Mark Scheme 4754
June 2005

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓
   - For error in follow through work, use ✓

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not been earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

c.a.o.	: correct answer only
b.o.d.	: benefit of doubt (where full work is not shown)
X ⎮ X	: work of no mark value between crosses
s.o.i.	: seen or implied
s.c.	: special case (as defined in the mark scheme)
w.w.w	: without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting prior to the batch 1 date, send a further sample of about 40 scripts, from complete centres. You should record the marks for these scripts on your marksheets. They will not be returned to you, but you will receive feedback on them. If all is well, you will then be given clearance to send your batch 1 scripts and marksheets to Cambridge.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

SECTION A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25,\ R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.927$ $f(\theta) = 7 + 5\cos(\theta - 0.927)$ $\Rightarrow$ Range is 2 to 12 Greatest value of $\dfrac{1}{7+3\cos\theta+4\sin\theta}$ is ½ .	**B1** **M1** **A1** **M1** **A1** **B1ft** **[6]**	R=5 tan α=4/3 oe ft their R 0.93 or 53.1° or better their cos (θ - 0. 927) = 1 or -1 used (*condone use of graphical calculator)* 2 and 12 seen cao simplified
2 $\sqrt{4+2x} = 2(1+\frac{1}{2}x)^{\frac{1}{2}}$ $= 2\{1+\frac{1}{2}.(\frac{1}{2}x)+\frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(\frac{1}{2}x)^2+\frac{\frac{1}{2}.(-\frac{1}{2}).(-\frac{3}{2})}{3!}(\frac{1}{2}x)^3+...\}$ $= k\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3+...\right)$ $= \left(2+\frac{1}{2}x-\frac{1}{16}x^2+\frac{1}{64}x^3+...\right)$ Valid for $-2 < x < 2$.	**M1** **M1** **A2,1,0** **A1cao** **B1cao** **[6]**	Taking out 4 oe correct binomial coefficients $\frac{1}{4}x, -\frac{1}{32}x^2, +\frac{1}{128}x^3$
3 $\sec^2\theta = 4$ $\Rightarrow \dfrac{1}{\cos^2\theta} = 4$ $\Rightarrow \cos^2\theta = ¼$ $\Rightarrow \cos\theta = ½$ or $-½$ $\Rightarrow \theta = \pi/3, 2\pi/3$ **OR** $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow \tan^2\theta = 3$ $\Rightarrow \tan\theta = \sqrt{3}$ or $-\sqrt{3}$ $\Rightarrow \theta = \pi/3, 2\pi/3$	**M1** **M1** **A1 A1** **M1** **M1** **A1 A1** **[4]**	sec θ = 1/cos θ used ± ½ allow unsupported answers ± √3 allow unsupported answers

4 $V = \int \pi y^2 dx$ $= \int_0^1 \pi(1+e^{-2x})dx$ $= \pi\left[x - \frac{1}{2}e^{-2x}\right]_0^1$ $= \pi(1 - ½ e^{-2} + ½)$ $= \pi(1½ - ½ e^{-2})$	**M1** **M1** **B1** **M1** **A1** **[5]**	Correct formula $k\int_0^1 (1+e^{-2x})dx$ $\left[x - \frac{1}{2}e^{-2x}\right]$ substituting limits. Must see *0* used. Condone omission of π o.e. but must be exact
5 $2\cos 2x = 2(2\cos^2 x - 1) = 4\cos^2 x - 2$ $\Rightarrow 4\cos^2 x - 2 = 1 + \cos x$ $\Rightarrow 4\cos^2 x - \cos x - 3 = 0$ $\Rightarrow (4\cos x + 3)(\cos x - 1) = 0$ $\Rightarrow \cos x = -3/4$ or 1 $\Rightarrow x = 138.6°$ or $221.4°$ or 0	**M1** **M1** **M1dep** **A1** **B1 B1** **B1** **[7]**	Any double angle formula used *getting a quadratic in cos x* attempt to solve for -3/4 and 1 139,221 or better www -1 extra solutions in range
6 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$ $= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$ $= 4$	**M1** **E1** **[2]**	Substituting for *x* and *y* in terms of *t* oe
(ii) EITHER $dx/dt = 1 + 1/t^2$, $dy/dt = 1 - 1/t^2$	**B1**	For both results
$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1-1/t^2}{1+1/t^2}$	**M1**	
$= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ *	**E1**	
OR $2y\frac{dy}{dx} - 2x = 0$	**B1**	
$\Rightarrow \frac{dy}{dx} = \frac{x}{y} = \frac{t-1/t}{t+1/t}$	**M1**	
$= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$	**E1**	
OR $y = \sqrt{(4+x^2)}$, $\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{4+x^2}}$	**B1**	
$= \frac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$ $= \frac{t-1/t}{\sqrt{(t+1/t^2)}} = \frac{t-1/t}{(t+1/t)}$	**M1**	
$= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$	**E1**	
$\Rightarrow$ $dy/dx = 0$ when $t = 1$ or -1 $t = 1, \Rightarrow (0, 2)$ $t = -1 \Rightarrow (0, -2)$	**M1** **A1 A1** **[6]**	

SECTION B

7 (i) $\int \frac{t}{1+t^2}dt = ½\ln(1+t^2)+c$ **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, $du = 2t dt$ $= \int \frac{1/2}{u}du$ $= ½ \ln u + c$ $= ½\ln(1+t^2)+c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k\ln(1+t^2)$ $½\ln(1+t^2)$ [$+c$] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow 1 = A(1+t^2) + (Bt+C)t$ $t = 0 \Rightarrow 1 = A$ coeff^t of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff^t of t $\Rightarrow 0 = C$ $\Rightarrow \frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	**M1** **M1** **A1** **A1** **A1** **[5]**	Equating numerators substituting or equating coeff^{t}s dep 1st M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow \int \frac{1}{M}dM = \int \frac{1}{t(1+t^2)}dt = \int [\frac{1}{t} - \frac{t}{1+t^2}]dt$ $\Rightarrow \ln M = \ln t - ½\ln(1+t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	**M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½\ln(1+t^2) + c$ combining $\ln t$ and $½\ln(1+t^2)$ $K = e^c$ o.e.
(iv) $t = 1$, $M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.
8 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (–15, 20, 30) $\Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow \overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$	**M1**	Scalar product with 1 vector in the plane OR vector x product oe
$\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$	**E1**	
$\Rightarrow$ $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane		
$\Rightarrow$ equation of plane is $2x + 3y + 2z = c$	**M1**	$2x + 3y +2z = c$ **or** an appropriate vector form
At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow$ $c = 2\times0 + 3\times10 +2\times30 = 90$	**M1dep**	substituting to find c **or** completely eliminating parameters
$\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**A1 cao** **[5]**	
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$	**B1**	
$\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$	**M1**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S
$=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$	**A1ft**	**Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
So T is $(-5, 16\frac{2}{3}, 25)$ *	**E1** **[4]**	
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$	**B1,B1**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\ldots \ldots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$
At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$	**M1** **A1**	Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ
1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**E1** **[5]**	oe www

COMPREHENSION

<table>
<tr><td>1. The masses are measured in units.
The ratio is dimensionless</td><td>B1
B1
[2]</td><td></td></tr>
<tr><td>2. Converting from base 5,
$3.03232 = 3 + \frac{0}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5}$
= 3.14144</td><td>M1

A1
[2]</td><td></td></tr>
<tr><td>3.
<table><tr><th>n</th><th>x_n</th></tr><tr><td>0</td><td>0.5</td></tr><tr><td>1</td><td>0.8</td></tr><tr><td>2</td><td>0.512</td></tr><tr><td>3</td><td>0.7995392</td></tr><tr><td>4</td><td>0.5128840565</td></tr><tr><td>5</td><td>0.7994688035</td></tr></table></td><td>B1</td><td>Condone variations in last digits</td></tr>
<tr><td>4.
$\frac{\phi}{1} = \frac{1}{\phi - 1}$
$\Rightarrow \phi^2 - \phi = 1 \;\; \Rightarrow \phi^2 - \phi - 1 = 0$
Using the quadratic formula gives
$\phi = \frac{1 \pm \sqrt{5}}{2}$</td><td>M1

E1</td><td>

Or complete verification B2</td></tr>
<tr><td>5.
$\frac{1}{\phi} = \frac{1}{\frac{1+\sqrt{5}}{2}} = \frac{2}{1+\sqrt{5}}$
$= \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$
$= \frac{2(\sqrt{5}-1)}{(\sqrt{5})^2 - 1} = \frac{2(\sqrt{5}-1)}{4} = \frac{\sqrt{5}-1}{2}$
OR
$\frac{1}{\phi} = \phi - 1$
$= \frac{\sqrt{5}+1}{2} - 1 = \frac{\sqrt{5}-1}{2}$</td><td>M1
M1
E1

M1
M1
E1
[3]</td><td>Must discount ±

Must discount ±
Substituting for ϕ and simplifying</td></tr>
</table>

6. Let $r = \frac{a_{n+1}}{a_n}$ and $r = \frac{a_n}{a_{n-1}}$	**M1**	For either ratio used
$a_{n+1} = 2a_n + 3a_{n-1}$		
dividing through by a_n. $\Rightarrow r = 2 + \frac{3}{r}$	**M1**	$r = 2 + \frac{3}{r}$
$\Rightarrow r^2 - 2r - 3 = 0$ $\Rightarrow (r-3)(r+1) = 0$	**A1**	
$\Rightarrow$ r = 3 (discounting -1)	**E1** **[4]**	SC B2 if dividing terms as far as $a_9/a_8 = a_{10}/a_9$ =3.00
7. The length of the next interval = l, where $\frac{0.0952...}{l} = 4.669...$	**M1**	
$\Rightarrow$ l = 0.0203	**A1**	
So next bifurcation at 3.5437… + 0.0203…≈ 3.564	**DM1** **A1** **[4]**	

Mark Scheme 4754
January 2006

Section A

1 $\frac{2x}{x-2} - \frac{4x}{x+1} = 3$ $\Rightarrow$ $2x(x+1) - 4x(x-2) = 3(x-2)(x+1)$ $\Rightarrow$ $2x^2 + 2x - 4x^2 + 8x = 3x^2 - 3x - 6$ $\Rightarrow$ $0 = 5x^2 - 13x - 6$ $= (5x+2)(x-3)$ $\Rightarrow$ $x = -2/5$ or 3.	M1 M1 A1 M1 A1 cao [5]	Clearing fractions expanding brackets oe factorising or formula
2 $dx/dt = 1 - 1/t$ $dy/dt = 1 + 1/t$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1+\frac{1}{t}}{1-\frac{1}{t}}$ When $t = 2$, $dy/dx = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$	B1 M1 A1 M1 A1 [5]	Either dx/dt or dy/dt soi www
3 $\overrightarrow{BA} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}2\\5\\-1\end{pmatrix}$ $\overrightarrow{BA}.\overrightarrow{BC} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\5\\-1\end{pmatrix} = (-4)\times 2 + 1\times 5 + (-3)\times(-1)$ $= -8 + 5 + 3 = 0$ $\Rightarrow$ angle ABC = 90° Area of triangle = ½ × BA × BC $= \frac{1}{2}\times\sqrt{(-4)^2+1^2+3^2}\times\sqrt{2^2+5^2+(-1)^2}$ $= \frac{1}{2} \times \sqrt{26} \times \sqrt{30}$ = 13.96 sq units	B1 M1 A1 M1 M1 A1 [6]	soi , condone wrong sense scalar product = 0 area of triangle formula oe length formula accept 14.0 and $\sqrt{195}$

4(i) $2\sin 2\theta + \cos 2\theta = 1$ $\Rightarrow 4\sin\theta\cos\theta + 1 - 2\sin^2\theta = 1$ $\Rightarrow 2\sin\theta(2\cos\theta - \sin\theta) = 0$ or $4\tan\theta - 2\tan^2\theta = 0$ $\Rightarrow \sin\theta = 0$ or $\tan\theta = 0$, $\theta = 0°, 180°$ or $2\cos\theta - \sin\theta = 0$ $\Rightarrow \tan\theta = 2$ $\Rightarrow \theta = 63.43°, 243.43°$ OR Using $R\sin(2\theta+\alpha)$ $R=\sqrt{5}$ and $\alpha=26.57°$ $2\theta + 26.57 = \arcsin 1/R$ $\theta = 0°, 180°$ $\theta = 63.43°, 243.43°$	M1 A1 A1 M1 A1, A1 [6] M1 A1 M1 A1 A1,A1 [6]	Using double angle formulae Correct simplification to factorisable or other form that leads to solutions 0° and 180° $\tan\theta = 2$ (-1 for extra solutions in range) (-1 for extra solutions in range)
5 (i) Plane has equation $x - y + 2z = c$ At $(2, -1, 4)$, $2 + 1 + 8 = c$ $\Rightarrow c = 11$. **(ii)** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$ $\Rightarrow 7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$ $\Rightarrow 2\lambda = -2$ $\Rightarrow \lambda = -1$ Coordinates are (6, 9, 7)	B1 M1 A1 M1 M1 A1 A1 [7]	$x - y + 2z = c$ finding c ft their equation from (i) ft their x-y+2z=c cao
6 (i) $\frac{1}{\sqrt{4-x^2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x^2)^{-\frac{1}{2}}$ $= \frac{1}{2}[1 + (-\frac{1}{2})(-\frac{1}{4}x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{1}{4}x^2)^2 + ...]$ $= \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + ...$ **(ii)** $\int_0^1 \frac{1}{\sqrt{4-x^2}}dx \approx \int_0^1 (\frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4)dx$ $= \left[\frac{1}{2}x + \frac{1}{48}x^3 + \frac{3}{1280}x^5\right]_0^1$ $= \frac{1}{2} + \frac{1}{48} + \frac{3}{1280}$ = 0.5232 (to 4 s.f.) **(iii)** $\int_0^1 \frac{1}{\sqrt{4-x^2}}dx = \left[\arcsin\frac{x}{2}\right]_0^1$ $= \pi/6 = 0.5236$	M1 M1 A1 A1 M1ft A1 B1 [7]	 Binomial coeffs correct Complete correct expression inside bracket cao

Section B

7(i) $A\hat{O}P = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \beta = \alpha + \theta$ $\Rightarrow \theta = \beta - \alpha$ $\tan\theta = \tan(\beta - \alpha)$ $= \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ $= \frac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}.\frac{y}{16}}$ $= \frac{16y - 10y}{160 + y^2}$ $= \frac{6y}{160 + y^2}$ * When $y = 6$, $\tan\theta = 36/196$ $\Rightarrow \theta = 10.4°$	M1 M1 E1 M1 A1 E1 M1 A1 cao [8]	Use of sum of angles in triangle OPT and AOP oe SC B1 for $\beta=\alpha+\theta$, $\theta=\beta-\alpha$ no justification Use of Compound angle formula Substituting values for tan α and tan β www accept radians
(ii) $\sec^2\theta \frac{d\theta}{dy} = \frac{(160 + y^2)6 - 6y.2y}{(160 + y^2)^2}$ $= \frac{6(160 + y^2 - 2y^2)}{(160 + y^2)^2}$ $\Rightarrow \frac{d\theta}{dy} = \frac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$ *	M1 M1 A1 A1 E1 [5]	$\sec^2\theta \frac{d\theta}{dy} = \ldots$ quotient rule correct expression simplifying numerator www
(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow y^2 = 160$ $\Rightarrow y = 12.65$ When $y = 12.65$, $\tan\theta = 0.237\ldots$ $\Rightarrow \theta = 13.3°$	M1 A1 M1 A1cao [4]	 oe accept radians

8 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt = a/x - 1$, $t = a/kx - 1/k$ $dt/dx = -a/kx^2$ $\Rightarrow dx/dt = -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y-y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y-y^2} dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}] dy = \int dt$ $\Rightarrow$ $½ \ln y - ½ \ln(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables $½ \ln y - ½ \ln(2 - y)$ ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$

Comprehension

1. It is the largest number in the Residual column in Table 5. **B1**

2. (i)

Acceptance percentage, *a*%		10%	14%	**12%**	**11%**	**10.5%**
Party	**Votes (%)**	**Seats**	**Seats**	**Seats**	**Seats**	**Seats**
P	30.2	**3**	**2**	**2**	**2**	**2**
Q	11.4	**1**	**0**	**0**	**1**	**1**
R	22.4	**2**	**1**	**1**	**2**	**2**
S	14.8	**1**	**1**	**1**	**1**	**1**
T	10.9	**1**	**0**	**0**	**0**	**1**
U	10.3	**1**	**0**	**0**	**0**	**0**
Total seats		**9**	**4**	**4**	**6**	**7**

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

10% & 14% **B1**
Trial **M1**
10.5% ($10.3<x\le10.9$) **A1**
Allocation **A1**

(ii)

	Round							
Party	1	2	3	4	5	6	7	Residual
P	30.2	**15.1**	**15.1**	**10.07**	**10.07**	**10.07**	**10.07**	**10.07**
Q	11.4	**11.4**	**11.4**	**11.4**	**11.4**	**5.7**	**5.7**	**5.7**
R	22.4	**22.4**	**11.2**	**11.2**	**11.2**	**11.2**	**7.47**	**7.47**
S	14.8	**14.8**	**14.8**	**14.8**	**7.4**	**7.4**	**7.4**	**7.4**
T	10.9	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**5.45**
U	10.3	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**
Seat allocated to	**P**	**R**	**P**	**S**	**Q**	**R**	**T**	

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

General method **M1** Round 2 correct **A1** Round 5 correct **A1**(condone minor arithmetic error) Residuals **A1** www Allocation **A1**cso

3. $\frac{11.2}{1+1} < 11 \leq \frac{11.2}{1} \Rightarrow 5.6 < 11 \leq 11.2$

M1, A1

for either or both
M1 only for 5.6<a≤11.2

4. (i) The end-points of the intervals are the largest values in successive columns of Table 5.(or two largest within a column)

B1

So in

2	$16.6 < a \leq 22.2$

22.2 is the largest number in Round 2. 16.6 is the largest number in Round 3.

