OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Paper A

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

NOTE

- This paper will be followed by **Paper B: Comprehension.**

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $\dfrac{2x}{x-2} - \dfrac{4x}{x+1} = 3$. [5]

2 A curve is defined parametrically by the equations

$$x = t - \ln t, \qquad y = t + \ln t \qquad (t > 0).$$

Find the gradient of the curve at the point where $t = 2$. [5]

3 A triangle ABC has vertices $A(-2, 4, 1)$, $B(2, 3, 4)$ and $C(4, 8, 3)$. By calculating a suitable scalar product, show that angle ABC is a right angle. Hence calculate the area of the triangle. [6]

4 Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

5 **(i)** Find the cartesian equation of the plane through the point $(2, -1, 4)$ with normal vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$ [3]

(ii) Find the coordinates of the point of intersection of this plane and the straight line with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 12 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$ [4]

6 **(i)** Find the first three non-zero terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^2}}$ for $|x| < 2$. [4]

(ii) Use this result to find an approximation for $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [2]

(iii) Given that $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\tfrac{1}{2}x\right) + c$, evaluate $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [1]

Section B (36 marks)

7 In a game of rugby, a kick is to be taken from a point P (see Fig. 7). P is a perpendicular distance y metres from the line TOA. Other distances and angles are as shown.

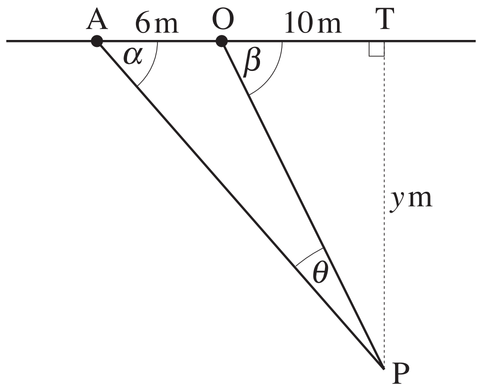

Fig. 7

(i) Show that $\theta = \beta - \alpha$, and hence that $\tan\theta = \dfrac{6y}{160 + y^2}$.

Calculate the angle θ when $y = 6$. [8]

(ii) By differentiating implicitly, show that $\dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$. [5]

(iii) Use this result to find the value of y that maximises the angle θ. Calculate this maximum value of θ. [You need not verify that this value is indeed a maximum.] [4]

[Question 8 is printed overleaf.]

8 Some years ago an island was populated by red squirrels and there were no grey squirrels. Then grey squirrels were introduced.

The population x, in thousands, of red squirrels is modelled by the equation

$$x = \frac{a}{1 + kt},$$

where t is the time in years, and a and k are constants. When $t = 0$, $x = 2.5$.

(i) Show that $\frac{dx}{dt} = -\frac{kx^2}{a}$. [3]

(ii) Given that the initial population of 2.5 thousand red squirrels reduces to 1.6 thousand after one year, calculate a and k. [3]

(iii) What is the long-term population of red squirrels predicted by this model? [1]

The population y, in thousands, of grey squirrels is modelled by the differential equation

$$\frac{dy}{dt} = 2y - y^2.$$

When $t = 0$, $y = 1$.

(iv) Express $\frac{1}{2y - y^2}$ in partial fractions. [4]

(v) Hence show by integration that $\ln\left(\frac{y}{2-y}\right) = 2t$.

Show that $y = \frac{2}{1 + e^{-2t}}$. [7]

(vi) What is the long-term population of grey squirrels predicted by this model? [1]

Candidate Name	Centre Number	Candidate Number

OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(B)

Applications of Advanced Mathematics (C4)

Paper B: Comprehension

Monday **23 JANUARY 2006** Afternoooon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces at the top of this page.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this section is 18.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
Total	

This question paper consists of 5 printed pages, 3 blank pages and an insert.

For Examiner's Use

1 Line 59 says "Again Party G just misses out; if there had been 7 seats G would have got the last one."

Where is the evidence for this in the article? [1]

....

2 6 parties, P, Q, R, S, T and U take part in an election for 7 seats. Their results are shown in the table below.

Party	**Votes (%)**
P	30.2
Q	11.4
R	22.4
S	14.8
T	10.9
U	10.3

(i) Use the Trial-and-Improvement method, starting with values of 10% and 14%, to find an acceptance percentage for 7 seats, and state the allocation of the seats. [4]

Acceptance percentage, *a*%		**10%**	**14%**			
Party	**Votes (%)**	**Seats**	**Seats**	**Seats**	**Seats**	**Seats**
P	30.2					
Q	11.4					
R	22.4					
S	14.8					
T	10.9					
U	10.3					
Total seats						

Seat Allocation P .... Q .... R .... S .... T .... U ....

For Examiner's Use

(ii) Now apply the d'Hondt Formula to the same figures to find the allocation of the seats. [5]

	Round							
Party	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Residual**
P	30.2							
Q	11.4							
R	22.4							
S	14.8							
T	10.9							
U	10.3							
Seat allocated to								

Seat Allocation P .... Q .... R .... S .... T .... U ....

3 In this question, use the figures for the example used in Table 5 in the article, the notation described in the section "Equivalence of the two methods" and the value of 11 found for *a* in Table 4.

Treating Party E as Party 5, verify that $\frac{V_5}{N_5 + 1} < a \leqslant \frac{V_5}{N_5}$. [2]

....

....

....

....

....

[Turn over

For Examiner's Use

4 Some of the intervals illustrated by the lines in the graph in Fig. 8 are given in this table.

Seats	Interval	Seats	Interval
1	$22.2 < a \leqslant 27.0$	5	
2	$16.6 < a \leqslant 22.2$	6	$10.6 < a \leqslant 11.1$
3		7	
4			

(i) Describe briefly, giving an example, the relationship between the end-points of these intervals and the values in Table 5, which is reproduced below. [2]

....

....

....

(ii) Complete the table *above*. [1]

	Round						
Party	**1**	**2**	**3**	**4**	**5**	**6**	**Residual**
A	22.2	22.2	11.1	11.1	11.1	11.1	7.4
B	6.1	6.1	6.1	6.1	6.1	6.1	6.1
C	27.0	13.5	13.5	13.5	9.0	9.0	9.0
D	16.6	16.6	16.6	8.3	8.3	8.3	8.3
E	11.2	11.2	11.2	11.2	11.2	5.6	5.6
F	3.7	3.7	3.7	3.7	3.7	3.7	3.7
G	10.6	10.6	10.6	10.6	10.6	10.6	10.6
H	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Seat allocated to	**C**	**A**	**D**	**C**	**E**	**A**	

Table 5

For Examiner's Use

5 The ends of the vertical lines in Fig. 8 are marked with circles. Those at the tops of the lines are filled in, e.g. ●, whereas those at the bottom are not, e.g. ○.

(i) Relate this distinction to the use of inequality signs. [1]

....

(ii) Show that the inequality on line 102 can be rearranged to give $0 \leqslant V_k - N_k a < a$. [1]

....

....

....

....

(iii) Hence justify the use of the inequality signs in line 102. [1]

....

....

....

....