OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754

Applications of Advanced Mathematics (C4)

INSTRUCTIONS

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes + up to 1 hour

The paper is in two parts:

Section A (1 hour 30 minutes)
Section B (up to 1 hour)

Supervisors are requested to ensure that Section B **is not issued** until Section A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Invigilators are not required to match up candidates' two parts. Part A and Part B should be sent to the examiner as two sets of scripts with candidates in the same order as the attendance register for each set.

This notice must be on the Invigilator's desk at all times during the afternoon of Thursday 16 June 2005.

These instructions consist of 1 printed page and 1 blank page.

SP (NF) S90260/1

Registered Charity Number: 1066969

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Section A

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this section is 72.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

Registered Charity 1066969
[Turn over

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

2 Find the first 4 terms in the binomial expansion of $\sqrt{4 + 2x}$. State the range of values of x for which the expansion is valid. [6]

3 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]

4 Fig. 4 shows a sketch of the region enclosed by the curve $\sqrt{1 + e^{-2x}}$, the x-axis, the y-axis and the line $x = 1$.

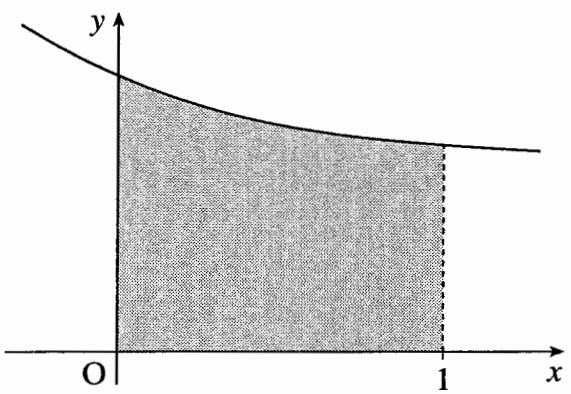

Fig. 4

Find the volume of the solid generated when this region is rotated through 360° about the x-axis. Give your answer in an exact form. [5]

5 Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

6 A curve has cartesian equation $y^2 - x^2 = 4$.

(i) Verify that

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t},$$

are parametric equations of the curve. [2]

(ii) Show that $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t^2+1}$. Hence find the coordinates of the stationary points of the curve. [6]

Section B (36 marks)

7 In a chemical process, the mass M grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dM}{dt} = \frac{M}{t(1+t^2)}.$$

(i) Find $\int \frac{t}{1+t^2}\,dt$. [3]

(ii) Find constants A, B and C such that

$$\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}.$$ [5]

(iii) Use integration, together with your results in parts **(i)** and **(ii)**, to show that

$$M = \frac{Kt}{\sqrt{1+t^2}},$$

where K is a constant. [6]

(iv) When $t = 1, M = 25$. Calculate K.

What is the mass of the chemical in the long term? [4]

[Turn over

8 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the line of the hole.

A 20 cm × 30 cm × 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to x-, y- and z-axes as shown in Fig. 8.1.

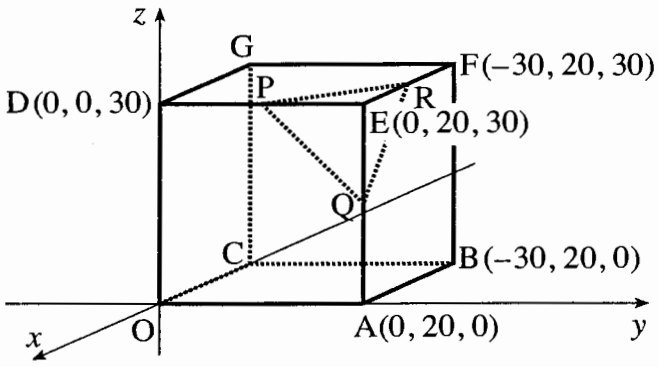

Fig. 8.1

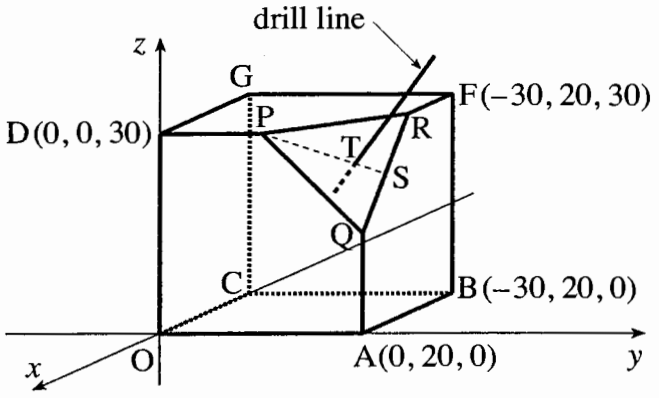

Fig. 8.2

First, a plane cut is made to remove the corner at E. The cut goes through the points P, Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Hence show that $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$. [4]

(ii) Show that the vector $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ is perpendicular to the plane through P, Q and R.

Hence find the cartesian equation of this plane. [5]

A hole is then drilled perpendicular to triangle PQR, as shown in Fig. 8.2. The hole passes through the triangle at the point T which divides the line PS in the ratio 2:1, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates $(-5, 16\frac{2}{3}, 25)$. [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. [5]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS 4754(B)

Applications of Advanced Mathematics (C4)

Section B: Comprehension

Thursday **16 JUNE 2005** Afternoon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces at the top of this page.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this section is 18.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
6	
7	
Total	

This question paper consists of 4 printed pages and an insert.

For Examiner's Use

1 Explain why the number 1836.108 for the ratio $\frac{\text{Rest mass of proton}}{\text{Rest mass of electron}}$ would be suitable for communication with other civilisations whereas neither the rest mass of the proton nor that of the electron would be. [2]

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2 A civilisation which works in base 5 sends out the first 6 digits of π as 3.032 32. Convert this to base 10. [2]

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3 Complete this table to show the next 3 values of the iteration

$$x_{n+1} = kx_n(1 - x_n)$$

in the case when $k = 3.2$ and $x_0 = 0.5$. Give your answers to calculator accuracy. [1]

n	x_n
0	0.5
1	0.8
2	0.512
3	
4	
5	

For Examiner's Use

4 Justify the statement that the equation in line 83,

$$\frac{\phi}{1} = \frac{1}{\phi - 1},$$

has the solution $\phi = \dfrac{1 \pm \sqrt{5}}{2}$. [2]

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5 Justify the statement in line 87 that

$$\frac{1}{\phi} = \frac{\sqrt{5} - 1}{2}.$$ [3]

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6 A sequence is defined by

$$a_{n+1} = 2a_n + 3a_{n-1} \quad \text{with } a_1 = 1 \text{ and } a_2 = 1.$$

Using the method on page 5, show that the value to which the ratio of successive terms converges is 3.

[4]

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For Examiner's Use

7 Use the information in the article, including the value of Feigenbaum's number given in line 142, to predict an approximate value of k at which the bifurcation from 8 to 16 outcomes occurs for the iterative equation

$$x_{n+1} = kx_n(1 - x_n).$$ [4]

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