B1

(ii)

Seats	a		**Seats**	a
1	$22.2 < a \leq 27.0$		5	$11.1 < a \leq 11.2$
2	$16.6 < a \leq 22.2$		6	$10.6 < a \leq 11.1$
3	$13.5 < a \leq 16.6$		7	$9.0 < a \leq 10.6$
4	$11.2 < a \leq 13.5$			

5. (i) ● means $\leq$, ○ means $<$ (greater or less than)
B1

(ii) $\frac{V_k}{N_k+1} < a$ $\quad$ $a \leq \frac{V_k}{N_k}$

$V_k < aN_k + a$ $\quad$ $aN_k \leq V_k$

$V_k - aN_k < a$ $\quad$ $0 \leq V_k - aN_k$

$$0 \leq V_k - aN_k < a$$

B1

(iii) The unused votes may be zero but must be less than a. **B1**

Mark Scheme 4754
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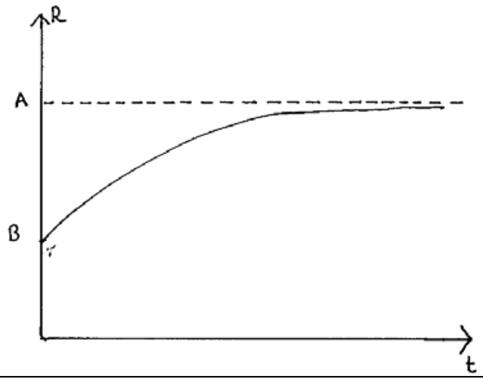
1 $\sin x - \sqrt{3}\cos x = R\sin(x-\alpha)$ $= R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ $\Rightarrow$ $R^2 = 1^2 + (\sqrt{3})^2 = 4$, $R = 2$ $\tan\alpha = \sqrt{3}/1 = \sqrt{3} \Rightarrow \alpha = \pi/3$ $\Rightarrow \sin x - \sqrt{3}\cos x = 2\sin(x - \pi/3)$ x coordinate of P is when $x - \pi/3 = \pi/2$ $\Rightarrow x = 5\pi/6$ $y = 2$ So coordinates are $(5\pi/6, 2)$	B1 M1 A1 M1 A1ft B1ft [6]	$R = 2$ $\tan\alpha = \sqrt{3}$ or $\sin\alpha = \sqrt{3}$/their R or $\cos\alpha = 1$/their R $\alpha = \pi/3$, 60° or 1.05 (or better) radians www Using x-their $\alpha = \pi/2$or 90° $\alpha \neq 0$ exact radians only (not $\pi/2$) their R (exact only)
2(i) $\frac{3+2x^2}{(1+x)^2(1-4x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{1-4x}$ $\Rightarrow$ $3+2x^2 = A(1+x)(1-4x) + B(1-4x) + C(1+x)^2$ $x = -1 \Rightarrow$ $5 = 5B \Rightarrow B = 1$ $x = ¼ \Rightarrow 3\frac{1}{8} = \frac{25}{16}C \Rightarrow C = 2$ coeff[t] of x^2: $2 = -4A + C \Rightarrow A = 0$.	M1 B1 B1 E1 [4]	Clearing fractions (or any 2 correct equations) $B = 1$ www $C = 2$ www $A = 0$ needs justification
(ii) $(1+x)^{-2} = 1 + (-2)x + (-2)(-3)x^2/2! + \ldots$ $= 1 - 2x + 3x^2 + \ldots$ $(1-4x)^{-1} = 1 + (-1)(-4x) + (-1)(-2)(-4x)^2/2! + \ldots$ $= 1 + 4x + 16x^2 + \ldots$ $\frac{3+2x^2}{(1+x)^2(1-4x)} = (1+x)^{-2} + 2(1-4x)^{-1}$ $\approx 1 - 2x + 3x^2 + 2(1 + 4x + 16x^2)$ $= 3 + 6x + 35x^2$	M1 A1 A1 A1ft [4]	Binomial series (coefficients unsimplified - for either) or $(3+2x^2)(1+x)^{-2}(1-4x)^{-1}$ expanded theirA,B,C and their expansions
3 $\sin(\theta + \alpha) = 2\sin\theta$ $\Rightarrow$ $\sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\sin\theta$ $\Rightarrow$ $\tan\theta\cos\alpha + \sin\alpha = 2\tan\theta$ $\Rightarrow$ $\sin\alpha = 2\tan\theta - \tan\theta\cos\alpha$ $= \tan\theta\,(2 - \cos\alpha)$ $\Rightarrow$ $\tan\theta = \frac{\sin\alpha}{2-\cos\alpha}$ * $\sin(\theta + 40°) = 2\sin\theta$ $\Rightarrow$ $\tan\theta = \frac{\sin 40}{2-\cos 40} = 0.5209$ $\Rightarrow$ $\theta = 27.5°, 207.5°$	M1 M1 M1 E1 M1 A1 A1 [7]	Using correct Compound angle formula in a valid equation dividing by $\cos\theta$ collecting terms in $\tan\theta$ or $\sin\theta$ or dividing by $\tan\theta$ oe www (can be all achieved for the method in reverse) $\tan\theta = \frac{\sin 40}{2-\cos 40}$ -1 if given in radians -1 extra solutions in the range

4 (a) $\frac{dx}{dt} = k\sqrt{x}$	M1 A1 [2]	$\frac{dx}{dt} = \ldots$ $k\sqrt{x}$
(b) $\frac{dy}{dt} = \frac{10000}{\sqrt{y}}$ $\Rightarrow \int \sqrt{y}dy = \int 10000dt$ $\Rightarrow \frac{2}{3}y^{\frac{3}{2}} = 10000t + c$ When t = 0, y = 900 $\Rightarrow$ 18000 = c $\Rightarrow y = [\frac{3}{2}(10000t + 18000)]^{\frac{2}{3}}$ $= (1500(10t+18))^{\frac{2}{3}}$ When t = 10, y = 3152	 M1 A1 B1 A1 M1 A1 [6]	 separating variables condone omission of c evaluating constant for their integral any correct expression for y = for method allow substituting t=10 in their expression cao
5 (i) $\int xe^{-2x}dx$ let $u = x$, dv/dx = e^{-2x} $\Rightarrow v = -½$ e^{-2x} $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ $= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$ $= -\frac{1}{4}e^{-2x}(1+2x) + c$ * or $\frac{d}{dx}[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c] = -\frac{1}{2}e^{-2x} + xe^{-2x} + \frac{1}{2}e^{-2x}$ = xe^{-2x}	M1 A1 E1 M1 A1 E1 [3]	Integration by parts with $u = x$, dv/dx = e^{-2x} $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ condone omission of c product rule
(ii) $V = \int_0^2 \pi y^2 dx$ $= \int_0^2 \pi(x^{1/2}e^{-x})^2 dx$ $= \pi\int_0^2 xe^{-2x}dx$ $= \pi\left[-\frac{1}{4}e^{-2x}(1+2x)\right]_0^2$ $= \pi(-¼\ e^{-4}.5 + ¼)$ $= \frac{1}{4}\pi(1 - \frac{5}{e^4})$ *	M1 A1 DM1 E1 [4]	Using formula condone omission of limits $y^2 = xe^{-2x}$ condone omission of limits and π condone omission of π (need limits)

Section B

6 (i) At E, $\theta = 2\pi$ $\Rightarrow$ $x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow$ $y = a(1 - \cos\pi) = 2a$	M1 A1 M1 A1 [4]	$\theta=\pi$, 180°, $\cos\theta=-1$
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{\sin\theta}{(1-\cos\theta)}$	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin\theta) = \cos\theta$, $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ both or equivalent www condone uncancelled a
(iii) $\tan 30° = 1/\sqrt{3}$ $\Rightarrow$ $\frac{\sin\theta}{(1-\cos\theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$ * When $\theta = 2\pi/3$, $\sin\theta = \sqrt{3}/2$ $(1-\cos\theta)/\sqrt{3} = (1+½)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + ½) = 3a/2$* OF = $a(2\pi/3 - \sqrt{3}/2)$	M1 E1 M1 E1 E1 B1 [6]	Or gradient=$1/\sqrt{3}$ $\sin\theta = \sqrt{3}/2$, $\cos\theta = -½$ or equiv.
(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ $= 20$ $\Rightarrow a = 2.22$ m	B1ft M1 A1 M1 A1 [5]	their OE -2their OF

7 (i) AE = √($15^2 + 20^2 + 0^2$) = 25	M1 A1 [2]	
(ii) $\overrightarrow{AE} = \begin{pmatrix}15\\-20\\0\end{pmatrix} = 5\begin{pmatrix}3\\-4\\0\end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\\0\end{pmatrix}$ BD = 15 ⇒ λ = 3 ⇒ D is (8, –19, 11)	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}15\\-20\\0\end{pmatrix}$ λ = 3 or 3/5 as appropriate
(iii) At A: –3 × 0 + 4 × 0 + 5 × 6 = 30 At B: –3 × (–1) + 4 × (–7) + 5 × 11 = 30 At C: –3 × (–8) + 4 × (–6) + 5 × 6 = 30 Normal is $\begin{pmatrix}-3\\4\\5\end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point OR M1 vector form of equation of plane eg r=0i+0j+6k+μ(i+7j-5k)+ν(8i+6j+0k) M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AE} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}15\\-20\\0\end{pmatrix} = 60 - 60 = 0$ $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AB} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}-1\\-7\\5\end{pmatrix} = -4 - 21 + 25 = 0$ ⇒ $\begin{pmatrix}4\\3\\5\end{pmatrix}$ is normal to plane Equation is $4x + 3y + 5z = 30$.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 $4x + 3y + 5z = \ldots$ 30 OR as * above OR M1 for subst 1 point in 4x+3y+5z= ,A1 for subst 2 further points =30 A1 correct equation , B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix}4\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\4\\5\end{pmatrix}$ $\cos\theta = \dfrac{4\times(-3)+3\times4+5\times5}{\sqrt{50}\times\sqrt{50}} = \dfrac{1}{2}$ ⇒ $\theta = 60°$	M1 M1 A1 A1 [4]	 Correct method for any 2 vectors their normals only (rearranged) or 120° cao

	Comprehension Paper 2		
Qu	**Answer**	**Mark**	**Comment**
1.	$\left(26+\dfrac{385}{1760}\right)\times 4$ minutes 1 hour 44 minutes 52.5 seconds	M1 A1	Accept all equivalent forms, with units. Allow ….52 and 53 seconds.
2.	$R = 259.6 - 0.391(T-1900)$ $\therefore\ 259.6 - 0.391(T-1900) = 0$ $\Rightarrow T = 2563.9$ *R* will become negative in 2563	M1 A1 A1	R=0 and attempting to solve. T=2563,2564,2563.9…any correct cao
3.	The value of *L* is 120.5 and this is over 2 hours or (120 minutes)	E1	or R>120.5minutes or showing there is no solution for 120=120.5+54.5e$^{-\ldots}$
4.(i)	Substituting $t=0$ in $R = L+(U-L)\mathrm{e}^{-kt}$ gives $R = L+(U-L)\times 1$ $= U$	M1 A1 E1	$\mathrm{e}^0 = 1$
4.(ii)	As $t\to\infty$, $\mathrm{e}^{-kt}\to 0$ and so $R \to L$	M1 E1	
5.(i)		M1 A1 A1	Increasing curve Asymptote *A* and *B* marked correctly
5.(ii)	Any field event: long jump, high jump, triple jump, pole vault, javelin, shot, discus, hammer, etc.	B1	
6.(i)	$t = 104$	B1	
6.(ii)	$R = 115+(175-115)\mathrm{e}^{-0.0467t^{0.797}}$ $R = 115+60\times \mathrm{e}^{-0.0467\times 104^{0.797}}$ $R = 115+60\times \mathrm{e}^{-1.892}$ $R = 124.047...$ 2 hours 4 minutes 3 seconds	M1 A1	Substituting their t 124, 124.05, etc.

Mark Scheme 4754
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Paper A – Section A

1 $\frac{1}{x}+\frac{x}{x+2}=1$ $\Rightarrow$ $x+2+x^2=x(x+2)$ $=x^2+2x$ $\Rightarrow$ $x=2$	M1 A1 DM1 A1 [4]	Clearing fractions solving cao
2(i) x: 0, 0.5, 1, 1.5, 2 y: 1, 1.0607, 1.4142, 2.0917, 3 $A \approx 0.5[(1+3)/2 + 1.0607 + 1.4142 + 2.0917]$ $= 3.28$ (3 s.f.)	B1 M1 A1 [3]	At least one value calculated correctly or 13.13…or 6.566… seen
(ii) 3.25 (or Chris) area should decrease with the number of strips used.	B1 B1 [2]	ft (i) or area should decrease as concave upwards
3(i) sin 60 = √3/2, cos 60 = 1/2 , sin 45 = 1/√2, cos 45 = 1/√2 sin(105°) = sin(60°+45°) = sin60°cos45° + cos60°sin45° $=\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}+\frac{1}{2}.\frac{1}{\sqrt{2}}$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}$ *	M1 M1 A1 E1 [4]	 splitting into 60° and 45°, and using the compound angle formulae
(ii) Angle B = 105° By the sine rule: $\frac{AC}{\sin B}=\frac{1}{\sin 30}$ $\Rightarrow$ AC = $\frac{\sin 105}{\sin 30}=\frac{\sqrt{3}+1}{2\sqrt{2}}.2$ $=\frac{\sqrt{3}+1}{\sqrt{2}}$ *	 M1 A1 E1 [3]	 Sine rule with exact values www
4 $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ $=\frac{1}{\cos 2\theta}$ $=\sec 2\theta$ $\sec 2\theta = 2 \Rightarrow \cos 2\theta = ½$ $\Rightarrow$ $2\theta = 60°, 300°$ $\Rightarrow$ $\theta = 30°, 150°$	M1 M1 M1 E1 M1 B1 B1 [7]	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $1+\tan^2\theta=\sec^2\theta$ used simplifying to a simple fraction in terms of $\sin\theta$ and/or $\cos\theta$ only $\cos^2\theta-\sin^2\theta=\cos 2\theta$ oe used or $1+\tan^2\theta = 2(1-\tan^2\theta) \Rightarrow \tan\theta = \pm 1/\sqrt{3}$oe 30° 150° and no others in range

5 $(1+3x)^{\frac{1}{3}} =$ $= 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(3x)^3+...$ $=1+x-x^2+\frac{5}{3}x^3+...$ Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	M1 B1 A2,1,0 B1 [5]	binomial expansion (at least 3 terms) correct binomial coefficients (all) x, $-x^2$, $5x^3/3$
6(i) $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$ $\Rightarrow$ $1 = A(x + 1) + B(2x + 1)$ $x = -1$: $1 = -B \Rightarrow B = -1$ $x = -½ : 1 = ½ A \Rightarrow A = 2$	M1 A1 A1 [3]	or cover up rule for either value
(ii) $\frac{dy}{dx}=\frac{y}{(2x+1)(x+1)}$ $\Rightarrow$ $\int\frac{1}{y}dy=\int\frac{1}{(2x+1)(x+1)}dx$ $=\int(\frac{2}{2x+1}-\frac{1}{x+1})dx$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + c$ When $x = 0$, $y = 2$ $\Rightarrow$ $\ln 2 = \ln 1 - \ln 1 + c \Rightarrow c = \ln 2$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + \ln 2$ $=\ln\frac{2(2x+1)}{x+1}$ $\Rightarrow$ $y=\frac{4x+2}{x+1}$ *	M1 A1 B1ft M1 E1 [5]	separating variables correctly condone omission of c. ft A,B from (i) calculating c , no incorrect log rules combining lns www

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding *dy/dθ and dx/dθ* correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ = 60 m	M1 A1 [2]	
(ii) $\overrightarrow{BA} = \begin{pmatrix}-40\\-40\\20\end{pmatrix} = 20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}} = -\dfrac{13}{3\sqrt{26}}$ $\Rightarrow \quad \theta = 148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ −13 oe eg -260 √9√26 oe eg 60√26 cao (or radians)
(iii) $\mathbf{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z = 0 \Rightarrow \lambda = 20$ $\Rightarrow \quad a = 40 + 3 \times 20 = 100$ $b = 0 + 4 \times 20 = 80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix}+\ldots$ $\ldots+\lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….$+\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix} = -12+10+2=0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2=0$ $\Rightarrow \begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x - 5y + 2z = c$ At B (say) $6 \times 40 - 5 \times 0 + 2 \times -20 = c$ $\Rightarrow c = 200$ so $6x - 5y + 2z = 200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)

Paper B Comprehension

1(i)	Leading digit: 1, 2, 3, 4, 5, 6, 7, 8, 9 Frequency: 6, 4, 2, 2, 2, 1, 1, 1, 1	B1 Table
(ii)	Leading digit: 1, 2, 3, 4, 5, 6, 7, 8, 9 Frequency: 7, 3, 2, 3, 1, 2, 1, 1, 0	M1 A1 Table
(iii)	Leading digit: 1, 2, 3, 4, 5, 6, 7, 8, 9 Frequency: 6.0, 3.5, 2.5, 1.9, 1.6, 1.3, 1.2, 1.0, 0.9	B1any 4 correct B1 other 4 correct
(iv)	Any sensible comment such as: • The general pattern of the frequencies/results is the same for all three tables. • Due to the small number of data items we cannot expect the pattern to follow Benford's Law very closely.	E1
2	Evidence of 4+3+4+2+2 from Table 4 frequencies is the same as 15 in Table 6	B1
3	$p_1 = p_3 + p_4 + p_5$: on multiplication by 3, numbers with a leading digit of 1 will be mapped to numbers with a leading digit of 3, 4 or 5 and no other numbers have this property.	B1 Multiplication B1… by 3
4	$\log_{10}(n+1) - \log_{10} n = \log_{10}\left(\frac{n+1}{n}\right) = \log_{10}\left(\frac{n}{n} + \frac{1}{n}\right) = \log_{10}\left(1 + \frac{1}{n}\right)$	M1 E1
5	Substitute $\mathrm{L}(4) = 2\times\mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$ in $\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$: this gives $\mathrm{L}(8) = \mathrm{L}(6) - \mathrm{L}(3) + \mathrm{L}(4) = \mathrm{L}(2) + 2\times\mathrm{L}(2) = 3\times\mathrm{L}(2)$	M1 M1 subst E1 (or alt M1 for 2 or more Ls used M1 use of at least 2 given results oe E1)
6	$a = 28$. All entries with leading digit 2 or 3 will, on multiplying by 5, have leading digit 1. None of the other original daily wages would have this property.	B1 B1
	$b = 9$. Similarly, all entries with leading digit 8 or 9 will, on multiplying by 5, have leading digit 4. None of the other original daily wages would have this property.	B1 B1
		Total 18

Mark Scheme 4754
June 2007

Section A

1 sin θ − 3 cos θ = R sin(θ − α) = R(sin θ cos α − cos θ sin α) $\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = 3$ $\Rightarrow$ $R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ $\tan\alpha = 3 \Rightarrow \alpha = 71.57°$ $\sqrt{10}\sin(\theta - 71.57°) = 1$ $\Rightarrow$ $\theta - 71.57° = \sin^{-1}(1/\sqrt{10})$ $\theta - 71.57° = 18.43°, 161.57°$ $\Rightarrow$ θ = 90° , 233.1°	 M1 B1 M1 A1 M1 B1 A1 [7]	 equating correct pairs oe ft www cao (71.6°or better) oe ft R, α www and no others in range (MR-1 for radians)
2 Normal vectors are $\begin{pmatrix}2\\3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\Rightarrow$ $\begin{pmatrix}2\\3\\4\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix} = 2 - 6 + 4 = 0$ $\Rightarrow$ planes are perpendicular.	B1 B1 M1 E1 [4]	
3 **(i)** $y = \ln x \Rightarrow x = e^y$ $\Rightarrow$ $V = \int_0^2 \pi x^2 dy$ $= \int_0^2 \pi (e^y)^2 dy = \int_0^2 \pi e^{2y} dy$ *	B1 M1 E1 [3]	
(ii) $\int_0^2 \pi e^{2y} dy = \pi\left[\frac{1}{2}e^{2y}\right]_0^2$ $= \frac{1}{2}\pi(e^4 - 1)$	B1 M1 A1 [3]	$\frac{1}{2}e^{2y}$ substituting limits in $k\pi e^{2y}$ or equivalent, but must be exact and evaluate e^0 as 1.
4 $x = \frac{1}{t} - 1 \Rightarrow \frac{1}{t} = x + 1$ $\Rightarrow$ $t = \frac{1}{x+1}$ $\Rightarrow$ $y = \dfrac{2 + \frac{1}{x+1}}{1 + \frac{1}{x+1}} = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2}$	M1 A1 M1 E1	Solving for t in terms of x or y Subst their t which must include a fraction, clearing subsidiary fractions/ changing the subject oe www
or $\frac{3+2x}{2+x} = \dfrac{3 + \frac{2-2t}{t}}{2 + \frac{1-t}{t}}$ $= \frac{3t+2-2t}{2t+1-t}$ $= \frac{t+2}{t+1} = y$	M1 A1 M1 E1 [4]	substituting for x or y in terms of t clearing subsidiary fractions/changing the subject

