OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4754(A)/01**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper A

TUESDAY 23 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Solve the equation $\frac{1}{x} + \frac{x}{x+2} = 1$. [4]

2 Fig. 2 shows part of the curve $y = \sqrt{1+x^3}$.

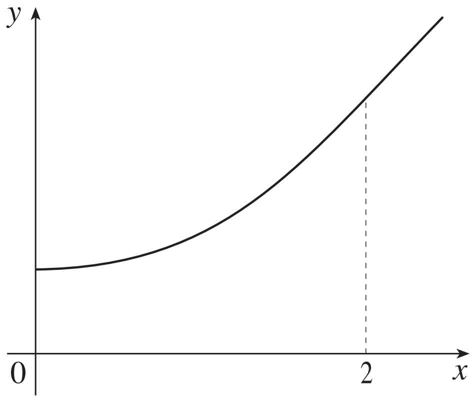

Fig. 2

(i) Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{1+x^3}\,dx$, giving your answer correct to 3 significant figures. [3]

(ii) Chris and Dave each estimate the value of this integral using the trapezium rule with 8 strips. Chris gets a result of 3.25, and Dave gets 3.30. One of these results is correct. Without performing the calculation, state with a reason which is correct. [2]

3 **(i)** Use the formula for $\sin(\theta + \phi)$, with $\theta = 45°$ and $\phi = 60°$, to show that $\sin 105° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$. [4]

(ii) In triangle ABC, angle BAC = 45°, angle ACB = 30° and AB = 1 unit (see Fig. 3).

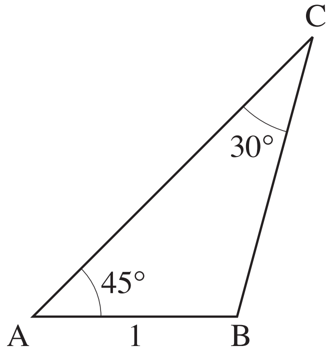

Fig. 3

Using the sine rule, together with the result in part (**i**), show that $\text{AC} = \dfrac{\sqrt{3}+1}{\sqrt{2}}$. [3]

4 Show that $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \sec 2\theta$.

Hence, or otherwise, solve the equation $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = 2$, for $0° \leqslant \theta \leqslant 180°$. [7]

5 Find the first four terms in the binomial expansion of $(1+3x)^{\frac{1}{3}}$.

State the range of values of x for which the expansion is valid. [5]

6 **(i)** Express $\dfrac{1}{(2x+1)(x+1)}$ in partial fractions. [3]

(ii) A curve passes through the point $(0, 2)$ and satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{(2x+1)(x+1)}.$$

Show by integration that $y = \dfrac{4x+2}{x+1}$. [5]

Section B (36 marks)

7 Fig. 7 shows the curve with parametric equations

$$x = \cos\theta,\ y = \sin\theta - \tfrac{1}{8}\sin 2\theta,\ 0 \leqslant \theta < 2\pi.$$

The curve crosses the x-axis at points $A(1, 0)$ and $B(-1, 0)$, and the positive y-axis at C. D is the maximum point of the curve, and E is the minimum point.

The solid of revolution formed when this curve is rotated through 360° about the x-axis is used to model the shape of an egg.

