OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\dfrac{x+1}{x^2(2x-1)}$ in partial fractions. **[5]**

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

3 Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3\sin x + 2\cos x, \quad 0 \leqslant x \leqslant \pi.$$ **[7]**

4 **(i)** Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, \mathrm{d}x$, giving your answer to 3 decimal places. **[3]**

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

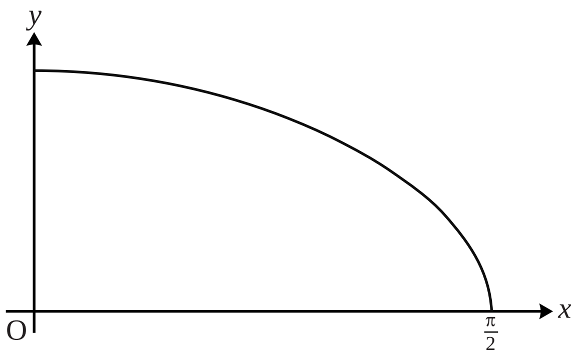

Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. **[1]**

5 Verify that the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points A(2, 0, 1), B(1, 2, 2) and C(0, –4, 1). Hence find the cartesian equation of the plane. **[5]**

6 Given the binomial expansion $(1 + qx)^p = 1 - x + 2x^2 + \ldots$, find the values of p and q. Hence state the set of values of x for which the expansion is valid. **[6]**

7 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ meet.

Find their point of intersection. **[5]**

Section B (36 marks)

8 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2, \; y = 4t, \quad -\sqrt{2} \leqslant t \leqslant \sqrt{2}\,.$$

P$(2t^2, 4t)$ is a point on the curve with parameter t. TS is the tangent to the curve at P, and PR is the line through P parallel to the x-axis. Q is the point (2, 0). The angles that PS and QP make with the positive x-direction are θ and ϕ respectively.

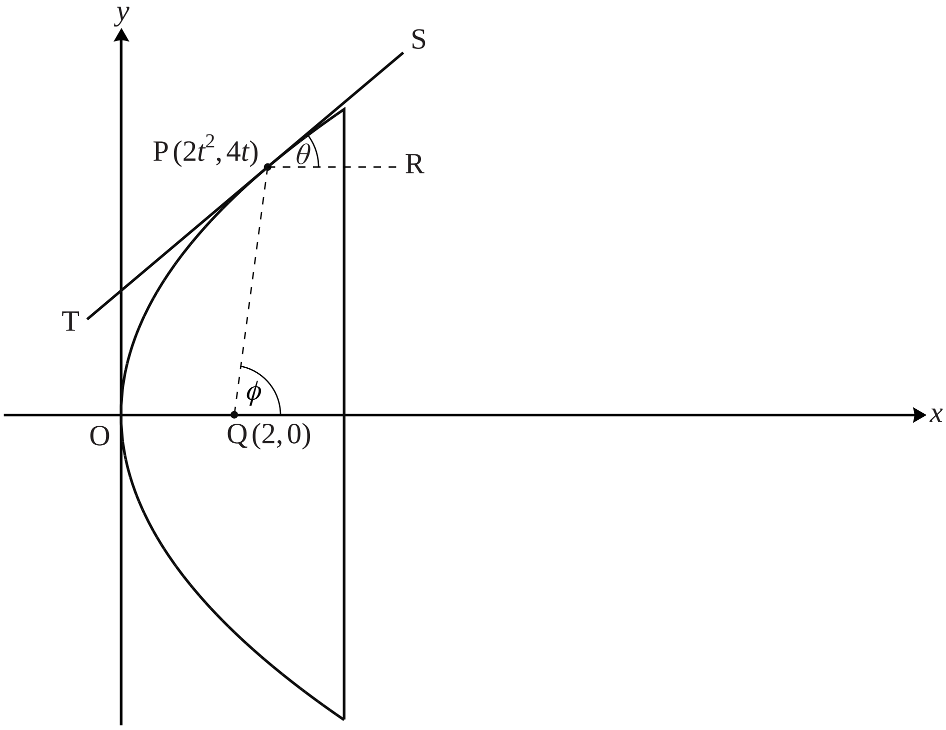

Fig. 8

(i) By considering the gradient of the tangent TS, show that $\tan\theta = \dfrac{1}{t}$. **[3]**

(ii) Find the gradient of the line QP in terms of t. Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ. **[8]**

[The above result shows that if a lamp bulb is placed at Q, then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x-axis.

(iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π. **[7]**

9

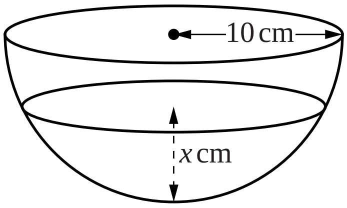

Fig. 9

Fig. 9 shows a hemispherical bowl, of radius 10 cm, filled with water to a depth of x cm. It can be shown that the volume of water, V cm^3, is given by

$$V = \pi(10x^2 - \tfrac{1}{3}x^3).$$

Water is poured into a leaking hemispherical bowl of radius 10 cm. Initially, the bowl is empty. After t seconds, the volume of water is changing at a rate, in cm^3 s^{-1}, given by the equation

$$\frac{dV}{dt} = k(20 - x),$$

where k is a constant.

(i) Find $\frac{dV}{dx}$, and hence show that $\pi x \frac{dx}{dt} = k$. **[4]**

(ii) Solve this differential equation, and hence show that the bowl fills completely after T seconds, where $T = \frac{50\pi}{k}$. **[5]**

Once the bowl is full, the supply of water to the bowl is switched off, and water then leaks out at a rate of kx cm^3 s^{-1}.

