$ $\Rightarrow x = 5\pi/6$ $y = 2$ So coordinates are $(5\pi/6, 2)$</p>	<p>B1 M1 A1 M1 A1ft B1ft [6]</p>	<p>$R = 2$ $\tan \alpha = \sqrt{3}$ or $\sin \alpha = \sqrt{3}/2$ or their R or $\cos \alpha = 1/2$ or their R $\alpha = \pi/3, 60^\circ$ or 1.05 (or better) radians www Using x-their $\alpha = \pi/2$ or 90° $\alpha \neq 0$ exact radians only (not $\pi/2$) their R (exact only)</p>
<p>2(i) $\frac{3+2x^2}{(1+x)^2(1-4x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{1-4x}$ $\Rightarrow 3+2x^2 = A(1+x)(1-4x) + B(1-4x) + C(1+x)^2$</p> <p>$x = -1 \Rightarrow 5 = 5B \Rightarrow B = 1$ $x = 1/4 \Rightarrow 3 \frac{1}{8} = \frac{25}{16}C \Rightarrow C = 2$ coeff^t of x^2: $2 = -4A + C \Rightarrow A = 0$.</p>	<p>M1 B1 B1 E1 [4]</p>	<p>Clearing fractions (or any 2 correct equations) $B = 1$ www $C = 2$ www $A = 0$ needs justification</p>
<p>(ii) $(1+x)^{-2} = 1 + (-2)x + (-2)(-3)x^2/2! + \dots$ $= 1 - 2x + 3x^2 + \dots$ $(1-4x)^{-1} = 1 + (-1)(-4x) + (-1)(-2)(-4x)^2/2! + \dots$ $= 1 + 4x + 16x^2 + \dots$</p> <p>$\frac{3+2x^2}{(1+x)^2(1-4x)} = (1+x)^{-2} + 2(1-4x)^{-1}$ $\approx 1 - 2x + 3x^2 + 2(1 + 4x + 16x^2)$ $= 3 + 6x + 35x^2$</p>	<p>M1 A1 A1 A1ft [4]</p>	<p>Binomial series (coefficients unsimplified - for either) or $(3+2x^2)(1+x)^{-2}(1-4x)^{-1}$ expanded their A, B, C and their expansions</p>
<p>3 $\sin(\theta + \alpha) = 2 \sin \theta$ $\Rightarrow \sin \theta \cos \alpha + \cos \theta \sin \alpha = 2 \sin \theta$ $\Rightarrow \tan \theta \cos \alpha + \sin \alpha = 2 \tan \theta$ $\Rightarrow \sin \alpha = 2 \tan \theta - \tan \theta \cos \alpha$ $= \tan \theta (2 - \cos \alpha)$ $\Rightarrow \tan \theta = \frac{\sin \alpha}{2 - \cos \alpha} *$ $\sin(\theta + 40^\circ) = 2 \sin \theta$ $\Rightarrow \tan \theta = \frac{\sin 40}{2 - \cos 40} = 0.5209$ $\Rightarrow \theta = 27.5^\circ, 207.5^\circ$</p>	<p>M1 M1 M1 E1 M1 A1 A1 [7]</p>	<p>Using correct Compound angle formula in a valid equation dividing by $\cos \theta$ collecting terms in $\tan \theta$ or $\sin \theta$ or dividing by $\tan \theta$ or $\sin \theta$ www (can be all achieved for the method in reverse) $\tan \theta = \frac{\sin 40}{2 - \cos 40}$ -1 if given in radians -1 extra solutions in the range</p>

