Mark Scheme 4754
January 2007

Paper A – Section A

1 $\frac{1}{x}+\frac{x}{x+2}=1$ $\Rightarrow$ $x+2+x^2=x(x+2)$ $=x^2+2x$ $\Rightarrow$ $x=2$	M1 A1 DM1 A1 [4]	Clearing fractions solving cao
2(i) x: 0, 0.5, 1, 1.5, 2 y: 1, 1.0607, 1.4142, 2.0917, 3 $A \approx 0.5[(1+3)/2 + 1.0607 + 1.4142 + 2.0917]$ $= 3.28$ (3 s.f.)	B1 M1 A1 [3]	At least one value calculated correctly or 13.13…or 6.566… seen
(ii) 3.25 (or Chris) area should decrease with the number of strips used.	B1 B1 [2]	ft (i) or area should decrease as concave upwards
3(i) sin 60 = √3/2, cos 60 = 1/2 , sin 45 = 1/√2, cos 45 = 1/√2 sin(105°) = sin(60°+45°) = sin60°cos45° + cos60°sin45° $=\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}+\frac{1}{2}.\frac{1}{\sqrt{2}}$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}$ *	M1 M1 A1 E1 [4]	 splitting into 60° and 45°, and using the compound angle formulae
(ii) Angle B = 105° By the sine rule: $\frac{AC}{\sin B}=\frac{1}{\sin 30}$ $\Rightarrow$ AC = $\frac{\sin 105}{\sin 30}=\frac{\sqrt{3}+1}{2\sqrt{2}}.2$ $=\frac{\sqrt{3}+1}{\sqrt{2}}$ *	 M1 A1 E1 [3]	 Sine rule with exact values www
4 $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ $=\frac{1}{\cos 2\theta}$ $=\sec 2\theta$ $\sec 2\theta = 2 \Rightarrow \cos 2\theta = ½$ $\Rightarrow$ $2\theta = 60°, 300°$ $\Rightarrow$ $\theta = 30°, 150°$	M1 M1 M1 E1 M1 B1 B1 [7]	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $1+\tan^2\theta=\sec^2\theta$ used simplifying to a simple fraction in terms of sinθ and/or cos θ only $\cos^2\theta-\sin^2\theta=\cos 2\theta$ oe used or $1+\tan^2\theta = 2(1-\tan^2\theta) \Rightarrow \tan\theta = \pm 1/\sqrt{3}$ oe 30° 150° and no others in range

5 $(1+3x)^{\frac{1}{3}} =$ $= 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(3x)^3+...$ $=1+x-x^2+\frac{5}{3}x^3+...$ Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	M1 B1 A2,1,0 B1 [5]	binomial expansion (at least 3 terms) correct binomial coefficients (all) x, $-x^2$, $5x^3/3$
6(i) $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$ $\Rightarrow$ $1 = A(x+1) + B(2x+1)$ $x = -1$: $1 = -B \Rightarrow B = -1$ $x = -½$: $1 = ½ A \Rightarrow A = 2$	M1 A1 A1 [3]	or cover up rule for either value
(ii) $\frac{dy}{dx}=\frac{y}{(2x+1)(x+1)}$ $\Rightarrow$ $\int\frac{1}{y}dy=\int\frac{1}{(2x+1)(x+1)}dx$ $=\int(\frac{2}{2x+1}-\frac{1}{x+1})dx$ $\Rightarrow$ $\ln y = \ln(2x+1) - \ln(x+1) + c$ When $x = 0$, $y = 2$ $\Rightarrow$ $\ln 2 = \ln 1 - \ln 1 + c \Rightarrow c = \ln 2$ $\Rightarrow$ $\ln y = \ln(2x+1) - \ln(x+1) + \ln 2$ $=\ln\frac{2(2x+1)}{x+1}$ $\Rightarrow$ $y=\frac{4x+2}{x+1}$ *	M1 A1 B1ft M1 E1 [5]	separating variables correctly condone omission of c. ft A,B from (i) calculating c , no incorrect log rules combining lns www

