OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

* 4 7 5 4 A *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Turn over

Section A (36 marks)

1 **(i)** Use the trapezium rule with four strips to estimate $\int_{-2}^{2} \sqrt{1+e^x}\,dx$, showing your working. **[4]**

Fig. 1 shows a sketch of $y = \sqrt{1+e^x}$.

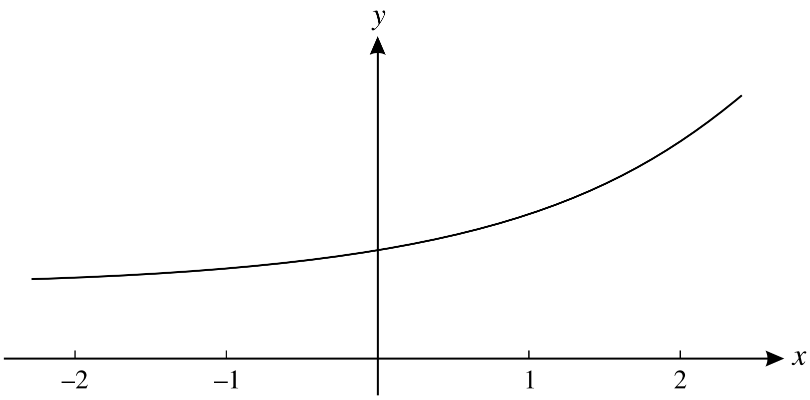

Fig. 1

(ii) Suppose that the trapezium rule is used with more strips than in part **(i)** to estimate $\int_{-2}^{2} \sqrt{1+e^x}\,dx$. State, with a reason but no further calculation, whether this would give a larger or smaller estimate. **[2]**

2 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \qquad y = \frac{1-t}{1+2t}.$$

Find t in terms of x. Hence find the cartesian equation of the curve, giving your answer as simply as possible. **[5]**

3 Find the first three terms in the binomial expansion of $\frac{1}{(3-2x)^3}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

4 The points A, B and C have coordinates (2, 0, −1), (4, 3, −6) and (9, 3, −4) respectively.

(i) Show that AB is perpendicular to BC. **[4]**

(ii) Find the area of triangle ABC. **[3]**

5 Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$. **[3]**

6 **(i)** Find the point of intersection of the line $\mathbf{r} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ and the plane $2x - 3y + z = 11$. **[4]**

(ii) Find the acute angle between the line and the normal to the plane. **[4]**

Section B (36 marks)

7 A particle is moving vertically downwards in a liquid. Initially its velocity is zero, and after t seconds it is $v\,\mathrm{m\,s^{-1}}$. Its terminal (long-term) velocity is $5\,\mathrm{m\,s^{-1}}$.

A model of the particle's motion is proposed. In this model, $v = 5\left(1 - \mathrm{e}^{-2t}\right)$.

(i) Show that this equation is consistent with the initial and terminal velocities. Calculate the velocity after 0.5 seconds as given by this model. **[3]**

(ii) Verify that v satisfies the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2v$. **[3]**

In a second model, v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 0.4v^2.$$

As before, when $t = 0$, $v = 0$.

(iii) Show that this differential equation may be written as

$$\frac{10}{(5-v)(5+v)}\frac{\mathrm{d}v}{\mathrm{d}t} = 4.$$

Using partial fractions, solve this differential equation to show that

$$t = \tfrac{1}{4}\ln\left(\frac{5+v}{5-v}\right).$$ **[8]**

This can be re-arranged to give $v = \dfrac{5\left(1 - \mathrm{e}^{-4t}\right)}{1 + \mathrm{e}^{-4t}}$. [You are **not** required to show this result.]

(iv) Verify that this model also gives a terminal velocity of $5\,\mathrm{m\,s^{-1}}$.

Calculate the velocity after 0.5 seconds as given by this model. **[3]**

The velocity of the particle after 0.5 seconds is measured as $3\,\mathrm{m\,s^{-1}}$.

(v) Which of the two models fits the data better? **[1]**

8 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at α to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all β.