<p>4 (a) $\frac{dx}{dt} = k\sqrt{x}$</p>	<p>M1 A1 [2]</p>	<p>$\frac{dx}{dt} = \dots$ $k\sqrt{x}$</p>
<p>(b) $\frac{dy}{dt} = \frac{10000}{\sqrt{y}}$</p> <p>$\Rightarrow \int \sqrt{y} dy = \int 10000 dt$</p> <p>$\Rightarrow \frac{2}{3} y^{\frac{3}{2}} = 10000t + c$</p> <p>When $t = 0, y = 900 \Rightarrow 18000 = c$</p> <p>$\Rightarrow y = \left[\frac{3}{2}(10000t + 18000) \right]^{\frac{2}{3}}$</p> <p>$= (1500(10t + 18))^{\frac{2}{3}}$</p> <p>When $t = 10, y = 3152$</p>	<p>M1 A1 B1 A1 M1 A1 [6]</p>	<p>separating variables condone omission of c evaluating constant for their integral any correct expression for $y =$ for method allow substituting $t=10$ in their expression cao</p>
<p>5 (i) $\int xe^{-2x} dx$ let $u = x, dv/dx = e^{-2x}$</p> <p>$\Rightarrow v = -\frac{1}{2} e^{-2x}$</p> <p>$= -\frac{1}{2} xe^{-2x} + \int \frac{1}{2} e^{-2x} dx$</p> <p>$= -\frac{1}{2} xe^{-2x} - \frac{1}{4} e^{-2x} + c$</p> <p>$= -\frac{1}{4} e^{-2x} (1 + 2x) + c$ *</p> <p>or $\frac{d}{dx} \left[-\frac{1}{2} xe^{-2x} - \frac{1}{4} e^{-2x} + c \right] = -\frac{1}{2} e^{-2x} + xe^{-2x} + \frac{1}{2} e^{-2x}$</p> <p>$= xe^{-2x}$</p>	<p>M1 A1 E1 M1 A1 E1 [3]</p>	<p>Integration by parts with $u = x, dv/dx = e^{-2x}$</p> <p>$= -\frac{1}{2} xe^{-2x} + \int \frac{1}{2} e^{-2x} dx$</p> <p>condone omission of c</p> <p>product rule</p>
<p>(ii) $V = \int_0^2 \pi y^2 dx$</p> <p>$= \int_0^2 \pi (x^{1/2} e^{-x})^2 dx$</p> <p>$= \pi \int_0^2 xe^{-2x} dx$</p> <p>$= \pi \left[-\frac{1}{4} e^{-2x} (1 + 2x) \right]_0^2$</p> <p>$= \pi \left(-\frac{1}{4} e^{-4} \cdot 5 + \frac{1}{4} \right)$</p> <p>$= \frac{1}{4} \pi \left(1 - \frac{5}{e^4} \right)$ *</p>	<p>M1 A1 DM1 E1 [4]</p>	<p>Using formula condone omission of limits</p> <p>$y^2 = xe^{-2x}$ condone omission of limits and π</p> <p>condone omission of π (need limits)</p>

Section B

<p>6 (i) At E, $\theta = 2\pi$ $\Rightarrow x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow y = a(1 - \cos \pi) = 2a$</p>	<p>M1 A1 M1 A1 [4]</p>	<p>$\theta = \pi, 180^\circ, \cos \theta = -1$</p>
<p>(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a \sin \theta}{a(1 - \cos \theta)}$ $= \frac{\sin \theta}{(1 - \cos \theta)}$</p>	<p>M1 M1 A1 [3]</p>	<p>$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin \theta) = \cos \theta, \frac{d}{d\theta}(\cos \theta) = -\sin \theta$ both or equivalent wwww condone uncanceled a</p>
<p>(iii) $\tan 30^\circ = 1/\sqrt{3}$ $\Rightarrow \frac{\sin \theta}{(1 - \cos \theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow \sin \theta = \frac{1}{\sqrt{3}}(1 - \cos \theta)^*$ When $\theta = 2\pi/3, \sin \theta = \sqrt{3}/2$ $(1 - \cos \theta)/\sqrt{3} = (1 + 1/2)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\text{BF} = a(1 + 1/2) = 3a/2^*$ $\text{OF} = a(2\pi/3 - \sqrt{3}/2)$</p>	<p>M1 E1 M1 E1 E1 B1 [6]</p>	<p>Or gradient = $1/\sqrt{3}$ $\sin \theta = \sqrt{3}/2, \cos \theta = -1/2$ or equiv.</p>
<p>(iv) $\text{BC} = 2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ $\text{AF} = \sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ $\text{AD} = \text{BC} + 2\text{AF}$ $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ $= 20$ $\Rightarrow a = 2.22 \text{ m}$</p>	<p>B1ft M1 A1 M1 A1 [5]</p>	<p>their OE -2their OF</p>

