

4754 (C4) Applications of Advanced Mathematics

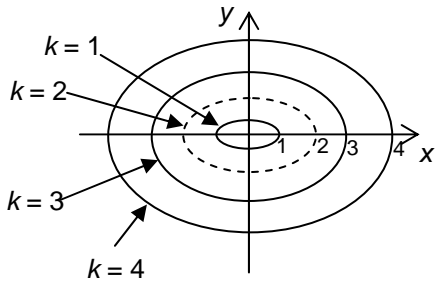
Section A

<p>1 $3 \cos \theta + 4 \sin \theta = R \cos(\theta - \alpha)$ $= R(\cos \theta \cos \alpha + \sin \theta \sin \alpha)$ $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$ $\tan \alpha = 4/3 \Rightarrow \alpha = 0.9273$</p> <p>$5 \cos(\theta - 0.9273) = 2$ $\Rightarrow \cos(\theta - 0.9273) = 2/5$ $\theta - 0.9273 = 1.1593, -1.1593$ $\Rightarrow \theta = 2.087, -0.232$</p>	<p>M1 B1 M1A1</p> <p>M1 A1 A1 [7]</p>	<p>$R = 5$ cwo</p> <p>and no others in the range</p>
<p>2(i) $(1-2x)^{-\frac{1}{2}} = 1 - \frac{1}{2}(-2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-2x)^2 + \dots$ $= 1 + x + \frac{3}{2}x^2 + \dots$ Valid for $-1 < -2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$</p>	<p>M1 A1</p> <p>A1 M1 A1 [5]</p>	<p>binomial expansion with $p = -\frac{1}{2}$ correct expression</p> <p>cao</p>
<p>(ii) $\frac{1+2x}{\sqrt{1-2x}} = (1+2x)(1+x+\frac{3}{2}x^2+\dots)$ $= 1+x+\frac{3}{2}x^2+2x+2x^2+\dots$ $= 1+3x+\frac{7}{2}x^2+\dots$</p>	<p>M1</p> <p>A1ft</p> <p>A1 [3]</p>	<p>substituting their $1+x+\frac{3}{2}x^2+\dots$ and expanding</p> <p>cao</p>
<p>3 $V = \int_1^2 \pi x^2 dy$ $y = 1 + x^2 \Rightarrow x^2 = y - 1$ $\Rightarrow V = \int_1^2 \pi(y-1) dy$ $= \pi \left[\frac{1}{2}y^2 - y \right]_1^2$ $= \pi(2 - 2 - \frac{1}{2} + 1)$ $= \frac{1}{2} \pi$</p>	<p>B1</p> <p>M1</p> <p>B1</p> <p>M1 A1 [5]</p>	<p>$\left[\frac{1}{2}y^2 - y \right]$</p> <p>substituting limits into integrand</p>

<p>4(i) $\sin(\theta + 45^\circ) = \cos \theta$ $\Rightarrow \sin \theta \cos 45 + \cos \theta \sin 45 = \cos \theta$ $\Rightarrow (1/\sqrt{2}) \sin \theta + (1/\sqrt{2}) \cos \theta = \cos \theta$ $\Rightarrow \sin \theta + \cos \theta = \sqrt{2} \cos \theta$ $\Rightarrow \sin \theta = (\sqrt{2} - 1) \cos \theta$ $\Rightarrow \frac{\sin \theta}{\cos \theta} = \tan \theta = \sqrt{2} - 1 *$</p>	<p>M1 B1 A1 M1 E1 [5]</p>	<p>compound angle formula $\sin 45 = 1/\sqrt{2}$, $\cos 45 = 1/\sqrt{2}$ collecting terms</p>
<p>(ii) $\tan \theta = \sqrt{2} - 1$ $\Rightarrow \theta = 22.5^\circ,$ 202.5°</p>	<p>B1 B1 [2]</p>	<p>and no others in the range</p>
<p>5 $\frac{4}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$ $= \frac{A(x^2 + 4) + (Bx + C)x}{x(x^2 + 4)}$ $\Rightarrow 4 = A(x^2 + 4) + (Bx + C)x$ $x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$ coefft of x^2: $0 = A + B \Rightarrow B = -1$ coeffts of x: $0 = C$ $\Rightarrow \frac{4}{x(x^2 + 4)} = \frac{1}{x} - \frac{x}{x^2 + 4}$</p>	<p>M1 M1 B1 DM1 A1 A1 [6]</p>	<p>correct partial fractions A=1 Substitution or equating coeffts B= -1 C= 0</p>
<p>6 $\operatorname{cosec} \theta = 3$ $\Rightarrow \sin \theta = 1/3$ $\Rightarrow \theta = 19.47^\circ,$ 160.53°</p>	<p>M1 A1 A1 [3]</p>	<p>and no others in the range</p>