5 $\mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1-\lambda\\2+2\lambda\\-1+3\lambda\end{pmatrix}$		
When $x = -1$, $1-\lambda = -1$, $\Rightarrow \lambda = 2$ $\Rightarrow y = 2 + 2\lambda = 6$, $z = -1 + 3\lambda = 5$ $\Rightarrow$ point lies on first line	M1 E1	Finding λ or μ checking other two coordinates
$\mathbf{r}=\begin{pmatrix}0\\6\\3\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-2\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\mu\\6\\3-2\mu\end{pmatrix}$ When $x = -1$, $\mu = -1$, $\Rightarrow y = 6$, $z = 3-2\mu = 5$ $\Rightarrow$ point lies on second line	 E1	 checking other two co-ordinates
Angle between $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ is θ, where	M1	Finding angle between correct vectors
$\cos\theta=\frac{-1\times1+2\times0+3\times-2}{\sqrt{14}.\sqrt{5}}$	M1	use of formula
$=-\frac{7}{\sqrt{70}}$	A1	$\pm\frac{7}{\sqrt{70}}$
$\Rightarrow \theta = 146.8°$ $\Rightarrow$ acute angle is 33.2°	A1cao [7]	Final answer must be acute angle
6(i) $A\approx 0.5[\frac{(1.1696+1.0655}{2}+1.1060]$ = 1.11 (3 s.f.)	M1 A1 cao [2]	Correct expression for trapezium rule
(ii) $(1+e^{-x})^{1/2}=1+\frac{1}{2}e^{-x}+\frac{\frac{1}{2}.-\frac{1}{2}}{2!}(e^{-x})^2+...$ $\approx 1+\frac{1}{2}e^{-x}-\frac{1}{8}e^{-2x}$ *	M1 A1 E1 [3]	Binomial expansion with p = ½ Correct coeffs
(iii) $I=\int_1^2(1+\frac{1}{2}e^{-x}-\frac{1}{8}e^{-2x})dx$ $=\left[x-\frac{1}{2}e^{-x}+\frac{1}{16}e^{-2x}\right]_1^2$ $=(2-\frac{1}{2}e^{-2}+\frac{1}{16}e^{-4})-(1-\frac{1}{2}e^{-1}+\frac{1}{16}e^{-2})$ = 1.9335 − 0.8245 = 1.11 (3 s.f.)	 M1 A1 A1 [3]	 integration substituting limits into correct expression

Section B

7 (a) (i) $P_{max} = \frac{2}{2-1} = 2$ $P_{min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \frac{dP}{dt} = -2(2-\sin t)^{-2}.-\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2\cos t = \frac{1}{2}\frac{4}{(2-\sin t)^2}\cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\ldots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow$ $1 = A(2P-1) + BP$ $P = 0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = ½ \Rightarrow 1 = A.0 + ½ B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P - P^2)\cos t$ $\Rightarrow \int \frac{1}{2P^2 - P}dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \int (\frac{2}{2P-1} - \frac{1}{P})dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow$ $\ln(2P-1) - \ln P = ½ \sin t + c$ When $t = 0$, $P = 1$ $\Rightarrow$ $\ln 1 - \ln 1 = ½ \sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln(\frac{2P-1}{P}) = \frac{1}{2}\sin t$ *	 M1 A1 A1 B1 E1 [5]	 separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) ½ $\sin t$ finding constant = 0
(iii) $P_{max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

8 (i) $\frac{dy}{dx}=\frac{10\cos\theta+10\cos 2\theta}{-10\sin\theta-10\sin 2\theta}$ $= -\frac{\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta}$ *	M1 E1	*dy/dθ ÷dx/dθ*
When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3+\cos 2\pi/3}{\sin\pi/3+\sin 2\pi/3}$ $= 0$ as $\cos \pi/3 = ½$, $\cos 2\pi/3 = -½$	B1	or solving $\cos\theta+\cos 2\theta=0$
At *A* $x = 10 \cos \pi/3 + 5 \cos 2\pi/3$ $= 2½$ $y = 10 \sin \pi/3 + 5 \sin 2\pi/3 = 15\sqrt{3}/2$	M1 A1 A1 [6]	substituting $\pi/3$ into *x* or *y* 2½ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2+y^2=(10\cos\theta+5\cos 2\theta)^2+(10\sin\theta+5\sin 2\theta)^2$ $= 100\cos^2\theta+100\cos\theta\cos 2\theta+25\cos^2 2\theta$ $+100\sin^2\theta+100\sin\theta\sin 2\theta+25\sin^2 2\theta$ $= 100 + 100\cos(2\theta - \theta) + 25$ $= 125 + 100\cos\theta$ *	B1 M1 DM1 E1 [4]	 expanding $\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = \cos(2\theta - \theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta+2\cos\theta - 1 = 0$ $\cos\theta=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm 2\sqrt{3}}{4}$	M1	quadratic formula
At B, $\cos\theta=\frac{-1+\sqrt{3}}{2}$	A1	or θ=68.53° or 1.20radians, correct root selected
$OB^2 = 125 + 50(-1 + \sqrt{3}) = 75 + 50\sqrt{3} = 161.6\ldots$ $\Rightarrow$ $OB = \sqrt{161.6\ldots} = 12.7$ (m)	M1 A1 [4]	or OB=$10\sin\theta+5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

Paper B Comprehension

1)	M $(a\pi, 2a)$, $\theta=\pi$ N $(4a\pi, 0)$, $\theta=4\pi$	B1 B1	
2)	Compare the equations with equations given in text, $x = a\theta - b\sin\theta$, $y = b\cos\theta$	M1	Seeing a=7, b=0.25
	Wavelength = $2\pi a = 14\pi$ (≈ 44) Height = $2b$ = 0.5	A1 B1	
3i)	Wavelength = 20 $\Rightarrow a = \dfrac{10}{\pi}$ (=3.18…) Height = 2 $\Rightarrow b = 1$	B1 B1	
ii)	In this case, the ratio is observed to be 12:8 Trough length : Peak length = $\pi a + 2b : \pi a - 2b$ and this is $(10 + 2 \times 1) : (10 - 2 \times 1)$ So the curve is consistent with the parametric equations	B1 M1 A1	substituting
4i)	$x = a\theta$, $y = b\cos\theta$ is the sine curve V and $x = a\theta - b\sin\theta$, $y = b\cos\theta$ is the curtate cycloid U. The sine curve is above mid-height for half its wavelength (or equivalent)	B1	
ii)	$d = a\theta - (a\theta - b\sin\theta)$ $\theta=\pi/2$, $d=\left(\dfrac{\pi a}{2}\right)-\left(\dfrac{\pi a}{2}-b\right)=b$	M1 E1	Subtraction Using $\theta=\pi/2$
iii)	Because b is small compared to a, the two curves are close together.	M1 E1	Comparison attempted Conclusion
5)	Measurements on the diagram give Wavelength $\approx$ 3.5cm, Height $\approx$ 0.8cm $\dfrac{\text{Wavelength}}{\text{Height}} \approx \dfrac{3.5}{0.8} = 4.375$ Since 4.375 < 7, the wave will have become unstable and broken.	B1 M1 E1	measurements/reading ratio [18]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4754(B)/01

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper B: Comprehension

INSERT

THURSDAY 14 JUNE 2007

Afternoon
Time: Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

This document consists of **8** printed pages.

Modelling sea waves

Introduction

There are many situations in which waves and oscillations occur in nature and often they are accurately modelled by the sine curve. However, this is not the case for sea waves as these come in a variety of shapes. The photograph in Fig. 1 shows an extreme form of sea wave being ridden by a surfer.

Fig. 1

At any time many parts of the world's oceans are experiencing storms. The strong winds create irregular *wind waves*. However, once a storm has passed, the waves form into a regular pattern, called *swell*. Swell waves are very stable; those resulting from a big storm would travel several times round the earth if they were not stopped by the land.

Fig. 2 illustrates a typical swell wave, but with the vertical scale exaggerated. The horizontal distance between successive *peaks* is the *wavelength*; the vertical distance from the lowest point in a *trough* to a peak is called the *height*. The height is twice the *amplitude* which is measured from the horizontal *line of mid-height*. The upper part is the *crest*. These terms are illustrated in Fig. 2.

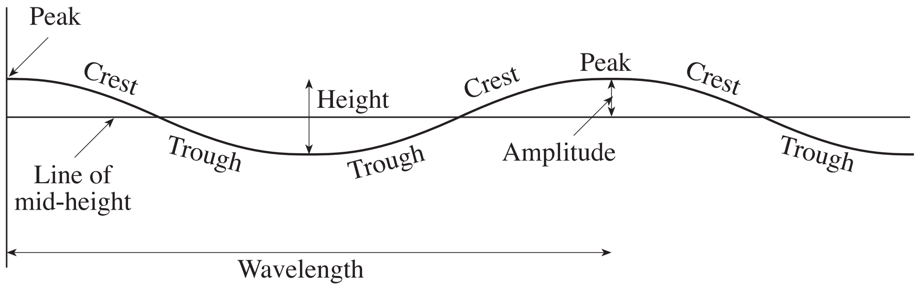

Fig. 2

The speed of a wave depends on the depth of the water; the deeper the water, the faster the wave. (This is, however, not true for very deep water, where the wave speed is independent of the depth.) This has a number of consequences for waves as they come into shallow water.

- Their speed decreases.
- Their wavelength shortens.
- Their height increases.

Observations show that, as their height increases, the waves become less symmetrical. The troughs become relatively long and the crests short and more pointed.

The profile of a wave approaching land is illustrated in Fig. 3. Eventually the top curls over and the wave "breaks".

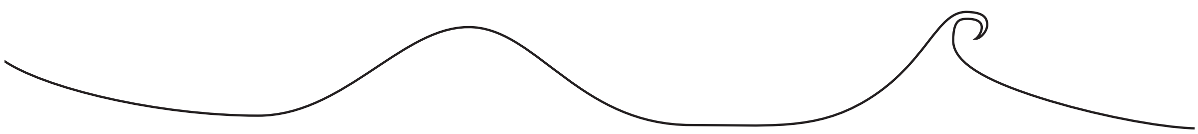

Fig. 3

If you stand at the edge of the sea you will see the water from each wave running up the shore towards you. You might think that this water had just travelled across the ocean. That would be wrong. When a wave travels across deep water, it is the shape that moves across the surface and not the water itself. It is only when the wave finally reaches land that the actual water moves any significant distance.

Experiments in wave tanks have shown that, except near the shore, each drop of water near the surface undergoes circular motion (or very nearly so). This has led people to investigate the possiblility that a form of cycloid would provide a better model than a sine curve for a sea wave.

Cycloids

There are several types of cycloid. In this article, the name *cycloid* refers to one of the family of curves which form the locus of a point on a circle rolling along a straight horizontal path.

Fig. 4 illustrates the basic cycloid; in this case the point is on the circumference of the circle.

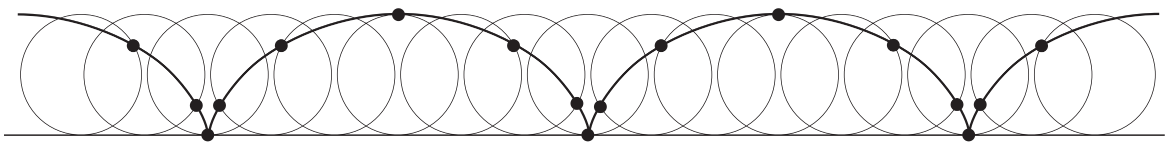

Fig. 4

Two variations on this basic cycloid are the prolate cycloid, illustrated in Fig. 5, and the curtate cycloid illustrated in Fig. 6. The prolate cycloid is the locus of a point attached to the circle but outside the circumference (like a point on the flange of a railway train's wheel); the curtate cycloid is the locus of a point inside the circumference of the circle.

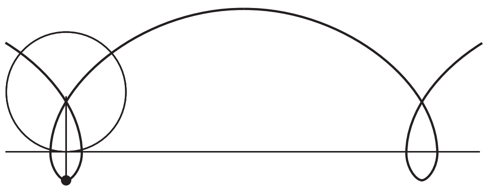

Fig. 5

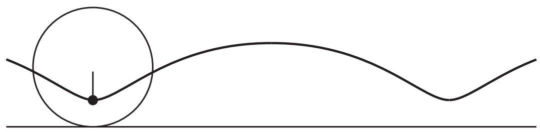

Fig. 6

When several cycles of the curtate cycloid are drawn "upside down", as in Fig. 7, the curve does indeed look like the profile of a wave in shallow water.

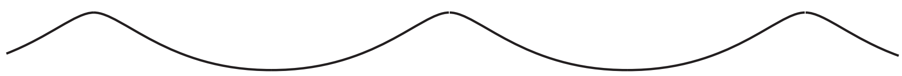

Fig. 7

The equation of a cycloid

The equation of a cycloid is usually given in parametric form.

Fig. 8.1 and Fig. 8.2 illustrate a circle rolling along the x-axis. The circle has centre Q and radius a. P and R are points on its circumference and angle PQR = θ, measured in radians. Fig. 8.1 shows the initial position of the circle with P at its lowest point; this is the same point as the origin, O. Some time later the circle has rolled to the position shown in Fig. 8.2 with R at its lowest point.

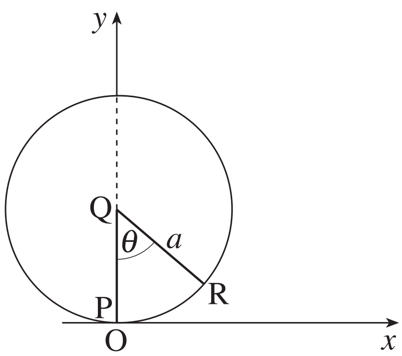

Fig. 8.1

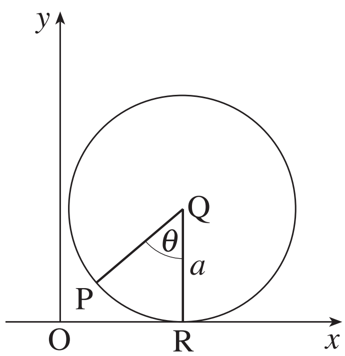

Fig. 8.2

In travelling to its new position, the circle has rolled the distance OR in Fig. 8.2. Since it has rolled along its circumference, this distance is the same as the arc length PR, and so is $a\theta$. Thus the coordinates of the centre, Q, in Fig. 8.2 are $(a\theta, a)$. To find the coordinates of the point P in Fig. 8.2, look at triangle QPZ in Fig. 9.

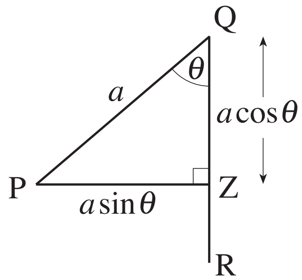

Fig. 9

You can see that

$$PZ = a \sin\theta \qquad \text{and} \qquad QZ = a\cos\theta.$$

Hence the coordinates of P are $(a\theta - a\sin\theta, a - a\cos\theta)$, and so the locus of the point P is described by the curve with parametric equations

$$x = a\theta - a\sin\theta, \quad y = a - a\cos\theta.$$

This is the basic cycloid.

These parametric equations can be generalised to

$$x = a\theta - b\sin\theta, \quad y = a - b\cos\theta,$$

where b is the distance of the moving point from the centre of the circle.

For	$b < a$	the curve is a *curtate cycloid*,
	$b = a$	the curve is a *basic cycloid*,
	$b > a$	the curve is a *prolate cycloid*.

The equivalent equations with the curve turned "upside down", and with the mid-height of the curve now on the x-axis, are

$$x = a\theta - b\sin\theta, \quad y = b\cos\theta.$$

(Notice that positive values of y are still measured vertically upwards.)

Modelling a particular wave

A question that now arises is how to fit an equation to a particular wave profile.

If you assume that the wave is a cycloid, there are two parameters to be found, a and b.