<p>7 (i) $AE = \sqrt{(15^2 + 20^2 + 0^2)} = 25$</p>	<p>M1 A1 [2]</p>	
<p>(ii) $\overline{AE} = \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 5 \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$</p> <p>Equation of BD is $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$</p> <p>$BD = 15 \Rightarrow \lambda = 3$ $\Rightarrow D$ is $(8, -19, 11)$</p>	<p>M1 A1 M1 A1cao [4]</p>	<p>Any correct form</p> <p>or $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix}$</p> <p>$\lambda = 3$ or $3/5$ as appropriate</p>
<p>(iii) At A: $-3 \times 0 + 4 \times 0 + 5 \times 6 = 30$ At B: $-3 \times (-1) + 4 \times (-7) + 5 \times 11 = 30$ At C: $-3 \times (-8) + 4 \times (-6) + 5 \times 6 = 30$</p> <p>Normal is $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$</p>	<p>M1 A2,1,0 B1 [4]</p>	<p>One verification</p> <p>(OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point</p> <p>OR M1 vector form of equation of plane eg $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}) + \nu(8\mathbf{i} + 6\mathbf{j} + 0\mathbf{k})$ M1 elimination of both parameters A1 equation of plane B1 Normal *)</p>
<p>(iv) $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \overline{AE} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 60 - 60 = 0$</p> <p>$\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \overline{AB} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix} = -4 - 21 + 25 = 0$</p> <p>$\Rightarrow \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to plane</p> <p>Equation is $4x + 3y + 5z = 30$.</p>	<p>M1 E1 M1 A1 [4]</p>	<p>scalar product with one vector in plane = 0</p> <p>scalar product with another vector in plane = 0</p> <p>$4x + 3y + 5z = \dots$ 30 OR as * above OR M1 for subst 1 point in $4x + 3y + 5z = \dots$, A1 for subst 2 further points = 30 A1 correct equation, B1 Normal</p>
<p>(v) Angle between planes is angle between normals $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$</p> <p>$\cos \theta = \frac{4 \times (-3) + 3 \times 4 + 5 \times 5}{\sqrt{50} \times \sqrt{50}} = \frac{1}{2}$</p> <p>$\Rightarrow \theta = 60^\circ$</p>	<p>M1 M1 A1 A1 [4]</p>	<p>Correct method for any 2 vectors their normals only (rearranged) or 120° cao</p>

Comprehension Paper 2			
Qu	Answer	Mark	Comment
1.	$\left(26 + \frac{385}{1760}\right) \times 4 \text{ minutes}$ 1 hour 44 minutes 52.5 seconds	M1 A1	Accept all equivalent forms, with units. Allow ...52 and 53 seconds.
2.	$R = 259.6 - 0.391(T - 1900)$ $\therefore 259.6 - 0.391(T - 1900) = 0$ $\Rightarrow T = 2563.9$ R will become negative in 2563	M1 A1 A1	$R=0$ and attempting to solve. $T=2563, 2564, 2563.9 \dots$ any correct cao
3.	The value of L is 120.5 and this is over 2 hours or (120 minutes)	E1	$R > 120.5$ minutes or showing there is no solution for $120 = 120.5 + 54.5e^{-kt}$
4.(i)	Substituting $t = 0$ in $R = L + (U - L)e^{-kt}$ gives $R = L + (U - L) \times 1$ $= U$	M1 A1 E1	$e^0 = 1$
4.(ii)	As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ and so $R \rightarrow L$	M1 E1	
5.(i)		M1 A1 A1	Increasing curve Asymptote A and B marked correctly
5.(ii)	Any field event: long jump, high jump, triple jump, pole vault, javelin, shot, discus, hammer, etc.	B1	
6.(i)	$t = 104$	B1	
6.(ii)	$R = 115 + (175 - 115)e^{-0.0467t^{0.797}}$ $R = 115 + 60 \times e^{-0.0467 \times 104^{0.797}}$ $R = 115 + 60 \times e^{-1.892}$ $R = 124.047 \dots$ 2 hours 4 minutes 3 seconds	M1 A1	Substituting their t 124, 124.05, etc.