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ $= 60$ m	M1 A1 [2]	
(ii) $\overrightarrow{BA} = \begin{pmatrix}-40\\-40\\20\end{pmatrix} = 20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}} = -\dfrac{13}{3\sqrt{26}}$ $\Rightarrow \quad \theta = 148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ -13 oe eg -260 $\sqrt{9}\sqrt{26}$ oe eg $60\sqrt{26}$ cao (or radians)
(iii) $\mathbf{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z = 0 \Rightarrow \lambda = 20$ $\Rightarrow \quad a = 40 + 3 \times 20 = 100$ $b = 0 + 4 \times 20 = 80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….$+\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix} = -12+10+2=0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2=0$ $\Rightarrow \begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x - 5y + 2z = c$ At B (say) $6 \times 40 - 5 \times 0 + 2 \times -20 = c$ $\Rightarrow c = 200$ so $6x - 5y + 2z = 200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)

Paper B Comprehension

<table>
<tr><td>1(i)</td><td>

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	6	4	2	2	2	1	1	1	1

</td><td>B1 Table</td></tr>
<tr><td>(ii)</td><td>

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	7	3	2	3	1	2	1	1	0

</td><td>M1 A1 Table</td></tr>
<tr><td>(iii)</td><td>

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	6.0	3.5	2.5	1.9	1.6	1.3	1.2	1.0	0.9

</td><td>B1any 4 correct
B1 other 4 correct</td></tr>
<tr><td>(iv)</td><td>Any sensible comment such as:
• The general pattern of the frequencies/results is the same for all three tables.
• Due to the small number of data items we cannot expect the pattern to follow Benford's Law very closely.</td><td>E1</td></tr>
<tr><td>2</td><td>Evidence of 4+3+4+2+2 from Table 4 frequencies is the same as 15 in Table 6</td><td>B1</td></tr>
<tr><td>3</td><td>$p_1 = p_3 + p_4 + p_5$: on multiplication by 3, numbers with a leading digit of 1 will be mapped to numbers with a leading digit of 3, 4 or 5 and no other numbers have this property.</td><td>B1 Multiplication

B1… by 3</td></tr>
<tr><td>4</td><td>$\log_{10}(n+1) - \log_{10} n = \log_{10}\left(\frac{n+1}{n}\right) = \log_{10}\left(\frac{n}{n} + \frac{1}{n}\right) = \log_{10}\left(1 + \frac{1}{n}\right)$</td><td>M1
E1</td></tr>
<tr><td>5</td><td>Substitute $\mathrm{L}(4) = 2 \times \mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$ in
$\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$:
this gives $\mathrm{L}(8) = \mathrm{L}(6) - \mathrm{L}(3) + \mathrm{L}(4) = \mathrm{L}(2) + 2 \times \mathrm{L}(2) = 3 \times \mathrm{L}(2)$</td><td>M1
M1 subst
E1
(or alt
M1 for 2 or more Ls used
M1 use of at least 2 given results oe
E1)</td></tr>
<tr><td>6</td><td>$a = 28$. All entries with leading digit 2 or 3 will, on multiplying by 5, have leading digit 1. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td>$b = 9$. Similarly, all entries with leading digit 8 or 9 will, on multiplying by 5, have leading digit 4. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td></td><td>Total 18</td></tr>
</table>

4754 - Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to that of last January. Candidates found it much more straightforward than the June 2006 paper. There was a wide range of responses but all questions were answered well by some candidates. There were some excellent scripts.
Candidates should be advised to read questions carefully. There were instances, particularly in the Comprehension, where instructions were not followed.
There was also some use of inefficient methods. Those that were competent at algebra and surds and were familiar with manipulating trigonometric formulae generally achieved good results. Some of the arithmetic in the trapezium rule and the integration of the polynomial was disappointing.
There was some evidence of shortage of time as a small proportion of candidates failed to complete question 8.

Comments on Individual Questions

Paper A

Section A

1 The algebraic fraction equation was almost always answered correctly.

2 (i) There seemed to be a lack of familiarity with the trapezium rule formula. Common errors were use of A=0.5h(y_0+y_4) +2(y_1+y_2 +y_3) but omission of the other brackets. Or alternatively omitting y_0 and using A=0.5h((y_1+y_4)+2 (y_2 +y_3)). Most obtained at least one ordinate correctly but there were many errors in the calculation of the answer.