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{x}{x^2-4}+\frac{2}{x+2}=\frac{x}{(x-2)(x+2)}+\frac{2}{x+2}$ $=\frac{x+2(x-2)}{(x+2)(x-2)}$ $=\frac{3x-4}{(x+2)(x-2)}$	M1 M1 A1 [3]	combining fractions correctly factorising and cancelling (may be $3x^2+2x-8$)
2 $V=\int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x})dx$ $=\pi\left[x+\frac{1}{2}e^{2x}\right]_0^1$ $=\pi(1+\frac{1}{2}e^2-\frac{1}{2})$ $=\frac{1}{2}\pi(1+e^2)$ *	M1 B1 M1 E1 [4]	must be π x their y^2 in terms of x $\left[x+\frac{1}{2}e^{2x}\right]$ only substituting both x limits in a function of x www
3 $\cos 2\theta = \sin\theta$ $\Rightarrow$ $1-2\sin^2\theta = \sin\theta$ $\Rightarrow$ $1-\sin\theta-2\sin^2\theta=0$ $\Rightarrow$ $(1-2\sin\theta)(1+\sin\theta)=0$ $\Rightarrow$ $\sin\theta = ½$ or -1 $\Rightarrow$ $\theta = \pi/6, 5\pi/6, 3\pi/2$	M1 M1 A1 M1 A1 A2,1,0 [7]	$\cos2\theta = 1-2\sin^2\theta$ oe substituted forming quadratic(in one variable) =0 correct quadratic www factorising or solving quadratic ½ , −1 oe www cao penalise extra solutions in the range
4 $\sec\theta = x/2$, $\tan\theta = y/3$ $\sec^2\theta = 1+\tan^2\theta$ $\Rightarrow$ $x^2/4 = 1 + y^2/9$ $\Rightarrow$ $x^2/4 - y^2/9 = 1$ * **OR** $x^2/4-y^2/9= 4\sec^2\theta/4-9\tan^2\theta/9$ $=\sec^2\theta-\tan^2\theta =1$	M1 M1 E1 [3]	$\sec^2\theta = 1+\tan^2\theta$ used (oe, e.g. converting to sines and cosines and using $\cos^2\theta + \sin^2\theta = 1$) eliminating θ (or x and y) www
5(i) $dx/du = 2u$, $dy/du = 6u^2$ $\Rightarrow$ $\frac{dy}{dx}=\frac{dy/du}{dx/du}=\frac{6u^2}{2u}$ $= 3u$ **OR** $y=2(x-1)^{3/2}$, $dy/dx=3(x-1)^{1/2}=3u$	B1 M1 A1 [3]	both $2u$ and $6u^2$ B1(y=f(x)), M1 differentiation, A1
(ii) At (5, 16), $u = 2$ $\Rightarrow$ $dy/dx = 6$	M1 A1 [2]	cao

6(i) $(1+4x^2)^{-\frac{1}{2}} = 1 - \frac{1}{2}.4x^2 + \frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(4x^2)^2 + ...$ $= 1 - 2x^2 + 6x^4 + ...$ Valid for $-1 < 4x^2 < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	M1 A1 A1 M1A1 [5]	binomial expansion with $p = -1/2$ $1 - 2x^2$ … $+ 6x^4$
(ii) $\frac{1-x^2}{\sqrt{1+4x^2}} = (1-x^2)(1-2x^2+6x^4+...)$ $= 1 - 2x^2 + 6x^4 - x^2 + 2x^4 + ...$ $= 1 - 3x^2 + 8x^4 + ...$	M1 A1 A1 [3]	substitituting their $1 - 2x^2 + 6x^4 + ...$ and expanding ft their expansion (of three terms) cao
7 $\sqrt{3}\sin x - \cos x = R\sin(x - \alpha)$ $= R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow \sqrt{3} = R\cos\alpha$, $1 = R\sin\alpha$ $\Rightarrow R^2 = 3 + 1 = 4 \Rightarrow R = 2$ $\tan\alpha = 1/\sqrt{3}$ $\Rightarrow \alpha = \pi/6$ $\Rightarrow y = 2\sin(x - \pi/6)$ Max when $x - \pi/6 = \pi/2 \Rightarrow x = \pi/6 + \pi/2 = 2\pi/3$ max value $y = 2$ So maximum is $(2\pi/3, 2)$	 M1 B1 M1 A1 B1 B1 [6]	 correct pairs soi $R = 2$ ft cao www cao ft their R SC B1 $(2, 2\pi/3)$ no working

Section B

8(i) At A: 3×0 + 2×0 + 20×(–15) + 300 = 0 At B: 3×100 + 2×0 + 20×(–30) + 300 = 0 At C: 3×0 + 2×100 + 20×(–25) + 300 = 0 So ABC has equation $3x + 2y + 20z + 300 = 0$	M1 A2,1,0 [3]	substituting co-ords into equation of plane… for ABC **OR** using two vectors in the plane form vector product M1A1 then $3x + 2y + 20z = c = -300$ A1 **OR** using vector equation of plane M1,elim both parameters M1, A1
(ii) $\overrightarrow{DE} = \begin{pmatrix}100\\0\\-10\end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix}0\\100\\5\end{pmatrix}$	B1B1	
$\begin{pmatrix}100\\0\\-10\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 100\times 2 + 0\times -1 + -10\times 20 = 200 - 200 = 0$	B1	need evaluation
$\begin{pmatrix}0\\100\\5\end{pmatrix}.\begin{pmatrix}2\\-1\\20\end{pmatrix} = 0\times 2 + 100\times -1 + 5\times 20 = -100 + 100 = 0$	B1	need evaluation
Equation of plane is $2x - y + 20z = c$ At D (say) $c = 20\times -40 = -800$ So equation is $2x - y + 20z + 800 = 0$	M1 A1 [6]	
(iii) Angle is θ, where $\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\20\end{pmatrix}.\begin{pmatrix}3\\2\\20\end{pmatrix}}{\sqrt{2^2 + (-1)^2 + 20^2}\sqrt{3^2 + 2^2 + 20^2}} = \dfrac{404}{\sqrt{405}\sqrt{413}}$ $\Rightarrow$ $\theta = 8.95°$	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
(iv) RS: $\mathbf{r} = \begin{pmatrix}15\\34\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$	B1	$\begin{pmatrix}15\\34\\0\end{pmatrix} + \ldots$
$= \begin{pmatrix}15+3\lambda\\34+2\lambda\\20\lambda\end{pmatrix}$	B1	$\ldots + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$
$\Rightarrow$ $3(15+3\lambda) + 2(34+2\lambda) + 20.20\lambda + 300 = 0$ $\Rightarrow$ $45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$	M1	solving with plane
$\Rightarrow$ $413 + 413\lambda = 0$ $\Rightarrow$ $\lambda = -1$	A1	$\lambda = -1$
so S is (12, 32, –20)	A1 [5]	cao

9(i) $v = \int 10e^{-\frac{1}{2}t}dt$ $= -20e^{-\frac{1}{2}t} + c$ when $t = 0, v = 0$ $\Rightarrow 0 = -20 + c$ $\Rightarrow c = 20$ so $v = 20 - 20e^{-\frac{1}{2}t}$	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
(ii) As $t \to \infty$ $e^{-1/2\,t} \to 0$ $\Rightarrow v \to 20$ So long term speed is 20 m s^{-1}	M1 A1 [2]	 ft (for their c>0, found)
(iii) $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5)+B(w-4)}{(w-4)(w+5)}$ $\Rightarrow 1 \equiv A(w+5) + B(w-4)$ $w = 4$: $1 = 9A \Rightarrow A = 1/9$ $w = -5$: $1 = -9B \Rightarrow B = -1/9$ $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	 cover up, substitution or equating coeffs 1/9 −1/9
(iv) $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow \int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2}dt$ $\Rightarrow \int [\frac{1}{9(w-4)} - \frac{1}{9(w+5)}]dw = \int -\frac{1}{2}dt$ $\Rightarrow \frac{1}{9}\ln(w-4) - \frac{1}{9}\ln(w+5) = -\frac{1}{2}t + c$ $\Rightarrow \frac{1}{9}\ln\frac{w-4}{w+5} = -\frac{1}{2}t + c$ When $t = 0$, $w = 10 \Rightarrow c = \frac{1}{9}\ln\frac{6}{15} = \frac{1}{9}\ln\frac{2}{5}$ $\Rightarrow \ln\frac{w-4}{w+5} = -\frac{9}{2}t + \ln\frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9}{2}t + \ln\frac{2}{5}} = \frac{2}{5}e^{-\frac{9}{2}t} = 0.4e^{-4.5t}$ *	 M1 M1 A1ft M1 M1 E1 [6]	 separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
(v) As $t \to \infty$ $e^{-4.5\,t} \to 0$ $\Rightarrow w - 4 \to 0$ So long term speed is 4 m s^{-1}.	M1 A1 [2]	

Comprehension

1. (i)

2	1	3
3	**2**	**1**
1	**3**	**2**

B1 cao

(ii)

2	3	**1**
3	**1**	**2**
1	2	**3**

B1 cao

2. Dividing the grid up into four 2 x 2 blocks gives

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

Lines drawn on diagram or reference to 2 x 2 blocks. **M1**

One (or more) block does not contain all 4 of the symbols 1, 2, 3 and 4. oe. **E1**

3.

1	2	3	4
4	**3**	**1**	**2**
2	**1**	**4**	**3**
3	**4**	**2**	**1**

<u>Many possible answers</u> **Row 2 correct** **B1**

Rest correct **B1**

4. **Either**

4	2	3	**1**
		2	4
		4	2
2	4	1	**3**

Or

4	2	3	1
		2	**4**
		4	**2**
2	4	1	3

B2

5. In the top row there are 9 ways of allocating a symbol to the left cell, then 8 for the next, 7 for the next and so on down to 1 for the right cell, giving **M1**

$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9!$ ways. **E1**

So there must be $9!\times$ the number of ways of completing the rest of the puzzle.

6.

(i)

Block side length, b	**Sudoku,** $s \times s$	M
1	1×1	-
2	4×4	12
3	9×9	**77**
4	16×16	**252**
5	$\mathbf{25 \times 25}$	**621**

$\mathbf{25 \times 25}$ **B1**

77, 252 and 621 **B1**

(ii) $M = b^4 - 4$

b^4 **B1**

- 4 **B1**

7.

(i)	There are neither 3s nor 5s among the givens.	M1
	So they are interchangeable and therefore there is no unique solution	E1
(ii)	The missing symbols form a 3×3 embedded Latin square.	M1
	There is not a unique arrangement of the numbers 1, 2 and 3 in this square.	E1
		[18]

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$ $\Rightarrow$ $3x + 2 = A(x^2 + 1) + (Bx + C)x$ $x = 0 \Rightarrow 2 = A$ coefft of x^2 : $0 = A + B \Rightarrow B = -2$ coefft of x: $3 = C$ $\Rightarrow$ $\frac{3x+2}{x(x^2+1)} = \frac{2}{x} + \frac{3-2x}{(x^2+1)}$	M1 M1 B1 M1 A1 A1 [6]	correct partial fractions equating coefficients at least one of B,C correct
2(i) $(1+2x)^{1/3} = 1 + \frac{1}{3}.2x + \frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(2x)^2 + \ldots$ $= 1 + \frac{2}{3}x - \frac{2}{18}4x^2 + \ldots$ $= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$ * Next term $= \frac{\frac{1}{3}.(-\frac{2}{3})(-\frac{5}{3})}{3!}(2x)^3$ $= \frac{40}{81}x^3$ Valid for $-1 < 2x < 1$ $\Rightarrow -½ < x < ½$	M1 A1 E1 M1 A1 B1 [6]	binomial expansion correct unsimplified expression simplification www
3 $4\mathbf{j} - 3\mathbf{k} = \lambda\,\mathbf{a} + \mu\,\mathbf{b}$ $= \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mu(4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow$ $0 = 2\lambda + 4\mu$ $4 = \lambda - 2\mu$ $-3 = -\lambda + \mu$ $\Rightarrow$ $\lambda = -2\mu,\ 2\lambda = 4 \Rightarrow \lambda = 2,\ \mu = -1$	M1 M1 A1 A1, A1 [5]	 equating components at least two correct equations
4 LHS = $\cot\beta - \cot\alpha$ $= \frac{\cos\beta}{\sin\beta} - \frac{\cos\alpha}{\sin\alpha}$ $= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $= \frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$ **OR** RHS$= \frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} = \frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $= \cot\beta - \cot\alpha$	M1 M1 E1 M1 M1 E1 [3]	cot = cos / sin combining fractions www using compound angle formula splitting fractions using cot=cos/sin

5(i) Normal vectors $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ Angle between planes is θ, where $\cos\theta = \dfrac{2\times1+(-1)\times0+1\times(-1)}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+0^2+(-1)^2}}$ $= 1/\sqrt{12}$ $\Rightarrow \theta = 73.2°$ or 1.28 rads	B1 M1 M1 A1 [4]	 scalar product finding invcos of scalar product divided by two modulae
(ii) $\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $= \begin{pmatrix}2+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}$ $\Rightarrow 2(2+2\lambda) - (-\lambda) + (1+\lambda) = 2$ $\Rightarrow 5 + 6\lambda = 2$ $\Rightarrow \lambda = -½$ So point of intersection is (1, ½ , ½)	B1 M1 A1 A1 [4]	
6(i) $\cos\theta + \sqrt{3}\sin\theta = r\cos(\theta-\alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$ $\tan\alpha = \sqrt{3}$ $\Rightarrow \alpha = \pi/3$	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs $\tan\alpha = \sqrt{3}$ o.e.
(ii) derivative of $\tan\theta$ is $\sec^2\theta$ $\int_0^{\frac{\pi}{3}} \dfrac{1}{(\cos\theta+\sqrt{3}\sin\theta)^2}d\theta = \int_0^{\frac{\pi}{3}} \dfrac{1}{4}\sec^2(\theta-\dfrac{\pi}{3})d\theta$ $= \left[\dfrac{1}{4}\tan(\theta-\dfrac{\pi}{3})\right]_0^{\frac{\pi}{3}}$ $= ¼ (0 - (-\sqrt{3}))$ $= \sqrt{3}/4$ *	B1 M1 A1 E1 [4]	 ft their α $\dfrac{1}{R^2}$ [tan ($\theta-\pi/3$)] ft their R,α(in radians) www

Section B

7(i) (A) 9 / 1.5 = 6 hours (B) 18/1.5 = 12 hours	B1 B1 [2]	
(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k dt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$ *	M1 A1 A1 M1 E1 [5]	separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with c) $A = e^c$
(iii) $98 = 50 + Ae^0$ $\Rightarrow A = 48$ Initially $\frac{d\theta}{dt} = -k(98-50) = -48k = -1.5$ $\Rightarrow k = 0.03125$*	M1 A1 M1 E1 [4]	
(iv) (A) $89 = 50 + 48e^{-0.03125t}$ $\Rightarrow 39/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(39/48)/(-0.03125) = 6.64$ hours (B) $80 = 50 + 48e^{-0.03125t}$ $\Rightarrow 30/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(30/48)/(-0.03125) = 15$ hours	M1 M1 A1 M1 A1 [5]	equating taking lns correctly for either
(v) Models disagree more for greater temperature loss	B1 [1]	

8(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin\theta$, $\frac{dx}{d\theta} = 2\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$	B1, B1 M1 A1 [4]	 substituting for theirs oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos\pi/3 - \sin\pi/6}{\cos\pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ $BC = 2\times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1,A1 B1ft [5]	 for either exact
(iii) (*A*) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ * (*B*) $\sin\theta = ½ (x - 2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼ (x - 2)^2$ $= 1 - ¼ x^2 + x - 1$ $= (x - ¼ x^2)$ * (*C*) Cartesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - ¼ x^2)$ $= x^3 - ¼ x^4$*	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2\sin\theta\cos\theta$ squaring and substituting for x
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi\int_0^4 (x^3 - \frac{1}{4}x^4)dx$ $= \pi\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]$ 12.8π or 40 or better.

Comprehension

1	$\frac{400\pi d}{1000} = 10$ $d = \frac{25}{\pi} = 7.96$	M1 E1	
2	$V = \pi 20^2 h + \frac{1}{2}\left(\pi 20^2 H - \pi 20^2 h\right)$ $= \frac{1}{2}\left(\pi 20^2 H + \pi 20^2 h\right)\text{cm}^3 = 200\pi\left(H+h\right)\text{ cm}^3$ $= \frac{1}{5}\pi\left(H+h\right)$ litres	M1 M1 E1	divide by 1000
3	$H = 5 + 40\tan 30^\circ$ or $H = h + 40\tan\theta$ $V = \frac{1}{5}\pi\left(H+h\right) = \frac{1}{5}\pi\left(10 + 40\tan 30^\circ\right)$ =20.8 litres	B1 M1 A1	or evaluated including substitution of values
4	$V = \frac{1}{2} \times 80 \times \left(40+5\right)$ $\times 30\text{ cm}^3 = 54\ 000\text{cm}^3$ = 54 litres	M1 M1 A1	×30
5	(i) Accurate algebraic simplification to give $y^2 - 160y + 400 = 0$ (ii) Use of quadratic formula (or other method) to find other root: $d = 157.5$ cm. This is greater than the height of the tank so not possible	B1 M1 A1 E1	
6	$y=10$ Substitute for y in (4): $V = \frac{1}{1000}\int_0^{100} 375\,dx$ $V = \frac{1}{1000} \times 37500 = 37.5$ *	B1 M1 E1 [18]	

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Section A

1 $4\cos\theta - \sin\theta = R\cos(\theta+\alpha)$ $= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 1$ $\Rightarrow R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ $\Rightarrow \alpha = 0.245$ $\sqrt{17}\cos(\theta + 0.245) = 3$ $\Rightarrow \cos(\theta + 0.245) = 3/\sqrt{17}$ $\Rightarrow \theta + 0.245 = 0.756, 5.527$ $\Rightarrow \theta = 0.511, 5.282$	M1 B1 M1 A1 M1 A1A1 [7]	correct pairs $R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ o.e. $\alpha = 0.245$ $\theta + 0.245 = \arccos 3/\sqrt{17}$ ft their R, α for method (penalise extra solutions in the range (-1))
2 $\dfrac{x}{(x+1)(2x+1)} = \dfrac{A}{x+1} + \dfrac{B}{(2x+1)}$ $\Rightarrow x = A(2x+1) + B(x+1)$ $x = -1 \Rightarrow -1 = -A \Rightarrow A = 1$ $x = -½ \Rightarrow -½ = ½B \Rightarrow B = -1$ $\Rightarrow \dfrac{x}{(x+1)(2x+1)} = \dfrac{1}{x+1} - \dfrac{1}{(2x+1)}$ $\Rightarrow \int \dfrac{x}{(x+1)(2x+1)}dx = \int \dfrac{1}{x+1} - \dfrac{1}{(2x+1)}dx$ $= \ln(x+1) - ½\ln(2x+1) + c$	M1 M1 A1 A1 B1 B1 A1 [7]	correct partial fractions substituting, equating coeffts or cover-up $A = 1$ $B = -1$ $\ln(x+1)$ ft their A $-½\ln(2x+1)$ ft their B cao – must have c
3 $\dfrac{dy}{dx} = 3x^2 y$ $\Rightarrow \int \dfrac{dy}{y} = \int 3x^2\,dx$ $\Rightarrow \ln y = x^3 + c$ When $x = 1$, $y = 1$, $\Rightarrow \ln 1 = 1 + c \Rightarrow c = -1$ $\Rightarrow \ln y = x^3 - 1$ $\Rightarrow y = e^{x^3-1}$	M1 A1 B1 A1 [4]	separating variables condone absence of c $c = -1$ oe o.e.
4 When $x = 0$, $y = 4$ $\Rightarrow V = \pi\int_0^4 x^2 dy$ $= \pi\int_0^4 (4-y)dy$ $= \pi\left[4y - \frac{1}{2}y^2\right]_0^4$ $= \pi(16 - 8) = 8\pi$	B1 M1 M1 B1 A1 [5]	must have integral, π, x^2 and dy soi must have π ,their (4-y), their numerical y limits $\left[4y - \frac{1}{2}y^2\right]$

5 $\frac{dy}{dt} = -a(1+t^2)^{-2}.2t$ $\frac{dx}{dt} = 3at^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at}{3at^2(1+t^2)^2}$ $= \frac{-2}{3t(1+t^2)^2}$ * At $(a, \frac{1}{2}a)$, $t = 1$ $\Rightarrow$ gradient = $\frac{-2}{3\times 2^2}$ = –1/6	M1 A1 B1 M1 E1 M1 A1 [7]	$(1+t^2)^{-2} \times kt$ for method ft finding t
6 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow 1 + \cot^2\theta - \cot\theta = 3$ * $\Rightarrow \cot^2\theta - \cot\theta - 2 = 0$ $\Rightarrow (\cot\theta - 2)(\cot\theta + 1) = 0$ $\Rightarrow \cot\theta = 2$, $\tan\theta = \frac{1}{2}$, $\theta = 26.57°$ $\cot\theta = -1$, $\tan\theta = -1$, $\theta = 135°$	E1 M1 A1 M1 A1 A1 [6]	clear use of $1+\cot^2\theta = \text{cosec}^2\theta$ factorising or formula roots 2, –1 cot = 1/tan used $\theta = 26.57°$ $\theta = 135°$ (penalise extra solutions in the range (-1))

