

4754 (C4) Applications of Advanced Mathematics

Section A

<p>1</p> $\frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ $\Rightarrow 3x+2 = A(x^2+1) + (Bx+C)x$ $x=0 \Rightarrow 2=A$ <p>coefft of x^2: $0 = A + B \Rightarrow B = -2$</p> <p>coefft of x: $3 = C$</p> $\Rightarrow \frac{3x+2}{x(x^2+1)} = \frac{2}{x} + \frac{3-2x}{x^2+1}$	<p>M1 M1 B1 M1 A1</p> <p>A1</p> <p>[6]</p>	<p>correct partial fractions</p> <p>equating coefficients at least one of B, C correct</p>
<p>2(i)</p> $(1+2x)^{1/3} = 1 + \frac{1}{3} \cdot 2x + \frac{\frac{1}{3} \cdot (-\frac{2}{3})}{2!} (2x)^2 + \dots$ $= 1 + \frac{2}{3}x - \frac{2}{18}4x^2 + \dots$ $= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \dots *$ <p>Next term $= \frac{\frac{1}{3} \cdot (-\frac{2}{3}) \cdot (-\frac{5}{3})}{3!} (2x)^3$</p> $= \frac{40}{81}x^3$ <p>Valid for $-1 < 2x < 1$</p> $\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	<p>M1 A1</p> <p>E1</p> <p>M1 A1</p> <p>B1 [6]</p>	<p>binomial expansion correct unsimplified expression</p> <p>simplification</p> <p>www</p>
<p>3</p> $4\mathbf{j} - 3\mathbf{k} = \lambda \mathbf{a} + \mu \mathbf{b}$ $= \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mu(4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow 0 = 2\lambda + 4\mu$ $4 = \lambda - 2\mu$ $-3 = -\lambda + \mu$ $\Rightarrow \lambda = -2\mu, 2\lambda = 4 \Rightarrow \lambda = 2, \mu = -1$	<p>M1 M1 A1</p> <p>A1, A1 [5]</p>	<p>equating components at least two correct equations</p>
<p>4</p> $\text{LHS} = \cot \beta - \cot \alpha$ $= \frac{\cos \beta}{\sin \beta} - \frac{\cos \alpha}{\sin \alpha}$ $= \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \sin \beta}$ $= \frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta}$ <p>OR</p> $\text{RHS} = \frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \sin \beta}$ $= \cot \beta - \cot \alpha$	<p>M1</p> <p>M1</p> <p>E1</p> <p>M1 M1 E1 [3]</p>	<p>cot = cos / sin</p> <p>combining fractions</p> <p>www</p> <p>using compound angle formula splitting fractions using cot=cos/sin</p>

<p>5(i) Normal vectors $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$</p> <p>Angle between planes is θ, where</p> $\cos \theta = \frac{2 \times 1 + (-1) \times 0 + 1 \times (-1)}{\sqrt{2^2 + (-1)^2 + 1^2} \sqrt{1^2 + 0^2 + (-1)^2}}$ $= 1/\sqrt{12}$ <p>$\Rightarrow \theta = 73.2^\circ$ or 1.28 rads</p>	<p>B1</p> <p>M1 M1</p> <p>A1 [4]</p>	<p>scalar product finding invcos of scalar product divided by two modulae</p>
<p>(ii) $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$</p> $= \begin{pmatrix} 2+2\lambda \\ -\lambda \\ 1+\lambda \end{pmatrix}$ <p>$\Rightarrow 2(2+2\lambda) - (-\lambda) + (1+\lambda) = 2$</p> <p>$\Rightarrow 5 + 6\lambda = 2$</p> <p>$\Rightarrow \lambda = -1/2$</p> <p>So point of intersection is $(1, 1/2, 1/2)$</p>	<p>B1</p> <p>M1</p> <p>A1 A1 [4]</p>	
<p>6(i) $\cos \theta + \sqrt{3} \sin \theta = r \cos(\theta - \alpha)$</p> $= R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$ <p>$\Rightarrow R \cos \alpha = 1, R \sin \alpha = \sqrt{3}$</p> <p>$\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$</p> <p>$\tan \alpha = \sqrt{3}$</p> <p>$\Rightarrow \alpha = \pi/3$</p>	<p>B1 M1</p> <p>M1 A1 [4]</p>	<p>$R = 2$ equating correct pairs</p> <p>$\tan \alpha = \sqrt{3}$ o.e.</p>
<p>(ii) derivative of $\tan \theta$ is $\sec^2 \theta$</p> $\int_0^{\pi/3} \frac{1}{(\cos \theta + \sqrt{3} \sin \theta)^2} d\theta = \int_0^{\pi/3} \frac{1}{4} \sec^2(\theta - \frac{\pi}{3}) d\theta$ $= \left[\frac{1}{4} \tan(\theta - \frac{\pi}{3}) \right]_0^{\pi/3}$ $= 1/4 (0 - (-\sqrt{3}))$ $= \sqrt{3}/4 *$	<p>B1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>[4]</p>	<p>ft their α</p> <p>$\frac{1}{R^2} [\tan(\theta - \pi/3)]$ ft their R, α (in radians)</p> <p>www</p>