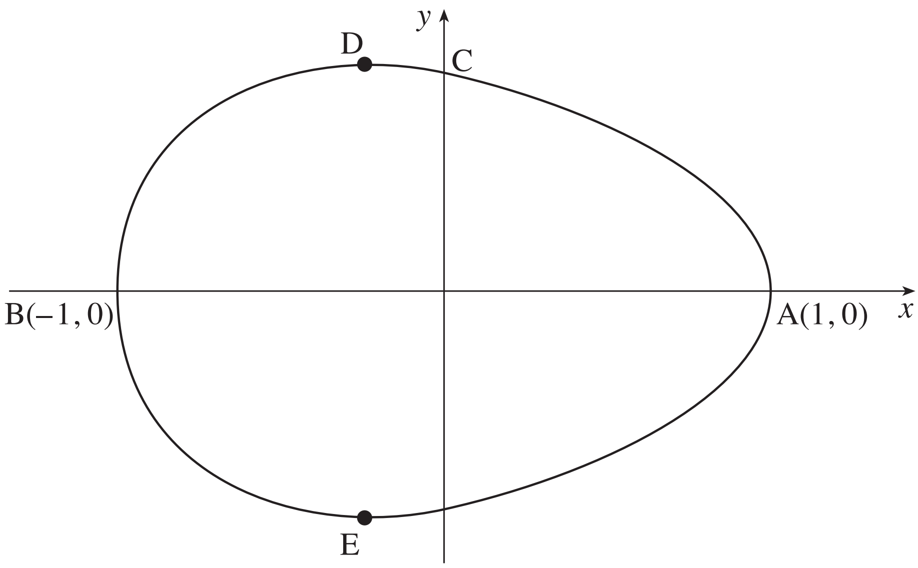

Fig. 7

(i) Show that, at the point A, $\theta = 0$. Write down the value of θ at the point B, and find the coordinates of C. [4]

(ii) Find $\dfrac{dy}{dx}$ in terms of θ.

Hence show that, at the point D,

$$2\cos^2\theta - 4\cos\theta - 1 = 0. \qquad [5]$$

(iii) Solve this equation, and hence find the y-coordinate of D, giving your answer correct to 2 decimal places. [5]

The cartesian equation of the curve (for $0 \leqslant \theta \leqslant \pi$) is

$$y = \tfrac{1}{4}(4 - x)\sqrt{1 - x^2}.$$

(iv) Show that the volume of the solid of revolution of this curve about the x-axis is given by

$$\tfrac{1}{16}\pi \int_{-1}^{1} \left(16 - 8x - 15x^2 + 8x^3 - x^4\right) dx.$$

Evaluate this integral. [6]

8 A pipeline is to be drilled under a river (see Fig. 8). With respect to axes Oxyz, with the x-axis pointing East, the y-axis North and the z-axis vertical, the pipeline is to consist of a straight section AB from the point A$(0, -40, 0)$ to the point B$(40, 0, -20)$ directly under the river, and another straight section BC. All lengths are in metres.

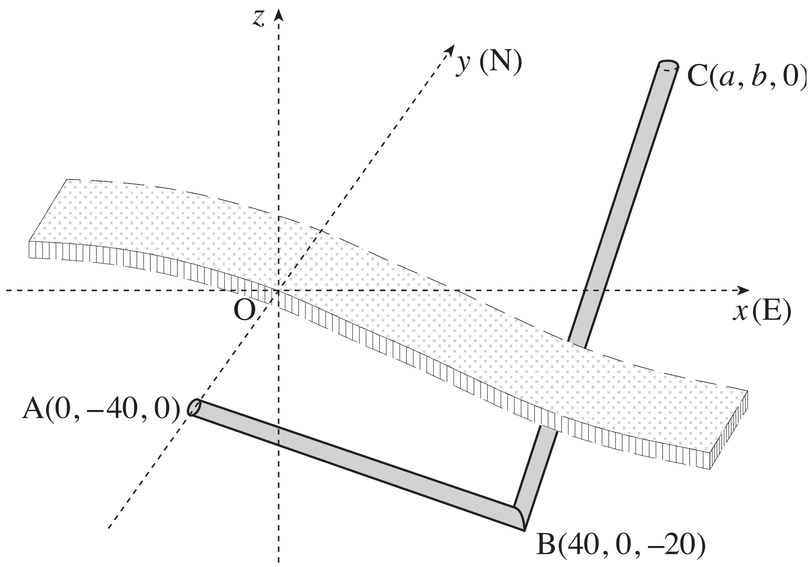

Fig. 8

(i) Calculate the distance AB. [2]

The section BC is to be drilled in the direction of the vector $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

(ii) Find the angle ABC between the sections AB and BC. [4]

The section BC reaches ground level at the point C$(a, b, 0)$.

(iii) Write down a vector equation of the line BC. Hence find a and b. [5]

(iv) Show that the vector $6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane ABC. Hence find the cartesian equation of this plane. [5]