Section B

7(i) $\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$	B1	
$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$	B1 [2]	or equivalent alternative
(ii) $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	B1	
$\cos\theta = \dfrac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}{\sqrt{2}\sqrt{5}} = \dfrac{1}{\sqrt{10}}$	B1 M1 M1	correct vectors (any multiples) scalar product used finding invcos of scalar product divided by two modulae
$\Rightarrow$ $\theta = 71.57°$	A1 [5]	72° or better
(iii) $\cos\phi = \dfrac{\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} . \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}}{\sqrt{2}\sqrt{9}} = \dfrac{2+1}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}}$	M1 A1	ft their **n** for method $\pm 1/\sqrt{2}$ oe exact
$\Rightarrow$ $\phi = 45°$ *	E1 [3]	
(iv) $\sin 71.57° = k \sin 45°$ $\Rightarrow$ $k = \sin 71.57° / \sin 45° = 1.34$	M1 A1 [2]	ft on their 71.57° oe
(v) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$		
$x = -2\mu$, $z = 2-\mu$	M1	soi
$x + z = -1$ $\Rightarrow$ $-2\mu + 2 - \mu = -1$	M1	subst in $x+z = -1$
$\Rightarrow$ $3\mu = 3$, $\mu = 1$	A1	
$\Rightarrow$ point of intersection is (–2, –2, 1)	A1	
distance travelled through glass = distance between (0, 0, 2) and (–2, –2, 1) $= \sqrt{(2^2 + 2^2 + 1^2)} = 3$ cm	B1 [5]	www dep on μ=1

8(i) (*A*) 360°÷24=15° CB/OB = sin 15° ⇒ CB = 1 sin 15° ⇒ AB = 2CB = 2 sin 15°*	M1 E1 [2]	AB=2AC or 2CB ∠AOC= 15° oe
(*B*) cos 30° = 1 – 2 $\sin^2$ 15° cos 30° = √3/2 ⇒ √3/2 = 1 – 2 $\sin^2$ 15° ⇒ 2 $\sin^2$ 15° = 1 – √3/2 = (2 – √3)/2 ⇒ $\sin^2$15° = (2 – √3)/4 ⇒ $\sin 15° = \sqrt{\dfrac{2-\sqrt{3}}{4}} = \dfrac{1}{2}\sqrt{2-\sqrt{3}}$ *	B1 B1 M1 E1 [4]	simplifying
(*C*) Perimeter = 12 × AB = 24 × ½ √(2 – √3) = 12√(2 – √3) circumference of circle > perimeter of polygon ⇒ 2π > 12√(2 – √3) ⇒ π > 6√(2 – √3)	M1 E1 [2]	
(ii) (*A*) tan 15° = FE/OF ⇒ FE = tan 15° ⇒ DE = 2FE = 2tan 15°	M1 E1 [2]	
(*B*) $\tan 30 = \dfrac{2\tan 15}{1-\tan^2 15} = \dfrac{2t}{1-t^2}$ tan 30 = 1/√3 ⇒ $\dfrac{2t}{1-t^2} = \dfrac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1-t^2$ ⇒ $t^2 + 2\sqrt{3}\,t - 1 = 0$ *	B1 M1 E1 [3]	
(*C*) $t = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = 2-\sqrt{3}$ circumference < perimeter ⇒ 2π < 24(2 – √3) ⇒ π < 12(2 – √3) *	M1 A1 M1 E1 [4]	using positive root from exact working
(iii) 6√(2 – √3) < π < 12(2 – √3) ⇒ 3.106 < π < 3.215	B1 B1 [2]	3.106, 3.215

Comprehension

1. $\frac{1}{4}\times[3+1+(-1)+(-2)]=0.25$ * **M1, E1**

2. (i) *b* is the benefit of shooting some soldiers from the other side while none of yours are shot**.** *w* is the benefit of having some of your own soldiers shot while not shooting any from the other side.

Since it is more beneficial to shoot some of the soldiers on the other side than it is to have your own soldiers shot, $b > w$. **E1**

(ii) *c* is the benefit from mutual co-operation (i.e. no shooting).
d is the benefit from mutual defection (soldiers on both sides are shot).
With mutual co-operation people don't get shot, while they do with mutual defection.
So $c > d$. **E1**

3. $\frac{1\times 2+(-2)\times(n-2)}{n}=-1.999$ **or equivalent** (allow n,n+2) **M1, A1**

$n = 6000$ so you have played 6000 rounds. **A1**

4. No. The inequality on line 132, $b + w < 2c$, would not be satisfied since $6+(-3) > 2\times 1$. **M1** *b*+*w*<2c and subst **A1** No,3>2oe

5. (i)

Round	You	Opponent	Your score	Opponent's score
1	C	D	-2	3
2	D	C	3	-2
3	C	D	-2	3
4	D	C	3	-2
5	C	D	-2	3
6	D	C	3	-2
7	C	D	-2	3
8	D	C	3	-2
…	…	…	…	…

M1 Cs and Ds in correct places**, A1** C=-2**, A1** D=3

(ii) $\frac{1}{2}\times[3+(-2)]=0.5$ DM1 A1ft their 3,-2

6. (i) All scores are increased by two points per round **B1**

(ii) The same player wins. No difference/change. The rank order of the players remains the same. **B1**

7. (i) They would agree to co-operate by spending less on advertising or by sharing equally. **B1**

(ii) Increased market share (or more money or more customers). **DB1**

4754 (C4) Applications of Advanced Mathematics

1		$\frac{1+2x}{(1-2x)^2}=(1+2x)(1-2x)^{-2}$	M1	binomial expansion power -2
		$=(1+2x)[1+(-2)(-2x)+\frac{(-2)(-3)}{1.2}(-2x)^2+...]$	A1	unsimplified,correct
		$=(1+2x)[1+4x+12x^2+...]$	A1	sufficient terms
		$=1+4x+12x^2+2x+8x^2+...$	M1	
		$=1+6x+20x^2+...$	A1	
		Valid for $-1 < -2x < 1$	M1	
		$\Rightarrow -½ < x < ½$	A1	
			[7]	
2		$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$	M1	oe eg converting either side into a one line fraction(s) involving sin θ and cos θ.
		$\Rightarrow \cot 2\theta=\frac{1}{\tan 2\theta}=\frac{1-\tan^2\theta}{2\tan\theta}$ *	E1	
		cot 2θ = 1 + tan θ		
		$\Rightarrow \frac{1-\tan^2\theta}{2\tan\theta}=1+\tan\theta$		
		$\Rightarrow 1-\tan^2\theta=2\tan\theta+2\tan^2\theta$		
		$\Rightarrow 3\tan^2\theta+2\tan\theta-1=0$	M1	quadratic = 0
		$\Rightarrow (3\tan\theta-1)(\tan\theta+1)=0$	M1	factorising or solving
		$\Rightarrow$ tan θ = 1/3, θ = 18.43°, 198.43°		
		or tan θ = –1, θ = 135°, 315°	A3,2,1, 0	18.43°, 198.43°, 135°, 315° –1 extra solutions in the range
			[7]	
3	**(i)**	$\frac{dy}{dt}=\frac{(1+t).2-2t.1}{(1+t)^2}=\frac{2}{(1+t)^2}$	M1A1	
		$\frac{dx}{dt}=2e^{2t}$	B1	
		$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	M1	
		$\Rightarrow \frac{dy}{dx}=\frac{\frac{2}{(1+t)^2}}{2e^{2t}}=\frac{1}{e^{2t}(1+t)^2}$	A1	
		$t = 0 \Rightarrow dy/dx = 1$	B1ft	
			[6]	

	(ii)	$2t = \ln x \Rightarrow t = \frac{1}{2}\ln x$ $\Rightarrow \quad y = \dfrac{\ln x}{1+\frac{1}{2}\ln x} = \dfrac{2\ln x}{2+\ln x}$	M1 A1 **[2]**	or t in terms of y
4	(i)	$\overrightarrow{AB} = \begin{pmatrix}-2\\-1\\-1\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-1\\-11\\3\end{pmatrix}$	B1 B1 **[2]**	
	(ii)	$\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-2\\-1\\-1\end{pmatrix} = -4+1+3=0$ $\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-1\\-11\\3\end{pmatrix} = -2+11-9=0$ $\Rightarrow$ plane is $2x - y - 3z = d$ $x = 1, y = 3, z = -2 \Rightarrow d = 2 - 3 + 6 = 5$ $\Rightarrow$ plane is $2x - y - 3z = 5$	M1 E1 E1 M1 A1 **[5]**	scalar product
5	(i)	$x = -5 + 3\lambda = 1 \Rightarrow \lambda = 2$ $y = 3 + 2 \times 0 = 3$ $z = 4 - 2 = 2$, so (1, 3, 2) lies on 1st line. $x = -1 + 2\mu = 1 \Rightarrow \mu = 1$ $y = 4 - 1 = 3$ $z = 2 + 0 = 2$, so (1, 3, 2) lies on 2nd line.	M1 E1 E1 **[3]**	finding λ or μ verifying two other coordinates for line 1 verifying two other coordinates for line 2
	(ii)	Angle between $\begin{pmatrix}3\\0\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\0\end{pmatrix}$ $\cos\theta = \dfrac{3\times2+0\times(-1)+(-1)\times0}{\sqrt{10}\sqrt{5}}$ $= 0.8485\ldots$ $\Rightarrow \quad \theta = 31.9°$	M1 M1 A1 A1 **[4]**	direction vectors only allow M1 for any vectors or 0.558 radians

6	**(i)**	BAC = 120 – 90 – (90 – θ) = θ – 60 $\Rightarrow$ BC = b sin(θ – 60) CD = AE = a sin θ $\Rightarrow$ h = BC + CD = a sin θ + b sin (θ – 60°) *	B1 M1 E1 **[3]**	
	(ii)	h = a sin θ + b sin (θ – 60°) = a sin θ + b (sinθ cos 60 – cosθ sin 60) = a sin θ + ½ b sin θ – √3/2 b cos θ $= (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta *$	M1 M1 E1 **[3]**	corr compound angle formula sin 60 = √3/2, cos 60 = ½ used
	(iii)	OB horizontal when h = 0 $\Rightarrow (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta = 0$ $\Rightarrow (a + \frac{1}{2}b)\sin\theta = \frac{\sqrt{3}}{2}b\cos\theta$ $\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}b}{a + \frac{1}{2}b}$ $\Rightarrow \tan\theta = \frac{\sqrt{3}b}{2a + b} *$	M1 M1 E1 **[3]**	$\frac{\sin\theta}{\cos\theta} = \tan\theta$
	(iv)	$2\sin\theta - \sqrt{3}\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow$ R cos α = 2, R sin α = √3 $\Rightarrow$ $R^2 = 2^2 + (\sqrt{3})^2 = 7$, R = √7 = 2.646 m tan α = √3/2, α = 40.9° So h = √7 sin(θ – 40.9°) $\Rightarrow$ h_{max} = √7 = 2.646 m when θ – 40.9° = 90° $\Rightarrow$ θ = 130.9°	M1 B1 M1A1 B1ft M1 A1 **[7]**	

7	(i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -1(1+\mathrm{e}^{-t})^{-2}.-\mathrm{e}^{-t}$ $= \frac{\mathrm{e}^{-t}}{(1+\mathrm{e}^{-t})^2}$ $1-x = 1-\frac{1}{1+\mathrm{e}^{-t}}$ $1-x = \frac{1+\mathrm{e}^{-t}-1}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}}$ $\Rightarrow$ $x(1-x) = \frac{1}{1+\mathrm{e}^{-t}}\frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{(1+\mathrm{e}^{-t})^2}$ $\Rightarrow$ $\frac{dx}{dt} = x(1-x)$ When $t = 0$, $x = \frac{1}{1+\mathrm{e}^0} = 0.5$	M1 A1 M1 A1 E1 B1 **[6]**	chain rule substituting for $x(1-x)$ $1-x = \frac{1+\mathrm{e}^{-t}-1}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}}$ [OR,M1 A1 for solving differential equation for t, B1 use of initial condition, M1 A1 making x the subject, E1 required form]
	(ii)	$\frac{1}{(1+\mathrm{e}^{-t})} = \frac{3}{4}$ $\Rightarrow$ $\mathrm{e}^{-t} = 1/3$ $\Rightarrow$ $t = -\ln 1/3 = 1.10$ years	M1 M1 A1 **[3]**	 correct log rules
	(iii)	$\frac{1}{x^2(1-x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1-x}$ $\Rightarrow$ $1 = A(1-x) + Bx(1-x) + Cx^2$ $x = 0 \Rightarrow A = 1$ $x = 1 \Rightarrow C = 1$ coefft of x^2: $0 = -B + C \Rightarrow B = 1$	 M1 M1 B(2,1,0) **[4]**	 clearing fractions substituting or equating coeffs for A,B or C $A = 1$, $B = 1$, $C = 1$ www
	(iv)	$\int \frac{\mathrm{d}x}{x^2(1-x)}\mathrm{d}x = \int \mathrm{d}t$ $\Rightarrow$ $t = \int(\frac{1}{x^2} + \frac{1}{x} + \frac{1}{1-x})\mathrm{d}x$ $= -1/x + \ln x - \ln(1-x) + c$ When $t = 0$, $x = ½ \Rightarrow 0 = -2 + \ln ½ - \ln ½ + c$ $\Rightarrow$ $c = 2$. $\Rightarrow$ $t = -1/x + \ln x - \ln(1-x) + 2$ $= 2 + \ln\frac{x}{1-x} - \frac{1}{x}$ *	M1 B1 B1 M1 E1 **[5]**	separating variables $-1/x + \ldots$ $\ln x - \ln(1-x)$ ft their A,B,C substituting initial conditions
	(v)	$t = 2 + \ln\frac{3/4}{1-3/4} - \frac{1}{3/4} = \ln 3 + \frac{2}{3} = 1.77$ yrs	M1A1 **[2]**	

1	15	B1	
2	THE MATHEMATICIAN	B1	
3	M H X I Q 3 or 4 correct – award 1 mark	B2	
4	Two from Ciphertext N has high frequency E would then correspond to ciphertext R which also has high frequency T would then correspond to ciphertext G which also has high frequency A is preceded by a string of six letters displaying low frequency	 B1 B1	 oe oe
5	The length of the keyword is a factor of both 84 and 40. The only common factors of 84 and 40 are 1,2 and 4 (and a keyword of length 1 can be dismissed in this context)	M1 E1	
6	Longer strings to analyse so letter frequency more transparent. Or there are fewer 2-letter keywords to check	B2	
7	OQH DRR EBG One or two accurate – award 1 mark	B2	
8 (i) **(ii)** **(iii)**	Evidence of intermediate H FACE Evidence of intermediate HCEG – award 2 marks Evidence of accurate application of one of the two decoding ciphers - award 1 mark $800 = (3 \times 266) + 2$; the second row gives T so plaintext is R	B1 B3 M1 A1	 Use of second row

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

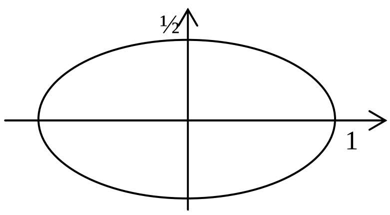
Advanced GCE **4754A**

Applications of Advanced Mathematics (C4) Paper A

Mark Scheme for June 2010

Section A

1 $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x}{(x-1)(x+1)}+\frac{2}{x+1}$	B1	$x^2 - 1 = (x + 1)(x - 1)$
$=\frac{x+2(x-1)}{(x-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{(3x-2)}{(x-1)(x+1)}$	A1	or $\frac{(3x-2)}{x^2-1}$ do not isw for incorrect subsequent cancelling
or $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x(x+1)+2(x^2-1)}{(x^2-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{3x^2+x-2}{(x^2-1)(x+1)}$		
$=\frac{(3x-2)(x+1)}{(x^2-1)(x+1)}$	B1	$(3x - 2)(x + 1)$
$=\frac{(3x-2)}{(x^2-1)}$	A1 [3]	accept denominator as x^2-1 or(x-1)(x+1) do not isw for incorrect subsequent cancelling
2(i) When $x = 0.5$, $y = 1.1180$	B1	4dp
$\Rightarrow$ $A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$	M1	
$= 0.25 \times 4.6059 = 1.151475$		(0.125 x 9.2118)
$= 1.151$ (3 d.p.)*	E1 [3]	need evidence
(ii) Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate.	B1	or use a diagram to show why
This becomes less with more strips. *or* Greater number of strips improves accuracy so becomes less	B1 [2]	
(iii) $V = \int_0^1 \pi y^2 dx$		
$= \int_0^1 \pi(1+x^2)dx$	M1	allow limits later
$= \pi\left[(x+x^3/3)\right]_0^1$	B1	$x + x^3/3$
$= 1\frac{1}{3}\pi$	A1 [3]	exact

3 $y = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$	M1	use of $\sin 2\theta$
$x = \cos 2\theta$ $\sin^2 2\theta + \cos^2 2\theta = 1$	M1	
$\Rightarrow x^2 + (2y)^2 = 1$ $\Rightarrow x^2 + 4y^2 = 1$ *	E1	
or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4(\sin\theta\cos\theta)^2$	M1	substitution
$= \cos^2 2\theta + \sin^2 2\theta$	M1	use of $\sin 2\theta$
$= 1$ *	E1	
or $\cos 2\theta = 2\cos^2\theta - 1$ $\cos^2\theta = (x+1)/2$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\sin^2\theta = (1-x)/2$	M1	for both
$y^2 = \sin^2\theta\cos^2\theta = \left(\frac{1-x}{2}\right)\left(\frac{x+1}{2}\right)$	M1	
$y^2 = (1-x^2)/4$ $x^2 + 4y^2 = 1$*	E1	
or $x = \cos 2\theta = \cos^2\theta - \sin^2\theta$ $x^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $y^2 = \sin^2\theta\cos^2\theta$	M1	correct use of double angle formulae
$x^2 + 4y^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + 4\sin^2\theta\cos^2\theta$ $= (\cos^2\theta + \sin^2\theta)^2$	M1	correct squaring and use of $\sin^2\theta + \cos^2\theta = 1$
$= 1$*	E1	
	M1 A1	ellipse correct intercepts
	[5]	