4754

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Section B

<p>7(i) (A) $9 / 1.5 = 6$ hours (B) $18/1.5 = 12$ hours</p>	<p>B1 B1 [2]</p>	
<p>(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k dt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$</p>	<p>M1 A1 A1 M1 E1 [5]</p>	<p>separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with c) $A=e^c$</p>
<p>(iii) $98 = 50 + Ae^0$ $\Rightarrow A = 48$ Initially $\frac{d\theta}{dt} = -k(98 - 50) = -48k = -1.5$ $\Rightarrow k = 0.03125^*$</p>	<p>M1 A1 M1 E1 [4]</p>	
<p>(iv) (A) $89 = 50 + 48e^{-0.03125t}$ $\Rightarrow 39/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(39/48)/(-0.03125) = 6.64$ hours (B) $80 = 50 + 48e^{-0.03125t}$ $\Rightarrow 30/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(30/48)/(-0.03125) = 15$ hours</p>	<p>M1 M1 A1 M1 A1 [5]</p>	<p>equating taking lns correctly for either</p>
<p>(v) Models disagree more for greater temperature loss</p>	<p>B1 [1]</p>	

<p>8(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin \theta, \frac{dx}{d\theta} = 2\cos \theta$</p> $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{dy}{dx} = \frac{2\cos 2\theta - 2\sin \theta}{2\cos \theta} = \frac{\cos 2\theta - \sin \theta}{\cos \theta}$	<p>B1, B1</p> <p>M1</p> <p>A1 [4]</p>	<p>substituting for theirs</p> <p>oe</p>
<p>(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos \pi/3 - \sin \pi/6}{\cos \pi/6}$</p> $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ <p>Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$</p> <p>BC = $2 \times 3\sqrt{3}/2 = 3\sqrt{3}$</p>	<p>E1</p> <p>M1 A1, A1</p> <p>B1ft [5]</p>	<p>for either exact</p>
<p>(iii) (A) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta^*$</p> <p>(B) $\sin\theta = \frac{1}{2}(x-2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - \frac{1}{4}(x-2)^2$ $= 1 - \frac{1}{4}x^2 + x - 1$ $= (x - \frac{1}{4}x^2)^*$</p> <p>(C) Cartesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - \frac{1}{4}x^2)$ $= x^3 - \frac{1}{4}x^4^*$</p>	<p>M1</p> <p>E1</p> <p>B1 M1</p> <p>E1</p> <p>M1</p> <p>E1 [7]</p>	<p>$\sin 2\theta = 2\sin\theta\cos\theta$</p> <p>squaring and substituting for x</p>
<p>(iv) $V = \int_0^4 \pi y^2 dx$</p> $= \pi \int_0^4 (x^3 - \frac{1}{4}x^4) dx$ $= \pi \left[\frac{1}{4}x^4 - \frac{1}{20}x^5 \right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2 \text{ (m}^3\text{)}$	<p>M1</p> <p>B1</p> <p>A1 [3]</p>	<p>need limits</p> $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5 \right]$ <p>12.8π or 40 or better.</p>

4754

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Comprehension

1	$\frac{400\pi d}{1000} = 10$ $d = \frac{25}{\pi} = 7.96$	M1 E1	
2	$V = \pi 20^2 h + \frac{1}{2}(\pi 20^2 H - \pi 20^2 h)$ $= \frac{1}{2}(\pi 20^2 H + \pi 20^2 h) \text{ cm}^3 = 200\pi(H + h) \text{ cm}^3$ $= \frac{1}{5}\pi(H + h) \text{ litres}$	M1 M1 E1	divide by 1000
3	$H = 5 + 40 \tan 30^\circ \text{ or } H = h + 40 \tan \theta$ $V = \frac{1}{5}\pi(H + h) = \frac{1}{5}\pi(10 + 40 \tan 30^\circ)$ $= 20.8 \text{ litres}$	B1 M1 A1	or evaluated including substitution of values
4	$V = \frac{1}{2} \times 80 \times (40 + 5)$ $\times 30 \text{ cm}^3 = 54\,000 \text{ cm}^3$ $= 54 \text{ litres}$	M1 M1 A1	$\times 30$
5	<p>(i) Accurate algebraic simplification to give $y^2 - 160y + 400 = 0$</p> <p>(ii) Use of quadratic formula (or other method) to find other root: $d = 157.5 \text{ cm}$. This is greater than the height of the tank so not possible</p>	B1 M1 A1 E1	
6	$y = 10$ Substitute for y in (4): $V = \frac{1}{1000} \int_0^{100} 375 dx$ $V = \frac{1}{1000} \times 37500 = 37.5 *$	B1 M1 E1	
		[18]	