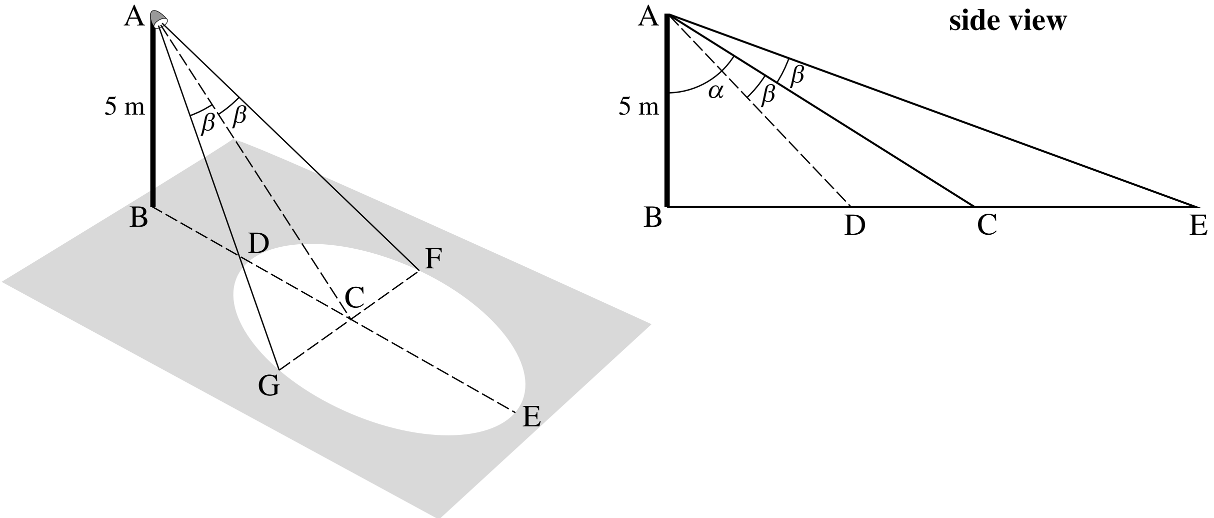

Fig. 8

In the following, all lengths are in metres.

(i) Find AC in terms of α, and hence show that GF = $10\sec\alpha\tan\beta$. **[3]**

(ii) Show that CE = $5\big(\tan(\alpha+\beta) - \tan\alpha\big)$.

Hence show that CE = $\dfrac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$. **[5]**

Similarly, it can be shown that CD = $\dfrac{5\tan\beta\sec^2\alpha}{1+\tan\alpha\tan\beta}$. [You are **not** required to derive this result.]

You are now given that $\alpha = 45°$ and that $\tan\beta = t$.

(iii) Find CE and CD in terms of t. Hence show that DE = $\dfrac{20t}{1-t^2}$. **[5]**

(iv) Show that GF = $10\sqrt{2}\,t$. **[2]**

For a certain value of β, DE = 2GF.

(v) Show that $t^2 = 1 - \dfrac{1}{\sqrt{2}}$.

Hence find this value of β. **[3]**

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ADVANCED GCE

MATHEMATICS (MEI) **4754B**

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Friday 14 January 2011
Afternoon

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES
- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- This paper should be attached to the candidate's paper A script before sending to the examiner.

Examiner's Use Only:	
1	
2	
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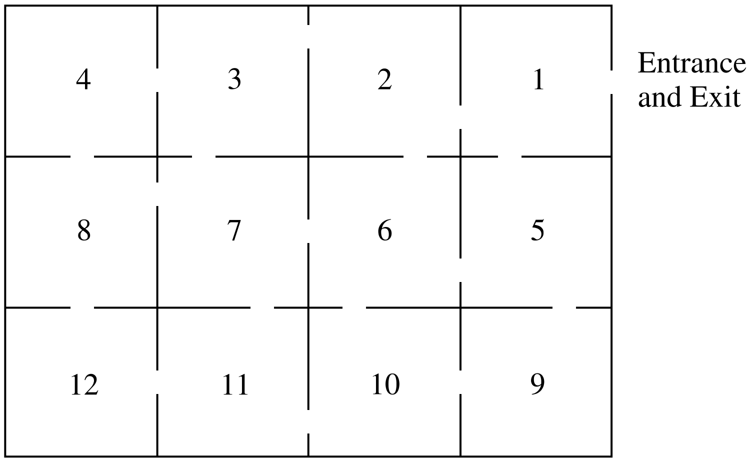
Total	

For Examiner's Use

1 The gallery shown below is a 3 by 4 grid of rectangular rooms.



(i) Mark on the diagram the positions of six guards so that the whole gallery can be observed. **[1]**

(ii) Give a counterexample to disprove the following proposition:

For an m by n grid of rectangular rooms, $\left\lfloor \dfrac{mn}{2} \right\rfloor$ guards are required. **[2]**

For Examiner's Use

2 **(i)** Show that $\dfrac{(2r+1)-(-1)^r}{4} = \left\lfloor \dfrac{r+1}{2} \right\rfloor$ in the case where $r = 4$. **[2]**

....

....

....