4 $\sqrt{4+x}=2(1+\frac{x}{4})^{\frac{1}{2}}$	M1	dealing with √4 (or terms in $4^{\frac{1}{2}}$, $4^{\frac{-1}{2}}$,…etc)
$= 2(1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}.-\frac{1}{2}}{2}(\frac{x}{4})^2+...)$	M1 A1	correct binomial coefficients correct **unsimplified** expression for $(1+x/4)^{\frac{1}{2}}$ or $(4+x)^{\frac{1}{2}}$
$= 2(1+\frac{1}{8}x-\frac{1}{128}x^2+...)$		
$= 2+\frac{1}{4}x-\frac{1}{64}x^2+...$	A1	cao
Valid for $-1 < x/4 < 1$ $\Rightarrow$ $-4 < x < 4$	B1 [5]	
5(i) $\frac{3}{(y-2)(y+1)}=\frac{A}{y-2}+\frac{B}{y+1}$ $=\frac{A(y+1)+B(y-2)}{(y-2)(y+1)}$		
$\Rightarrow$ $3 = A(y + 1) + B(y - 2)$ $y = 2 \Rightarrow 3 = 3A \Rightarrow A = 1$ $y = -1 \Rightarrow 3 = -3B \Rightarrow B = -1$	M1 A1 A1 [3]	substituting, equating coeffs or cover up
(ii) $\frac{dy}{dx}=x^2(y-2)(y+1)$		
$\Rightarrow$ $\int\frac{3\,dy}{(y-2)(y+1)}=\int 3x^2\,dx$	M1	separating variables
$\Rightarrow$ $\int(\frac{1}{(y-2)}-\frac{1}{y+1})\,dy=\int 3x^2\,dx$		
$\Rightarrow$ $\ln(y-2)-\ln(y+1)=x^3+c$	B1ft B1	$\ln(y-2)-\ln(y+1)$ ft their A,B x^3+c
$\Rightarrow$ $\ln\left(\frac{y-2}{y+1}\right)=x^3+c$		
$\Rightarrow$ $\frac{y-2}{y+1}=e^{x^3+c}=e^{x^3}.e^c=Ae^{x^3}$ *	M1 E1 [5]	anti-logging including c www
6 $\tan(\theta+45)=\frac{\tan\theta+\tan 45}{1-\tan\theta\tan 45}$	M1	oe using sin/cos
$=\frac{\tan\theta+1}{1-\tan\theta}$	A1	
$\Rightarrow$ $\frac{\tan\theta+1}{1-\tan\theta}=1-2\tan\theta$		
$\Rightarrow$ $1+\tan\theta=(1-2\tan\theta)(1-\tan\theta)$ $=1-3\tan\theta+2\tan^2\theta$ $\Rightarrow$ $0=2\tan^2\theta-4\tan\theta=2\tan\theta(\tan\theta-2)$	M1 A1 M1	multiplying up and expanding any correct one line equation solving quadratic for $\tan\theta$ oe
$\Rightarrow$ $\tan\theta=0$ or 2 $\Rightarrow$ $\theta=0$ or 63.43	A1A1 [7]	www -1 extra solutions in the range

Section B

7(i) $\overrightarrow{AB} = \begin{pmatrix} 100-(-200) \\ 200-100 \\ 100-0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ *	E1	
AB = $\sqrt{(300^2 + 100^2 + 100^2)}$ = 332 m	M1 A1 [3]	accept surds
(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$	B1B1	oe
Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	M1	…and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
$\Rightarrow \cos\theta = \frac{3\times0+1\times0+1\times1}{\sqrt{11}\sqrt{1}} = \frac{1}{\sqrt{11}}$	M1 A1	complete scalar product method(including cosine) for correct vectors
$\Rightarrow \theta = 72.45°$	A1 [6]	72.5º or better, accept 1.26 radians
(iii) Meets plane of layer when $(-200+300\lambda) + 2(100+100\lambda) + 3\times100\lambda = 320$	M1	
$\Rightarrow 800\lambda = 320$ $\Rightarrow \lambda = 2/5$	A1	
$\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$	M1	
so meets layer at (–80, 140, 40)	A1 [4]	
(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$	B1	
Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$		
$\Rightarrow \cos\theta = \frac{3\times1+1\times2+1\times3}{\sqrt{11}\sqrt{14}} = \frac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$	M1A1	complete method
$\Rightarrow \theta = 49.86°$	A1	
$\Rightarrow$ angle with layer = 40.1°	A1 [5]	ft 90-theirθ accept radians

8(i) At A, $y = 0 \Rightarrow 4\cos\theta = 0$, $\theta = \pi/2$ At B, $\cos\theta = -1$, $\Rightarrow \theta = \pi$ x-coord of A $= 2\times\pi/2 - \sin\pi/2 = \pi - 1$ x-coord of B $= 2\times\pi - \sin\pi = 2\pi$ $\Rightarrow$ OA $= \pi - 1$, AC $= 2\pi - \pi + 1 = \pi + 1$ $\Rightarrow$ ratio is $(\pi - 1):(\pi + 1)$ *	B1 B1 M1 A1 E1 [5]	 for either A or B/C for both A and B/C
(ii) $\frac{dy}{d\theta} = -4\sin\theta$ $\frac{dx}{d\theta} = 2 - \cos\theta$ $\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= -\frac{4\sin\theta}{2-\cos\theta}$ At A, gradient $= -\frac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	B1 M1 A1 A1 [4]	either dx/dθ or dy/dθ www
(iii) $\frac{dy}{dx} = 1 \Rightarrow -\frac{4\sin\theta}{2-\cos\theta} = 1$ $\Rightarrow -4\sin\theta = 2 - \cos\theta$ $\Rightarrow \cos\theta - 4\sin\theta = 2$ *	M1 E1 [2]	their dy/dx =1
(iv) $\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$ $= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1$, $R\sin\alpha = 4$ $\Rightarrow R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$ $\tan\alpha = 4$, $\alpha = 1.326$ $\Rightarrow \sqrt{17}\cos(\theta + 1.326) = 2$ $\Rightarrow \cos(\theta + 1.326) = 2/\sqrt{17}$ $\Rightarrow \theta + 1.326 = 1.064, 5.219, 7.348$ $\Rightarrow \theta = (-0.262), 3.89, 6.02$	 M1 B1 M1 A1 M1 A1 A1 [7]	 corr pairs accept 76.0º , 1.33 radians inv cos (2/√17) ft their R for method -1 extra solutions in the range

RECOGNISING ACHIEVEMENT

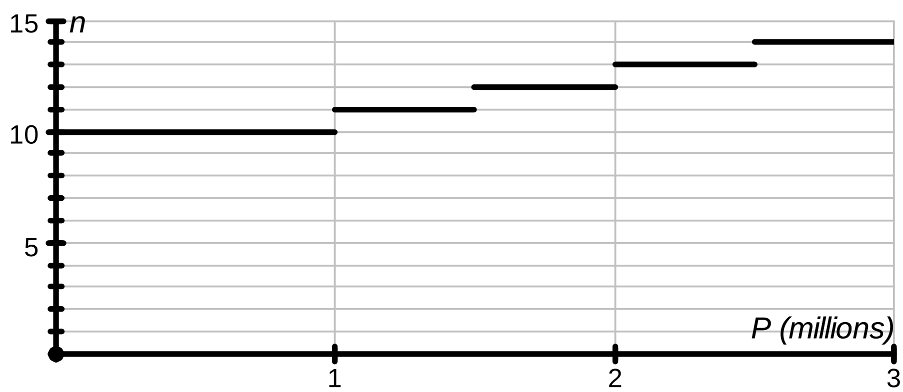
GCE

Mathematics (MEI)

Advanced GCE **4754B**

Applications of Advanced Mathematics (C4) Paper B: Comprehension Greener Travel

Mark Scheme for June 2010

1.		Rail: $307 \times 0.0602 = 18.4814 = 18.5$ kg (3 sf) Road: $300 \times 0.2095 \div 1.58 = 39.77... = 39.8$ kg (3 sf) Reduction = 21.3 kg	**B1** for either **B1**
2.		$y = \frac{1}{10^4}(x^3 - 100x^2 - 10000x + 2100100)$.. $\Rightarrow \frac{dy}{dx} = \frac{1}{10^4}(3x^2 - 200x - 10\ 000)$ $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 200x - 10\ 000 = 0$ $(3x+100)(x-100) = 0$ $x = 100$ (or $x = -\frac{100}{3}$) The graph shows the minimum emission occurs at speed of 100 km hour^{-1} Substituting $x = 100$ gives $y = 110.01$ Minimum rate of emission is 110 grams per km.	**M1** **A1** **M1** solving quadratic **A1** **A1** or $\frac{d^2y}{dx^2}$ justify min **A1**
3.	**(i)**	Substituting $p = 250, d = 279, s = 4$ in $E = (10 + 0.0015p)d + 200s$ $\Rightarrow E = 3694.625$ (in kg) So emissions are 3.7 tonnes to 2 s.f. *	**M1** subst **E1**
	(ii)	Emission rate = 1.5 g km^{-1} Distance = 279 km Emissions = 1.5×279 = 418.5 g = 0.42 kg (2 s.f.), and so is less than ½ kg. *or* p=251 in formula gives E=3695.0435, difference=0.4185kg<0.5kg	 **E1**
4.	**(i)**	 **Approximate Step graph** **Correct with scales shown**	 **M1** **A1**
	(ii)	There is a basic service of 10 trains a day for up to 1 million passengers per year. For every half million extra passengers above 1 million, an extra daily train is provided.	**B1** **B1**

5.		100 miles = 1.609344 $\times$ 100 km = 160 km 934 m 40 cm So it appears to give the answer to the nearest 10 cm (option B).	**M1** **A1** **A1** **[18]**

Section A

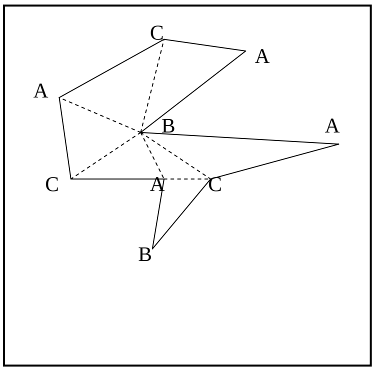
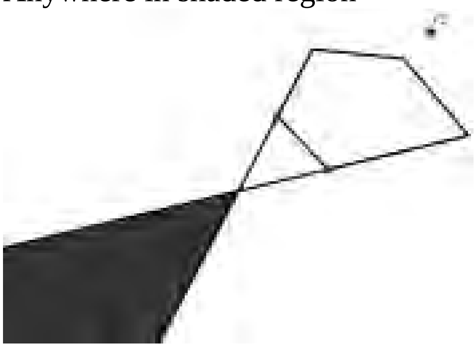
Question	Mark	Comment
1(i) x: -2, -1, 0, 1, 2 y: 1.0655, 1.1696, 1.4142, 1.9283, 2.8964 $A \approx ½\times1\{1.0655+2.8964+2(1.1696+1.4142+1.9283)\}$ $= 6.493$	B2,1,0 M1 A1 [4]	table values formula 6.5 or better www
(ii) Smaller, as the trapezium rule is an over-estimate in this case and the error is less with more strips	B1 B1 [2]	
2 $x=\frac{1}{1+t} \Rightarrow 1+t=\frac{1}{x}$ $\Rightarrow t=\frac{1}{x}-1$ $y=\frac{1-t}{1+2t}=\frac{1-\frac{1}{x}+1}{1+\frac{2}{x}-2}$ $=\frac{2-\frac{1}{x}}{\frac{2}{x}-1}=\frac{2x-1}{2-x}$	M1 A1 M1 M1 A1 [5]	attempt to solve for t oe substituting for t in terms of x clearing subsidiary fractions
3 $(3-2x)^{-3}=3^{-3}(1-\frac{2}{3}x)^{-3}$ $=\frac{1}{27}(1+(-3)(-\frac{2}{3}x)+\frac{(-3)(-4)}{2}(-\frac{2}{3}x)^2+\ldots)$ $=\frac{1}{27}(1+2x+\frac{8}{3}x^2+\ldots)$ $=\frac{1}{27}+\frac{2}{27}x+\frac{8}{81}x^2+\ldots$ Valid for $-1<-\frac{2}{3}x<1$ $\Rightarrow -\frac{3}{2}<x<\frac{3}{2}$	M1 B1 B2,1,0 A1 M1 A1 [7]	dealing with the '3' correct binomial coeffs 1, 2, 8/3 oe cao

4(i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{AB}.\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}.\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 2\times5+3\times0+(-5)\times2=0$ $\Rightarrow$ AB is perpendicular to BC.	B1 B1 M1E1 [4]	
(ii) AB = $\sqrt{(2^2 + 3^2 + (-5)^2)} = \sqrt{38}$ BC = $\sqrt{(5^2 + 0^2 + 2^2)} = \sqrt{29}$ Area = ½ ×$\sqrt{38}$ × $\sqrt{29}$ = ½ $\sqrt{1102}$ or 16.6 units^2	M1 B1 A1 [3]	complete method ft lengths of both AB, BC oe www
5 LHS = $\frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ $= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$ $= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$	M1 M1 E1 [3]	one correct double angle formula used cancelling cos θ's
6(i) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8-3\lambda \\ -2 \\ 6+\lambda \end{pmatrix}$ Substituting into plane equation: $2(-8-3\lambda) - 3(-2) + 6 + \lambda = 11$ $\Rightarrow$ $-16 - 6\lambda + 6 + 6 + \lambda = 11$ $\Rightarrow$ $5\lambda = -15$, $\lambda = -3$ So point of intersection is (1, –2, 3)	B1 M1 A1 A1ft [4]	
(ii) Angle between $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ $\cos\theta = \frac{2\times(-3)+(-3)\times0+1\times1}{\sqrt{14}\sqrt{10}}$ = (–)0.423 $\Rightarrow$ acute angle = 65°	B1 M1 A1 A1 [4]	allow M1 for a complete method only for any vectors

Section B

7(i) When $t = 0$, $v = 5(1 - e^0) = 0$ As $t \to \infty$, $e^{-2t} \to 0$, $\Rightarrow$ $v \to 5$ When $t = 0.5$, $v = 3.16$ m s^{-1}	E1 E1 B1 [3]	
(ii) $\frac{dv}{dt} = 5\times(-2)e^{-2t} = 10e^{-2t}$ $10 - 2v = 10 - 10(1 - e^{-2t}) = 10e^{-2t}$ $\Rightarrow$ $\frac{dv}{dt} = 10 - 2v$	B1 M1 E1 [3]	
(iii) $\frac{dv}{dt} = 10 - 0.4v^2$ $\Rightarrow$ $\frac{10}{100 - 4v^2}\frac{dv}{dt} = 1$ $\Rightarrow$ $\frac{10}{25 - v^2}\frac{dv}{dt} = 4$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)}\frac{dv}{dt} = 4$ * $\frac{10}{(5-v)(5+v)} = \frac{A}{5-v} + \frac{B}{5+v}$ $\Rightarrow$ $10 = A(5 + v) + B(5 - v)$ $v = 5 \Rightarrow 10 = 10A \Rightarrow A = 1$ $v = -5 \Rightarrow 10 = 10B \Rightarrow B = 1$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)} = \frac{1}{5-v} + \frac{1}{5+v}$ $\Rightarrow$ $\int (\frac{1}{5-v} + \frac{1}{5+v})\,dv = 4\int dt$ $\Rightarrow$ $\ln(5+v) - \ln(5-v) = 4t + c$ when $t = 0$, $v = 0$, $\Rightarrow 0 = 4\times0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln\left(\frac{5+v}{5-v}\right) = 4t$ $\Rightarrow$ $t = \frac{1}{4}\ln\left(\frac{5+v}{5-v}\right)$ *	M1 E1 M1 A1 M1 A1 A1 E1 [8]	for both A=1,B=1 separating variables correctly and indicating integration ft their A,B, condone absence of c ft finding c from an expression of correct form
(iv) When $t \to \infty$, $e^{-4t} \to 0$, $\Rightarrow$ $v \to 5/1 = 5$ when $t = 0.5$, $t = \frac{5(1-e^{-2})}{1+e^{-2}} = 3.8\,\text{m s}^{-1}$	E1 M1A1 [3]	
(v) The first model	E1 [1]	www

8(i) $AC = 5\sec\alpha$ $\Rightarrow$ $CF = AC\tan\beta$ $= 5\sec\alpha\tan\beta$ $\Rightarrow$ $GF = 2CF = 10\sec\alpha\tan\beta$ *	B1 M1 E1 [3]	oe ACtanβ
(ii) $CE = BE - BC$ $= 5\tan(\alpha+\beta) - 5\tan\alpha$ $= 5(\tan(\alpha+\beta) - \tan\alpha)$ $= 5\left(\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} - \tan\alpha\right)$ $= 5\left(\dfrac{\tan\alpha+\tan\beta-\tan\alpha+\tan^2\alpha\tan\beta}{1-\tan\alpha\tan\beta}\right)$ $= \dfrac{5(1+\tan^2\alpha)\tan\beta}{1-\tan\alpha\tan\beta}$ $= \dfrac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$ *	 E1 M1 M1 DM1 E1 [5]	 compound angle formula combining fractions $\sec^2 = 1 + \tan^2$
(iii) $\sec^2 45° = 2$, $\tan 45° = 1$ $\Rightarrow$ $CE = \dfrac{5t\times 2}{1-t} = \dfrac{10t}{1-t}$ $CD = \dfrac{10t}{1+t}$ $\Rightarrow$ $DE = \dfrac{10t}{1-t} + \dfrac{10t}{1+t} = 10t(\dfrac{1}{1-t} + \dfrac{1}{1+t})$ $= 10t\left(\dfrac{1+t+1-t}{(1-t)(1+t)}\right) = \dfrac{20t}{1-t^2}$ *	B1 M1 A1 M1 E1 [5]	used substitution for both in CE or CD oe for both adding their CE and CD
(iv) $\cos 45° = 1/\sqrt{2} \Rightarrow \sec\alpha = \sqrt{2}$ $\Rightarrow$ $GF = 10\sqrt{2}\tan\beta = 10\sqrt{2}\,t$	M1 E1 [2]	
(v) $DE = 2GF$ $\Rightarrow$ $\dfrac{20t}{1-t^2} = 20\sqrt{2}t$ $\Rightarrow$ $1 - t^2 = 1/\sqrt{2} \Rightarrow t^2 = 1 - 1/\sqrt{2}$ * $\Rightarrow$ $t = 0.541$ $\Rightarrow$ $\beta = 28.4°$	 E1 M1 A1 [3]	 invtan t

Qn	Answer	Marks
1(i)	6 correct marks	B1
1(ii)	Either state both m and n odd or give a diagram (doorways between rooms not necessary) justification	B1 B1ft
2(i)	$\frac{9-1}{4} = 2 = \left\lfloor \frac{4+1}{2} \right\rfloor$	B2 (B1 for LHS correct)
2(ii)	$\begin{array}{c\|c\|c\|c\|c\|c} x & 1 & 2 & 3 & 4 & 5 \\ \hline \left\lceil \frac{x}{2} \right\rceil & 1 & 1 & 2 & 2 & 3 \end{array}$	B2,1,0
3.	If each of A, B and C appeared at least four times then the total number of vertices would have to be at least 3×4=12	E2
4(i)		M1 allow if one error A1
4(ii)	Two points labelled B above clearly marked (or f.t. from (i))	A1
5(i)	True. Two cameras at the vertices labelled A or at the vertices labelled B would cover the entire gallery	A1 M1 for either
5(ii)	False. One camera at either vertex labelled A would be sufficient (or C on RHS)	A1 M1
6	Anywhere in shaded region  correct construction correct shading	 M1 A1