(ii) Those without correct, or almost correct, answers in the first part could not make a valid comment about which of Chris or Dave was correct in their calculations. There were some poor explanations given, such as 'the trapezium rule always overestimates results'.

3 Most candidates correctly used the compound angle formula as the first stage. Those that used *sin/cos* 45° as $\sqrt{2}/2$ rather than $1/\sqrt{2}$ could not always deal with cancelling $(\sqrt{6}+\sqrt{2})/4$. The sine rule was usually correct.

4 There were some efficient solutions but weaker candidates found it difficult to see ahead to what was needed. In some cases poor knowledge of trigonometric identities and their rearrangement was the problem. Some tried to work on both sides simultaneously - some more clearly than others.
There were some confused starts using incorrect identities in the second part but many did obtain the first solution. The solution θ=150° was often lost - in some cases due to missing the negative square root.

5 This was well answered. The improvement seen in the binomial expansion was pleasing although this was possibly due to the first number in the bracket being a 1. There were still some candidates who used *x* rather than *3x* throughout the calculation and many could not deal successfully with the range for the validity.

6 The partial fractions were almost always correct.
The second part was less successful. Some separated the variables to *ydy*=…. Many integrated 2/ (2*x*+1) as 2ln (2*x*+1) and there were many instances of the omission of the constant. Poor use of the laws of logarithms meant that c was often not found correctly. For example, lny=ln(2x+1) – ln(x+1) +c leading to y= (2x+1)/(x+1) +c was common. Those that found c before combining their logs were more successful. 2ln(2x+1) – ln(x+1) = ln 2(2x+1)/(x+1) was also a common error.

Section B

7 (i) This was usually correctly answered although some candidates used long methods to show that $\theta=0$ at A and others gave the value of θ at B in degrees.

(ii) There were many errors in *dy/dθ*- usually the coefficient of *cos 2θ* being incorrect - and there were also sign errors. Most knew that they had to equate *dy/dx* to zero but made errors in their simplification to the given equation.

(iii) Some omitted this or tried to factorise and then abandoned the attempt. Of those that did use the formula, a common mistake was to solve the quadratic equation for *cos θ* but then to use this as *θ* in the expression for *y*.

(iv) This was disappointing. The first part was usually correct but a significant number failed to integrate the polynomial. Of those that did integrate, many surprisingly made numerical errors when substituting the limits.

8 (i) Most candidates correctly found the distance AB.

(ii) Many failed to find the required angle ABC.

(iii) This proved to be very successful for many. Those that gave the required vector equation in terms of *a* and *b*, however, could rarely make progress. A few found *a* and *b* successfully without explicitly writing down the equation of the line.

(iv) Once again too many candidates failed to realise that in order to prove that a vector is perpendicular to a plane it is necessary to show it is perpendicular to **two** vectors in the plane. Others did not evaluate their dot product, merely stating it was zero. Most used the Cartesian form of the equation with success. There were still some candidates who approached this from the vector equation of the plane and they were more likely to make errors.

Paper B

Comprehension

1 The tables in (i) and (ii) were usually correct but there were occasional slips. In (iii) candidates often failed to calculate using Benford's Law. It was unclear what their methods were in (iii) but they may have been trying to use Fig.9.

2 This was often successful but it was not always clear which tables the candidates were referring to.

3 Some failed to explain about the multiplication of leading digits. For those that did, the multiplication factor quoted did not always work for the complete range. Multiplying 3,4 and 5 by 3.5 or 4 was commonly seen.

4 Usually correct although log (n+1) –log n=log(n+1)/log n was seen.

5 The approach encouraged by the question was not always used. There were some very long and often confused solutions involving changing all ‘L’ expressions to strings of ‘p’ equations and eliminating.

6 Candidates often seemed not to have read this question carefully. There were many good solutions, but too often the proportions were calculated rather than using the frequencies in the table.