(iii) Show that, t seconds later, $\pi(20 - x)\frac{dx}{dt} = -k$. **[3]**

(iv) Solve this differential equation.

Hence show that the bowl empties in $3T$ seconds. **[6]**

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Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

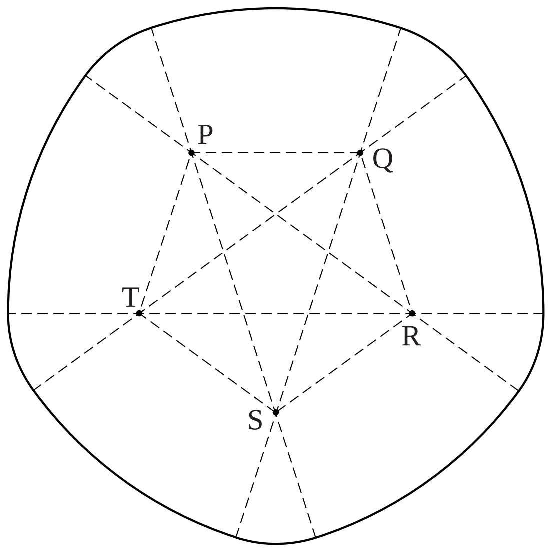
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

1 In lines 22 and 23 it says "arcs can be added to any regular polygon with an odd number of sides to make a curve of constant width". State why the method described cannot be applied to a regular polygon with an even number of sides. **[1]**

1	

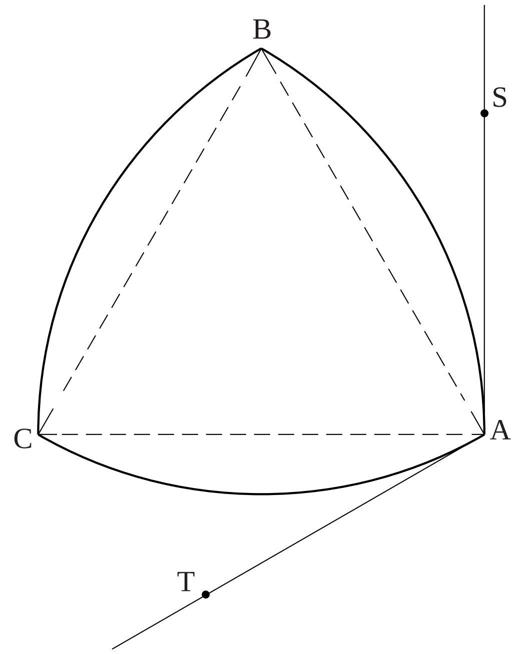
2 **(i)** On the curve of constant width below, indicate clearly the arcs that were constructed with centre P. **[1]**

(ii) Given that this curve has perimeter 70 cm, calculate its width, correct to 3 significant figures. **[2]**

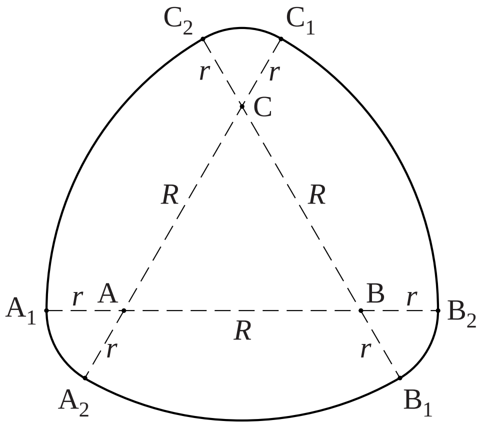
2 (i)	
2 (ii)	

3 The diagram below shows two tangents, AS and AT, at vertex A on a Reuleaux triangle. State the angle SAT justifying your answer carefully. **[3]**

3



4 For the curve in Fig. 7b (copied below) the width, l, is $R + 2r$.



(i) Prove that the perimeter is πl. **[3]**

(ii) You are given that, in the case where $r = \frac{R}{2}$, the area enclosed by this curve is $R^2\left(\frac{5\pi - 2\sqrt{3}}{4}\right)$.

Show that this area falls in the range indicated in lines 28 and 29. **[3]**

4 (i)	
4 (ii)	

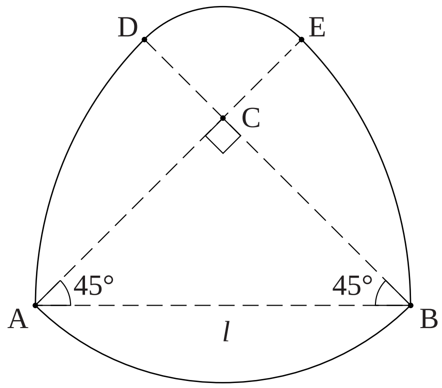
5 Fig. 11b is copied below.

(i) Show that CE has length $\frac{(2-\sqrt{2})}{2}\,l$. **[2]**

(ii) Hence show that the square path traced out by point C (see line 67) has side length $(\sqrt{2}-1)\,l$. **[2]**

(iii) A square hole of side length 50 mm is to be cut in a sheet of plastic, using the method described in lines 69 to 71. Calculate the side length of the square hole needed in the guide plate, giving your answer correct to the nearest millimetre. **[1]**

5 (i)



5 (ii)

5 (iii)

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