1 $\frac{1}{(2x+1)(x^2+1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+1}$ $\Rightarrow$ $1 = A(x^2+1) + (Bx+C)(2x+1)$ $x = -½$: $1 = 1¼ A \Rightarrow A = 4/5$ coeff of x^2: $0 = A + 2B \Rightarrow B = -2/5$ constants: $1 = A + C \Rightarrow C = 1/5$	M1 M1 B1 B1 B1 [5]	correct form of partial fractions mult up and equating or substituting oe soi www www www	for omission of B or C on numerator, M0, M1, then (x= -1/2, A= 4/5) B1, B0, B0 is possible. for $\frac{A+Dx}{2x+1} + \frac{Bx+C}{x^2+1}$, M1,M1 then B1 for both A=4/5 and D=0, B1, B1 is possible. isw for incorrect assembly of final partial fractions following correct A,B & C. condone omission of brackets for second M1 only if the brackets are implied by subsequent working.
2 $(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2 + ...$ $= 1 + x - x^2 + \ldots$ Valid for $-1 \le 3x \le 1$ $\Rightarrow$ $-1/3 \le x \le 1/3$	M1 A1 A1 M1 A1 [5]	correct binomial coefficients $1 + x \ldots$ $\ldots - x^2$ or $\lvert 3x \rvert \le 1$ oe or $\lvert x \rvert \le 1/3$ (correct final answer scores M1A1)	ie 1, 1/3, (1/3)(-2/3)/2 not nCr form simplified www in this part simplified www in this part, ignore subsequent terms using $(3x)^2$ as $3x^2$ can score M1B1B0 condone omission of brackets if $3x^2$ is used as $9x^2$ do not allow MR for power 3 or -1/3 or similar **condone inequality signs throughout** or say < at one end and ≤ at the other condone $-1/3 \le \lvert x \rvert \le 1/3$, $x \le 1/3$ is M0A0 the last two marks are not dependent on the first three
3 $2\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 3$ $\Rightarrow R^2 = 2^2 + 3^2 = 13, R = \sqrt{13}$ $\tan\alpha = 3/2$, $\Rightarrow \alpha = 0.983$ minimum $1 - \sqrt{13}$, maximum $1 + \sqrt{13}$	M1 B1 M1 A1 B1 B1 [6]	correct pairs $R = \sqrt{13}$ or 3.61 or better 0.98 or better or –2.61, 4.61 or better	condone wrong sign at this stage correct division, ft from first M1 radians only accept multiples of π that round to 0.98 allow B1, B1ft for $1-\sqrt{R}$ and $1+\sqrt{R}$ for their R to 2dp or better

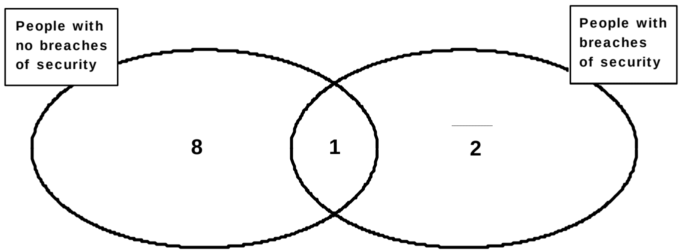
4(i) $x = 2\sin\theta$, $y = \cos 2\theta$ When $\theta = \pi/3$, $x = 2\sin\pi/3 = \sqrt{3}$ $y = \cos 2\pi/3 = -½$ **EITHER** $dx/d\theta = 2\cos\theta$, $dy/d\theta = -2\sin 2\theta$ $\Rightarrow \frac{dy}{dx} = \frac{-\sin 2\theta}{\cos\theta}$ $= \frac{-\sin 2\pi/3}{\cos\pi/3} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ ………………………………………………………….... **OR** expressing y in terms of x, $y=1-x^2/2$ $\frac{dy}{dx} = -x$ or $-2\sin\theta$ $= -\sqrt{3}$	B1 B1 M1 A1 A1 …….. M1 A1 A1 [5]	$x = \sqrt{3}$ $y = -½$ $dy/dx = (dy/d\theta) / (dx/d\theta)$ used any correct equivalent form exact www ………………………………………… exact www	exact only (isw all dec answers following exact ans) ft their derivatives if right way up (condone one further minor slip if intention clear) condone poor notation can isw if incorrect simplification
(ii) $y = 1 - 2\sin^2\theta = 1 - 2(x/2)^2 = 1 - ½x^2$	M1A1 [2]	or reference to (i) if used there	for M1, need correct trig identity and attempt to substitute for x allow SC B1 for $y=\cos 2\arcsin(x/2)$ or equivalent

5 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow$ $1 + \cot^2\theta = 1 + 2\cot\theta$ $\Rightarrow$ $\cot^2\theta - 2\cot\theta = 0$ $\Rightarrow$ $\cot\theta(\cot\theta - 2) = 0$ $\Rightarrow$ $\cot\theta = 0$, **and** $\cot\theta = 2$, $\tan\theta = ½$ $\Rightarrow$ $\theta = 26.6°, -153.4°, -90°, 90°$ ……………………………………………………….. **OR** $\frac{1}{\sin^2\theta} = 1 + \frac{2\cos\theta}{\sin\theta} = \frac{\sin\theta + 2\cos\theta}{\sin\theta}$ $\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta - 1 = 0$ $\Rightarrow 2\sin\theta\cos\theta - \cos^2\theta = 0$ $\Rightarrow \cos\theta\,(2\sin\theta - \cos\theta) = 0$ $\Rightarrow \cos\theta = 0$, **and** $\tan\theta = ½$ $\theta = 26.6°, -153.4°, -90°, 90°$	M1 M1 M1 B3,2,1,0 M1 M1 M1 B3,2,1,0 [6]	correct trig identity used factorising oe **both** needed and $\cot\theta = 1/\tan\theta$ soi –90°, 90°, 27°, –153° or better www …………………………………… correct trig equivalents and a one line equation (or common denominator) formed use of Pythagoras and factorising **both** needed and $\tan\theta = \sin\theta/\cos\theta$ oe soi accept 27º, -153º as above ……………………………………… answers, no working, award B3,2,1,0 (it is possible to score say M1 then B3 ow)	(use of $1-\cot^2\theta$ could lead to M0 M1 M1 B1) allow if $\cot\theta = 0$ not seen (ie quadratic equation followed by $\cot\theta - 2 = 0$ or $\cot\theta = 2$) (omission of $\cot\theta = 0$ could gain M1, M1, M0, B1) …………………………………………... as above allow if $\cos\theta = 0$ not seen (as above) ……………………………………… **in both cases**, -1 if **extra** solutions in the range are given (dependent on at least B1 being scored)-not their incorrect solutions eg 26.6°,-153.4°, 0°,180°,-180° would obtain B1 -1 MR if answers given in radians (-π/2,π/2,0.464, -2.68 (-1.57.1.57) or multiples of π that round to these, or better) (dependent on at least B1 being scored) to lose both of these, at least B2 would need to be scored.
6 Vol = vol of rev of curve + vol of rev of line vol of rev of curve = $\int_0^2 \pi x^2\,dy$ $= \int_0^2 \pi \frac{y}{2}\,dy$ $= \pi\left[\frac{y^2}{4}\right]_0^2$ $= \pi$ height of cone = 3 – 2 = 1 so vol of cone = 1/3 π 1^2 x1 = π/3 so total vol = 4π/3	M1 M1 B1 A1 B1 B1 A1 [7]	(soi) at any stage substituting $x^2 = y/2$ $\left[\frac{y^2}{4}\right]$ h=1 soi www cao	 for M1 need π, substitution for x^2, (dy soi), intention to integrate and correct limits even if π missing or limits incorrect or missing cao OR $\pi\int_2^3 (3-y)^2 dy$ M1(even if expanded incorrectly) =π/3 A1 www

Section B

7(i) $\overrightarrow{AB}=\begin{pmatrix}-4\\0\\-2\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}-2\\4\\1\end{pmatrix}$	B1B1		condone rows
$\cos BAC=\dfrac{\begin{pmatrix}-4\\0\\-2\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix}}{AB.AC}=\dfrac{(-4).(-2)+0.4+(-2).1}{\sqrt{20}\sqrt{21}}$	M1 M1	dot product evaluated **cos** BAC= dot product / |AB|. |AC|	**substituted,** ft their vectors AB, AC for method only need to see method for modulae as far as √…
$= 0.293$	A1	0.293 or cos ABC=correct numerical expression as RHS above, or better	use of vectors BA and CA could obtain B0 B0 M1 M1 A1 A1
⇒ BAC = 73.0°	A1 [6]	or rounds to 73.0º (accept 73º www)	(or 1.27 radians)
(ii) A: $x + y - 2z + d = 2 - 6 + d = 0$ ⇒ $d = 4$ B: $-2 + 0 - 2\times1 + 4 = 0$ C: $0 + 4 - 2\times4 + 4 = 0$	M1 DM1 A1	substituting one point evaluating for other two points d = 4 www	alternatively, finding the equation of the plane using any valid method (eg from vector equation, M1 A1 for using valid equation and eliminating both parameters, A1 for required form, or using vector cross product to get $x+y-2z=c$ oe M1 A1,finding c and required form, A1, or showing that two vectors in the plane are perpendicular to normal vector M1 A1 and finding d, A1) oe
Normal $\mathbf{n}=\begin{pmatrix}1\\1\\-2\end{pmatrix}$	B1	stated or used as normal anywhere in part (ii)	
$\mathbf{n}.\begin{pmatrix}0\\0\\1\end{pmatrix}=\dfrac{-2}{\sqrt{6}}=\cos\theta$	M1 A1	finding angle between normal vector and **k** allow ±2/√6 or 144.7º for A1	(may have deliberately made +ve to find acute angle)
⇒ θ = 144.7° ⇒ acute angle = 35.3°	A1 [7]	or rounds to 35.3º	do not need to find 144.7º explicitly (or 0.615 radians)
(iii) At D, $-2 + 4 - 2k + 4 = 0$ ⇒ $2k = 6$, $k = 3$ *	M1 A1	substituting into plane equation **AG**	
$\overrightarrow{CD}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}=\dfrac{1}{2}\overrightarrow{AB}$	M1	$\overrightarrow{CD}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}$	finding vector CD (or vector DC)
⇒ CD is parallel to AB	A1		or DC parallel to AB or BA oe (or hence two parallel sides, if clear which) but A0 if their vector CD is vector DC
CD : AB = 1 : 2	B1 [5]	mark final answer **www** allow CD:AB=1/2, √5:√20 oe, AB is twice CD oe	for B1 allow vector CD used as vector DC

8(i) $\frac{dV}{dt} = -kx$ $V = 1/3\, x^3 \Rightarrow dV/dx = x^2$ $\frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt} = x^2\frac{dx}{dt}$ $\Rightarrow x^2\frac{dx}{dt} = -kx$ $\Rightarrow x\frac{dx}{dt} = -k$ *	B1 M1 A1 [3]	oe eg $dx/dt = dx/dV \,.\, dV/dt = 1/x^2 .\, -kx = -k/x$ **AG**	
(ii) $x\frac{dx}{dt} = -k \Rightarrow \int x\,dx = \int -k\,dt$ $\Rightarrow \frac{1}{2}x^2 = -kt + c$ When $t = 0$, $x = 10 \Rightarrow 50 = c$ $\Rightarrow \frac{1}{2}x^2 = 50 - kt$ $\Rightarrow x = \sqrt{(100 - 2kt)}$ *	M1 A1 B1 A1 [4]	separating variables and intention to integrate condone absence of c finding c correctly ft their integral of form $ax^2 = bt + c$ where a,b non zero constants **AG**	
(iii) When $t = 50$, $x = 0$ $\Rightarrow 0 = 100 - 100k \Rightarrow k = 1$	M1 A1 [2]		
(iv) $dV/dt = 1 - kx = 1 - x$ $\Rightarrow x^2 dx/dt = 1 - x$ $\Rightarrow \frac{dx}{dt} = \frac{1-x}{x^2}$ *	M1 A1 [2]	for $dV/dt = 1 - kx$ or better **AG**	
(v) $\frac{1}{1-x} - x - 1 = \frac{1-(1-x)x-(1-x)}{1-x}$ $= \frac{1-x+x^2-1+x}{1-x} = \frac{x^2}{1-x}$ * $\int \frac{x^2}{1-x}dx = \int dt \Rightarrow \int (\frac{1}{1-x} - x - 1)dx = t + c$ $\Rightarrow -\ln(1-x) - \frac{1}{2}x^2 - x = t + c$ When $t = 0$, $x = 0 \Rightarrow c = -\ln 1 - 0 - 0 = 0$ $\Rightarrow t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x$ *	M1 A1 M1 A1 B1 A1 [6]	combining to single fraction **AG** separating variables & subst for $x^2/(1-x)$ and intending to integrate condone absence of c finding c for equation of correct form eg $c = 0$, or $\pm$ ln1 (allow c=0 without evaluation here) cao **AG**	or long division or cross multiplying check signs need both sides of integral accept ln $(1/(1-x))$ as $-\ln(1-x)$ www ie $a\ln(1-x)+bx^2+dx=et+c$ a,b,d,e non zero constants do not allow if c=0 without evaluation
(vi) understanding that ln (1/0) or 1/0 is undefined oe	B1 [1]	www	ln (1/0) = ln 0, $1/0 = \infty$ and ln $(1/0) = \infty$ are all B0

Question			Answer	Marks	Guidance	
1			$\frac{16}{250} = 6.4\%$ * or $\frac{16}{250}$ x 100 = 6.4*	B1 **[1]**	or $\frac{250-(64+170)}{250}$ =6.4% oe	need evaluation
2	(i)		The smallest possible PIN that does not begin with zero is 1000 and the largest is 9999, giving 9000. However the 9 numbers 1111, 2222, … 9999 are disallowed. The other disallowed numbers are 1234, 2345, … 6789 (6 numbers) And 9876, 8765, … 3210 (7 numbers). So, in all, there are 9000 – (9 + 6 + 7) = 8978 possible PINs	M1 M1 A1 **[3]**	from a correct starting point (eg 10,000 or 9000), clear attempt to eliminate (or not include) numbers starting with 0 clear attempt to eliminate **all three** of these categories (with approx correct values in each category) **if unclear, M0** M marks not dependent SC 8978 www B3	Alt1) for M1 (no 0 start), nos starting with 1,2,7,8,9 give 1000-2, nos starting with 3,4,5,6 give 1000-3 =5(1000-2)+4(1000-3)=8978 M1,A1 or2) eg starting with1, 1,not2,any,any+1,2,not3,any +1,2,3,not4 =900+90+9=999-(1111term)=998 can lead to 5(900+90+9-1)+4(900+90+9-2)=8978 oe
2	(ii)		$\frac{6\ 700\ 000\ 000}{8978} = 746\ 269$ The average is about 750 000.	M1 A1 **[2]**	ft from (i) ft	 accept 2sf (or 1sf) only for A1
3				M1 A1 **[2]**	numbers total 11 all correct	

Question			Answer	Marks	Guidance	
4			100 000 transactions from 80 people over 3½ years with 365 days per year $\frac{100\ 000}{(80\times3.5\times365)}(=0.978...)$ Approximately 1 transaction per person per day	M1 A1 **[2]**	 cao	allow approximate number of days in a year eg 360 for M1 A1
5			**Allow any one of the following for 1 mark** An attack can happen without a breach of the card's security. The probabilities that a successful attack followed or did not follow a breach of card security are so close that a court would look for other evidence before reaching a decision. In many cases of unauthorised withdrawals the banks refund the money. The banks' software does not detect all the attacks that occur.	B1 **[1]**	only accept versions of these statements	
6	(i)		(table below)	B1 B2 **[3]**	for top row 480, 20, 500 all five other entries correct	(500 000 is given) allow B1 for three or four correct from 499460,40,499500,499940,60

Transactions	**Authorised**	**Un-authorised**	**Total**
Queried	480	20	500
Not queried	499 460	40	499 500
Total	499 940	60	500 000

Question			Answer	Marks	Guidance	
6	(ii)		$\frac{480}{40}=12$ or 12 to 1	B1 [1]	ft from (i)	their 480: their 40 isw accept unsimplified answers
6	(iii)		(table below)			NB they are not required to complete the table.
				M1	ft from (i)	{2500or 5xtheir 500}-(their 60-5) [=their 2445]
				DM1		their 2445 ft from (i) :5
			$\frac{2445}{5}=489$ or 489 to 1	A1 [3]	cao	

Transactions	Authorised	Un-authorised	Total
Queried	2 445	55	2 500
Not queried	497 495	5	497 500
Total	499 940	60	500 000

Question			Answer	Marks	Guidance
1			$\frac{4x}{x+1}-\frac{3}{2x+1}=1$		
			$\Rightarrow\quad 4x(2x+1)-3(x+1)=(x+1)(2x+1)$	M1	Multiplying throughout by $(2x + 1)(x + 1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout) Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved Do not condone omission of brackets unless it is clear from subsequent work that they were assumed eg $4x(2x + 1) - 3(x + 1) = (x + 1)(2x \underline{- 1})$ gets M1 $4x(2x + 1) - 3(x + 1) = 1$ gets M0 $4x(x + 1)(2x + 1) - 3(x + 1)(2x + 1) = (x + 1)(2x + 1)$ gets M0 $4x(2x + 1) - 3(x + 1) = (x + 1)$ gets M1, just, for slip in omission of $(2x + 1)$
			$\Rightarrow\quad 8x^2 + 4x - 3x - 3 = 2x^2 + 3x + 1$ $\Rightarrow\quad 6x^2 - 2x - 4 = 0$	DM1	Multiplying out, collecting like terms and forming quadratic = 0. Follow through from their equation provided the algebra is not significantly eased and it is a quadratic. Condone a further sign or numerical error or minor slip when rearranging.
			$\Rightarrow\quad 3x^2 - x - 2 = 0$	A1	or $6x^2 - 2x - 4 = 0$ oe www, (not fortuitously obtained - check for double errors)
			$\Rightarrow\quad (3x + 2)(x - 1) = 0$	M1	Solving their three term quadratic provided $b^2 - 4ac \geq 0$. Use of <u>correct</u> quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving their correct x^2 and constant terms when factors multiplied out) or comp the square oe. soi
			$\Rightarrow\quad x = -2/3$ or 1	A1	cao for both obtained www (accept $-4/6$ oe, or exact decimal equivalent (condone -0.667or better))
				[5]	SC B1 $x = 1$ with or without any working

Question			Answer	Marks	Guidance
2			$(1+2x)^{1/2} = 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(2x)^2 + \frac{\frac{1}{2}.(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3 + ...$	M1	Do not MR for $n \neq 1/2$ All four correct binomial coeffs (not nCr form) soi Accept **unsimplified** coefficients if a subsequent error when simplifying.
				B1	Condone absence of brackets only if followed by correct work eg $2x^2 = 4x^2$ must be soi for second B mark. $1 + x$ www
			$= 1 + x - ½ x^2 + ½ x^3 + …$	B1 B1	… $- ½ x^2$ www … $+ ½ x^3$ www If there is an error in say the third coeff of the expansion, M0, B1, B0, B1 can be scored
			Valid for $\|x\| < 1/2$ or $-1/2 < x < 1/2$	B1	Independent of expansion $\|x\| \leq 1/2$ and $-1/2 \leq x \leq 1/2$ are actually correct in this case so we will accept them. Condone a combination of inequalities. Condone also, say $-1/2 < \|x\| < 1/2$ but not $x < 1/2$ or $-1 < 2x < 1$ or $-1/2 > x > 1/2$
				[5]	