(ii) The ceiling function, $\lceil x \rceil$, is defined as the smallest integer greater than or equal to x.

Complete the following table. **[2]**

x	1	2	3	4	5
$\left\lceil \dfrac{x}{2} \right\rceil$					

3 Justify the statement in lines 79 and 80. **[2]**

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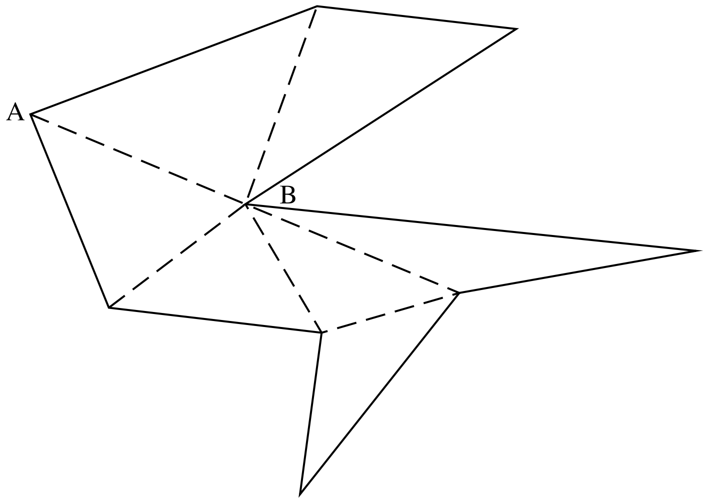
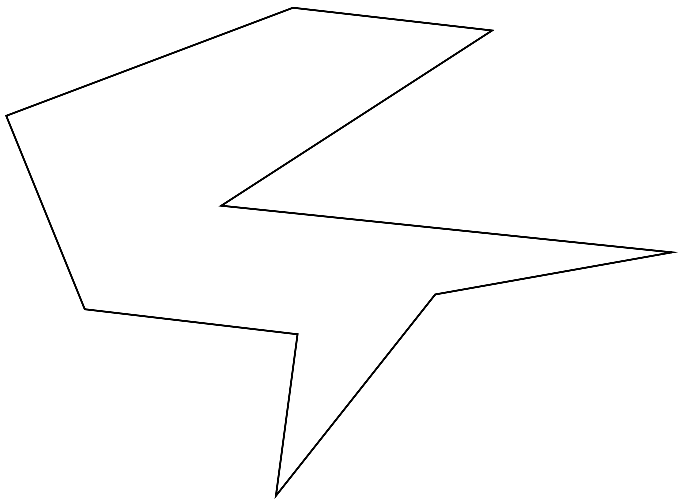
For Examiner's Use

4 **(i)** Following the procedure in Fig. 6, complete the labelling of the polygon shown below. **[2]**

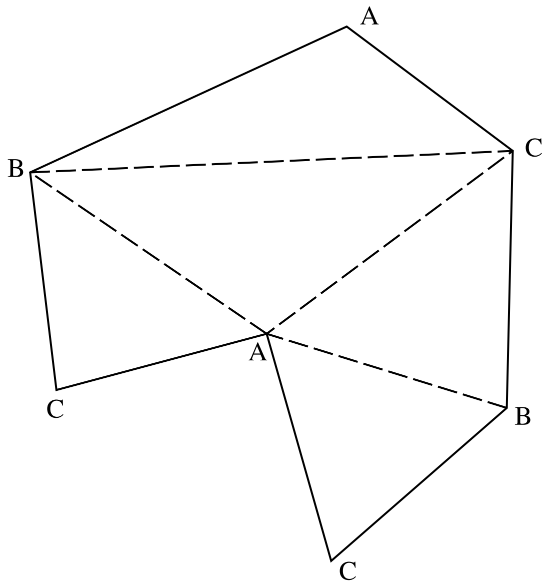


(ii) In order to use the minimum number of cameras, show on the diagram below where your answer to part **(i)** indicates the cameras should be placed. **[1]**



For Examiner's Use

5 With reference to the labelled triangulation shown below, state with a reason whether each of the following statements is true or false.



(i) The triangulation shows that 2 cameras are sufficient. **[2]**

....

Reason: ....

....

(ii) The triangulation shows that 2 cameras are necessary. **[2]**

....

Reason: ....

....

For Examiner's Use

6 On lines 96 and 97 it says "If the cameras did not need to be mounted on the walls, but could be positioned further away from the building, then fewer cameras would usually suffice."

Using the diagram below, which shows a pentagon and one external camera, C, indicate by shading the region in which a second camera must be positioned so that all the walls could be observed by the two cameras. **[2]**