Section B

<p>7(i) $\overline{CD} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}$ $\overline{CB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$.</p>	<p>B1 B1 [2]</p>	
<p>(ii) $\sqrt{(-6)^2 + 6^2 + 24^2}$ = 25.46 cm</p>	<p>M1 A1 [2]</p>	
<p>(iii) $\overline{CD} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = -24 + 0 + 24 = 0$ $\overline{CB} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 0 + 0 + 0 = 0$ \Rightarrow plane BCDE is $4x + z = c$ At C (say) $4 \times 15 + 0 = c \Rightarrow c = 60$ \Rightarrow plane BCDE is $4x + z = 60$</p>	<p>M1 A1 B1 M1 A1 [5]</p>	<p>using scalar product or other equivalent methods</p>
<p>(iv) OG: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 24 \end{pmatrix}$ AF: $\mathbf{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$ At (5, 10, 40), $3\lambda = 5 \Rightarrow \lambda = 5/3$ $\Rightarrow 6\lambda = 10, 24\lambda = 40$, so consistent. At (5, 10, 40), $3\mu = 5 \Rightarrow \mu = 5/3$ $\Rightarrow 20 - 6\mu = 10, 24\mu = 40$, so consistent. So lines meet at (5, 10, 40)*</p>	<p>B1 B1 M1 E1 E1 [5]</p>	<p>evaluating parameter and checking consistency. [or other methods, e.g. solving]</p>
<p>(v) $h=40$ POABC: $V = 1/3 \times 20 \times 15 \times 40$ $= 4000 \text{ cm}^3$. PDEFG: $V = 1/3 \times 8 \times 6 \times (40-24)$ $= 256 \text{ cm}^3$ \Rightarrow vol of ornament = $4000 - 256 = 3744 \text{ cm}^3$</p>	<p>B1 M1 A1 A1 [4]</p>	<p>soi $1/3 \times w \times d \times h$ used for either –condone one error both volumes correct cao</p>

<p>8(i) $\cos \theta = \frac{x}{k}, \sin \theta = \frac{2y}{k}$ $\cos^2 \theta + \sin^2 \theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2 *$</p>	<p>M1 M1 E1 [3]</p>	<p>Used substitution</p>
<p>(ii) $\frac{dx}{d\theta} = -k \sin \theta, \frac{dy}{d\theta} = \frac{1}{2}k \cos \theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k \cos \theta}{k \sin \theta}$ $= -\frac{1}{2} \cot \theta$ $-\frac{x}{4y} = -\frac{2k \cos \theta}{4k \sin \theta} = -\frac{1}{2} \cot \theta = \frac{dy}{dx}$</p>	<p>M1 A1 E1</p>	<p>oe</p>
<p>or, by differentiating implicitly $2x + 8y \, dy/dx = 0$ $\Rightarrow \, dy/dx = -2x/8y = -x/4y*$</p>	<p>M1 A1 E1 [3]</p>	
<p>(iii) $k = 2$</p>	<p>B1 [1]</p>	
<p>(iv)</p> 	<p>B1 B1 B1 [3]</p>	<p>1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct</p>
<p>(v) grad of stream path = $-1/\text{grad of contour}$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x} *$</p>	<p>M1 E1 [2]</p>	
<p>(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int \frac{dy}{y} = \int \frac{4dx}{x}$ $\Rightarrow \ln y = 4 \ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$.</p> <p>When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16 *$</p>	<p>M1 A1 M1 M1 A1 E1 [6]</p>	<p>Separating variables $\ln y = 4 \ln x (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant</p> <p>www</p>

Paper B Comprehension 4754 (C4)

1	4, 1, 5, 6, 11, 17	B1 B1	for 11 and 17 for 1 and 4
2	Even, odd, odd, even, odd, odd recurs 100 th term is therefore even	M1 A1	for reason www
3	$\phi^6 = (3\phi + 2) + (5\phi + 3) = 8\phi + 5$	B1	
4	$1 - EH = 1 - CG = 1 - (\phi - 1)$ $= 2 - \phi = 2 - \left(\frac{1 + \sqrt{5}}{2}\right)$ $= \frac{3 - \sqrt{5}}{2}$	M1 A1 A1	oe
5	(i) Gradients $-\frac{1}{\phi}$ and $\frac{1}{\phi - 1}$ (ii) Product of gradients: $-\frac{1}{\phi} \times \frac{1}{\phi - 1} = -\frac{1}{\phi^2 - \phi}$ $= -\frac{1}{1} = -1$	B1 B1 M1 E1	
6	$\frac{\phi + 1}{2\phi - 1} = \frac{\frac{1 + \sqrt{5}}{2} + 1}{1 + \sqrt{5} - 1}$ $= \frac{3 + \sqrt{5}}{2\sqrt{5}}$ $= \frac{(3 + \sqrt{5})\sqrt{5}}{2\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5} + 5}{10}$	M1 A1 E1	
7	$a + (a + d) = a + 2d \Rightarrow a = d$ $(a + d) + (a + 2d) = a + 3d \Rightarrow a = 0$ $a = d = 0$ *	M1 M1 E1 [18]	