Question			Answer	Marks	Guidance
3	(i)		$dV/dt = k\sqrt{V}$ $V = (½\, kt + c)^2$ $\Rightarrow$ $dV/dt = 2(½\, kt + c).½\, k$ $= k(½\, kt + c)$ $= k\sqrt{V}$	B1 M1 A1 A1 **[4]**	cao condone different *k* (allow MR B1 for = kV^2) 2(1/2 *kt* + *c*) × constant multiple of *k* (or from multiplying out oe; or implicit differentiation) cao www any equivalent form (including unsimplified) Allow SCB2 if $V=(1/2\, kt + c)^2$ fully obtained by integration including convincing change of constant if used Can score B1 M0 SCB2
	(ii)		$(½\, k + c)^2 = 10\,000 \Rightarrow ½\, k + c = 100$ $(k + c)^2 = 40\,000 \Rightarrow k + c = 200$ $\Rightarrow$ $½\, k = 100$ $\Rightarrow$ $k = 200, c = 0$ $\Rightarrow V = (100t)^2 = 10000t^2$	B1 B1 M1 A1 **[4]**	substituting any one from $t = 1$, $V = 10,000$ or $t = 0$, $V = 0$ or $t = 2$, $V = 40,000$ into squared form or rooted form of equation (Allow −/±100 or −/±200) substituting any other from above Solving correct equations for both www (possible solutions are (200,0), (−200,0), (600, −400), (−600,400) (some from –ve root)) either form www SC B2 for $V = (100t)^2$ oe stated without justification SCB4 if justification eg showing substitution SC those working with $(k + c)^2 = 30,000$ can score a maximum of B1B0 M1A0 (leads to $k \approx 146$, $c \approx 26.8$)

Question			Answer	Marks	Guidance
4			LHS = $\sec^2\theta + \text{cosec}^2\theta$		
			$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$	M1	**Use** of $\sec\theta = 1/\cos\theta$ and $\text{cosec}\theta = 1/\sin\theta$ not just stating
			$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$	M1	adding
			$= \dfrac{1}{\cos^2\theta\sin^2\theta}$	A1	use of $\cos^2\theta + \sin^2\theta = 1$ soi
			$=\sec^2\theta\,\text{cosec}^2\theta$	A1	**AG**
			…………………………………………………………		
			OR		
			$\sec^2\theta+\text{cosec}^2\theta=\tan^2\theta+1+\cot^2\theta+1=\sin^2\theta/\cos^2\theta+\cos^2\theta/\sin^2\theta+2$	M1	correct formulae oe
			$= \dfrac{\cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1	adding
			$= \dfrac{(\cos^2\theta + \sin^2\theta)^2}{\sin^2\theta\cos^2\theta} = \dfrac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta\,\text{cosec}^2\theta$	A1 A1	use of Pythagoras **AG**
			…………………………………………………………		
			OR working with both sides		
			Eg LHS $\sec^2\theta + \text{cosec}^2\theta = \tan^2\theta+1+\cot^2\theta+1=\tan^2\theta+\cot^2\theta+2$	M1	Correct formulae used on one side
			RHS $=(1+\tan^2\theta)(1+\cot^2\theta)= 1+ \tan^2\theta+ \cot^2\theta+\tan^2\theta\cot^2\theta$	M1	Use of same formulae on other side
			$=\tan^2\theta + \cot^2\theta + 2 =$ LHS	A1	Use of $\tan\theta\cot\theta = 1$ oe, dependent on both method marks
				A1	Showing equal
				[4]	

Question			Answer	Marks	Guidance
5			$\sin(x + 45°) = \sin x \cos 45° + \cos x \sin 45°$	M1	Use of **correct** compound angle formula
			$= \sin x\,.1/\sqrt{2} + \cos x\,.\,1/\sqrt{2}$	A1	
			$= (1/\sqrt{2})(\sin x + \cos x) = 2\cos x$		
			$\Rightarrow \quad \sin x + \cos x = 2\sqrt{2}\cos x$ *	A1	**Since AG ,** $\sin x \cos 45° + \cos x \sin 45° = 2\cos x$ $\sin x + \cos x = 2\sqrt{2}\cos x$ only gets M1 need the second line or statement of $\cos 45° = \sin 45° = 1/\sqrt{2}$ oe as an intermediate step to get A1 A1
			$\Rightarrow \quad \sin x = (2\sqrt{2} - 1)\cos x$		
			$\Rightarrow \quad \tan x = 2\sqrt{2} - 1$	M1	terms collected and $\tan x = \sin x / \cos x$ used
			$\Rightarrow \quad x = 61.32°$,	A1	for first correct solution
			$241.32°$	A1	for second correct solution and no others in the range 2dp but allow overspecification ignore solutions outside the range SC A1 for both 61.3° and 241.3° SC A1 for both 1.07 and 4.21 radians (or better) SC A1 for incorrect answers that round to 61.3° and 180° + their ans eg 61.33° and 241.33° Do not award SC marks if there are extra solutions in the range.
				[6]	

Question			Answer	Marks	Guidance
6			$\frac{dy}{dx} = \frac{y}{x(x+1)}$		
			$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x(x+1)}\,dx$	M1	correctly separating variables and intending to integrate (ie need to see attempt at integration or integral signs)
			$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$	M1	partial fractions soi
			$\Rightarrow 1 = A(x+1) + Bx$		
			$x = 0 \Rightarrow A = 1$	A1	$A = 1$ www
			$x = -1 \Rightarrow 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$\Rightarrow \ln y = \int (\frac{1}{x} - \frac{1}{x+1})\,dx = \ln x - \ln(x+1) + c$	A1	ft their A,B condone absence of c or ln$c$
			$x = 1, y = 1 \Rightarrow 0 = 0 - \ln 2 + c \Rightarrow c = \ln 2$	B1	evaluating their c **at any stage** dependent on x and y terms all being logs of correct form but **do not award following incorrect log rules,** ft their A,B**.** c could be say a decimal. (eg $y = x/(x+1) + c$ then c being found is B0)
			$\Rightarrow \ln y = \ln x - \ln(x+1) + \ln 2 = \ln(2x/(x+1))$	M1	correctly combining lns and antilogging throughout (must have included the constant term). Apply this strictly. Do not allow if c is included as an afterthought unless completely convinced. ft A,B Logs must be of correct form ie not following say $\int \frac{1}{x(x+1)}dx = \ln(x^2 + x)$ unless ft from partial fractions and $B = 1$
			$\Rightarrow y = 2x/(x+1)$	A1	cao www ($y = e^{.693}\left(\frac{x}{x+1}\right)$ loses final A1) **NB** evaluating c and log work can be in either order. eg $y = cx/(x+1)$, at $x = 1$, $y = 1$, $c = 2$
				[8]	

Question			Answer	Marks	Guidance
7	(i)		$\theta = -\pi/2$: O (0, 0) $\theta = 0$: P (2, 0) $\theta = \pi/2$: O (0, 0)	B1 B1 B1 **[3]**	Origin or O, condone omission of (0, 0) or O Or, say at P $x = 2$, $y = 0$, need P stated Origin or O, condone omission of (0,0) or O
7	(ii)		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{2\cos 2\theta}{-2\sin\theta} = -\frac{\cos 2\theta}{\sin\theta}$ When $\theta = \pi/2$ $dy/dx = -\cos \pi/ \sin\pi/2 = 1$ When $\theta = -\pi/2$ $dy/dx = -\cos(-\pi)/\sin(-\pi/2) = -1$ **Either** $1\times -1 = -1$ so perpendicular **Or** gradient tangent $=1 \Rightarrow$ meets axis at 45° , similarly, gradient $= -1 \Rightarrow$ meets axis at 45° oe	M1 A1 M1 A1 A1 **[5]**	their $dy/d\theta$ /$dx/d\theta$ any equivalent form www (not from $-2\cos2\theta/2\sin\theta$) subst $\theta = \pi/2$ in their equation Obtaining $dy/dx = 1$,and $dy/dx = -1$ **shown** (or explaining using symmetry of curve) **www** justification that tangents are perpendicular **www** dependent on previous A1
7	(iii)		At Q, $\sin 2\theta = 1 \Rightarrow 2\theta = \pi/2$, $\theta = \pi/4$ $\Rightarrow$ coordinates of Q are $(2\cos \pi/4, \sin \pi/2)$ $= (\sqrt{2}, 1)$	M1 A1 A1 **[3]**	or, using the derivative, $\cos 2\theta = 0$ soi or their $dy/dx = 0$ to find θ. If the only error is in the sign or the coeff of the derivative in (ii), allow full marks in this part (condone $\theta = 45°$) www (exact only) accept $2/\sqrt{2}$
7	(iv)		$\sin^2\theta = (1 - \cos^2\theta) = 1 - \frac{1}{4}x^2$ $\Rightarrow$ $y = \sin 2\theta = 2\sin\theta\cos\theta$ $= (\pm)\, x\sqrt{(1 - \frac{1}{4}x^2)}$ $\Rightarrow$ $y^2 = x^2(1 - \frac{1}{4}x^2)$*	B1 M1 A1 A1 **[4]**	oe, eg may be $x^2 =$….. **Use** of $\sin2\theta = 2\sin\theta\cos\theta$ subst for x **or** $y^2 = 4\sin^2\theta\cos^2\theta$ (squaring) either order oe squaring **or** subst for x either order oe **AG**

Question			Answer	Marks	Guidance
7	(v)		$V = \int_0^2 \pi x^2 (1 - \frac{1}{4}x^2)\,dx$	M1	integral including correct limits but ft their '2' from (i) (limits may appear later) condone omission of dx if intention clear
			$= \int_0^2 (\pi x^2 - \frac{1}{4}\pi x^4)\,dx$		
			$= \pi\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$	B1	$\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]$ ie allow if no π and/or incorrect/no limits (or equivalent by parts)
			$= \pi\left[\frac{8}{3} - \frac{32}{20}\right]$	A1	substituting limits into correct expression (including π) ft their '2'
			$= 16\pi/15$	A1 **[4]**	cao oe, 3.35 or better (any multiple of π must round to 3.35 or better)
8	(i)		$\overrightarrow{AA'} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$	B1	finding $\overrightarrow{AA'}$ or $\overrightarrow{A'A}$ by subtraction, **subtraction must be seen** B0 if $\overrightarrow{AA'}$, $\overrightarrow{A'A}$ confused Assume they have found $\overrightarrow{AA'}$ if no label
			This vector is normal to $x + 2y - 3z = 0$	B1	reference to normal or ***n***, or perpendicular to $x + 2y - 3z = 0$, or statement that vector matches coefficients of plane and is therefore perpendicular, or showing AA' is perpendicular to **two** vectors in the plane
			M is (1½, 3, 2½) $x + 2y - 3z = 1½ + 6 - 7½ = 0$	M1	for finding M correctly (can be implied by two correct coordinates)
			$\Rightarrow$ M lies in plane	A1 **[4]**	showing numerical subst of M in plane = 0

Question			Answer	Marks	Guidance
8	(ii)		$\mathbf{r}=\begin{pmatrix}1\\2\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1+\lambda\\2-\lambda\\4+2\lambda\end{pmatrix}$ meets plane when		
			$1+\lambda+2(2-\lambda)-3(4+2\lambda)=0$	M1	subst of **AB** in the plane
			$\Rightarrow -7-7\lambda=0,\ \lambda=-1$	A1	
			So B is (0, 3, 2)	A1	cao
			$\overrightarrow{A'B}=\begin{pmatrix}0\\3\\2\end{pmatrix}-\begin{pmatrix}2\\4\\1\end{pmatrix}=\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	or $\overrightarrow{BA'}$, ft **only** on their B (condone $\overrightarrow{A'B}$ used as $\overrightarrow{BA'}$ or no label) (can be implied by two correct coordinates)
			Eqn of line A'B is $\mathbf{r}=\begin{pmatrix}2\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	B1 ft	$\begin{pmatrix}2\\4\\1\end{pmatrix}$ or their B +……
				A1 ft **[6]**	…λ × their $\overrightarrow{A'B}$ (or $\overrightarrow{BA'}$) ft only their B correctly
8	(iii)		Angle between $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	correct vectors but ft their $\overrightarrow{A'B}$. Allow say, $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and/or $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ condone a minor slip if intention is clear
			$\Rightarrow \cos\theta=\dfrac{1.(-2)+(-1).(-1)+2.1}{\sqrt{6}.\sqrt{6}}$	M1	correct formula (including cosθ) for their direction vectors from (ii) condone a minor slip if intention is clear
			= 1/6	A1	±1/6 or 99.6º from appropriate vectors only soi **Do not allow either A mark if the correct *B* was found fortuitously in (ii)**
			$\Rightarrow \theta=80.4°$	A1 **[4]**	cao or better

Question			Answer	Marks	Guidance
8	(iv)		Equation of BC is $\mathbf{r} = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix} = \begin{pmatrix}2-2\lambda\\4-\lambda\\1+\lambda\end{pmatrix}$		**NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$
			Crosses Oxz plane when $y = 0$	M1	For putting $y = 0$ in their line BC and solving for λ
			$\Rightarrow \lambda = 4$	A1	Do not allow either A mark if *B* was found fortuitously in (ii) for A marks need fully correct work only **NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$ leads to $\mu = -3$
			$\Rightarrow \mathbf{r} = \begin{pmatrix}-6\\0\\5\end{pmatrix}$ so $(-6, 0, 5)$	A1	cao
				[3]	

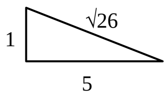
Question			Answer	Marks	Guidance
1			$\frac{2x}{x+1} - \frac{1}{x-1} = 1$		
			$\Rightarrow \quad 2x(x-1) - (x+1) = (x+1)(x-1)$	M1	mult throughout by $(x + 1)(x - 1)$ or combining fractions and mult up oe (can retain denominator throughout). Condone a single computational error provided that there is no conceptual error. Do not condone omission of brackets unless it is clear from subsequent work that they were assumed.
			$\Rightarrow \quad 2x^2 - 3x - 1 = x^2 - 1$	A1	any fully correct multiplied out form (including say, if 1's correctly cancelled) soi
			$\Rightarrow \quad x^2 - 3x = 0 = x(x - 3)$	DM1	dependent on first M1.For any method leading to both solutions. Collecting like terms and forming quadratic (= 0) and attempting to solve *(provided that it is a quadratic and $b^2 - 4ac \geq 0$).Using either correct quadratic equation formula (can be error when substituting), factorising (giving correct x^2 and constant terms when factors multiplied out), completing the square oe soi.*
			$\Rightarrow \quad x = 0$ or 3	A1	for both solutions www.
				[4]	**SC** B1 for $x = 0$ (or $x = 3$) without any working **SC** B2 for $x = 0$ (or $x = 3$) without above algebra but showing that they satisfy equation **SC** M1A1M0 SCB1 for first two stages followed by stating $x = 0$ **SC** M1A1M0 SCB1 for first two stages and cancelling x to obtain $x = 3$ only.

Question			Answer	Marks	Guidance
2			$\sqrt[3]{1-2x}=(1-2x)^{1/3}$	B1	n = 1/3 only. Do not MR for $n \neq 1/3$
			$=1+\frac{1}{3}(-2x)+\frac{\frac{1}{3}(-\frac{2}{3})}{2!}(-2x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(-2x)^3+...$	M1	all four **correct unsimplified** binomial coeffs (not nCr) soi condone absence of brackets only if it is clear from subsequent work that they were assumed
				B1	$1-\frac{2}{3}x$ www in this term
			$=1-\frac{2}{3}x-\frac{4}{9}x^2-\frac{40}{81}x^3+...$	B1	…. $-\frac{4}{9}x^2$ www in this term (not if used $2x$ for $(-2x)$ throughout)
				B1	…..$-\frac{40}{81}x^3$ www in this term If there is an error in say the third coeff of the expansion then M0 B1B0B1 is possible.
			Valid for $-½ < x < ½$ or $\lvert x\rvert < ½$	B1	Independent of expansion Allow ≤'s (valid in this case) or a combination. Condone also, say, $–½ < \lvert x\rvert < ½$ but not $x < ½$ or $–1 < 2x < 1$ or $–½ > x > ½$
				[6]	

Question			Answer	Marks	Guidance
3	(i)		$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{2\cos 2\theta}{\cos\theta}$	M1 A1	their dy/dθ / their dx/dθ www correct (can isw)
			When $\theta = \pi/6$ = $\frac{dy}{dx}=\frac{2\cos(\pi/3)}{\cos(\pi/6)}$	DM1	subst $\theta = \pi/6$ in theirs
			$=\frac{1}{\sqrt{3}/2}=\frac{2}{\sqrt{3}}$	A1	oe exact only, www (but not 1/√3/2)
			....		....
			OR		
			$y = 2x\sqrt{(1-x^2)}$	M1	full method for differentiation including product rule and function of a function oe
			$\frac{dy}{dx}=-2x^2(1-x^2)^{-1/2}+2(1-x^2)^{1/2}$	A1	oe cao (condone lack of consideration of sign)
			$at\theta=\pi/6, \sin\pi/6=1/2$	DM1	subst sin $\pi/6$ = ½ in theirs
			$\frac{dy}{dx}=\frac{-2}{4}(1-\frac{1}{4})^{-1/2}+2(\frac{3}{4})^{1/2}=\frac{2}{\sqrt{3}}$	A1	oe ,exact only, www (but not 1/√3/2)
				[4]	
3	(ii)		$y = \sin 2\theta = 2\sin\theta\cos\theta$	M1	using $\sin 2\theta = 2\sin\theta\cos\theta$
			$\Rightarrow y^2 = 4\sin^2\theta\cos^2\theta = 4x^2(1-x^2)$	M1	using $\cos^2\theta = 1-\sin^2\theta$ to eliminate $\cos\theta$
			$= 4x^2 - 4x^4$ *	A1	**AG** need to see sufficient working or A0.
				[3]	

Question			Answer	Marks	Guidance
4	(a)		$V=\int_0^2 \pi y^2 \mathrm{d}x=\int_0^2 \pi(1+\mathrm{e}^{2x})\mathrm{d}x$	M1	$\int_0^2 \pi(1+\mathrm{e}^{2x})\mathrm{d}x$ limits must appear but may be later condone omission of dx if intention clear
			$=\pi\left[x+\frac{1}{2}\mathrm{e}^{2x}\right]_0^2$	B1	$\left[x+\frac{1}{2}\mathrm{e}^{2x}\right]$ independent of π and limits
			$=\pi(2+½\,\mathrm{e}^4-½)$	DM1	dependent on first M1.Need **both** limits substituted in their integral of the form $ax+b\,e^{2x}$, where a, b non-zero constants. Accept answers including e^0 for M1. Condone absence of π for M1 at this stage
			$=½\,\pi(3+\mathrm{e}^4)$	A1 **[4]**	cao exact only
4	(b)	(i)	$x=0$, $y=1.4142$; $x=2$, $y=7.4564$	B1	1.414, 7.456 or better
			$A=0.5/2\{(1.4142+7.4564)$ $+2(1.9283+2.8964+4.5919)\}$	M1	correct formula seen (can be implied by correct intermediate step eg 27.7038../4)
			$=6.926$	A1 **[3]**	6.926 or 6.93 (do not allow more dp)
4	(b)	(ii)	8 strips: 6.823, 16 strips: 6.797		
			Trapezium rule overestimates this area, but the overestimate gets less as the no of strips increases.	B1 **[1]**	oe

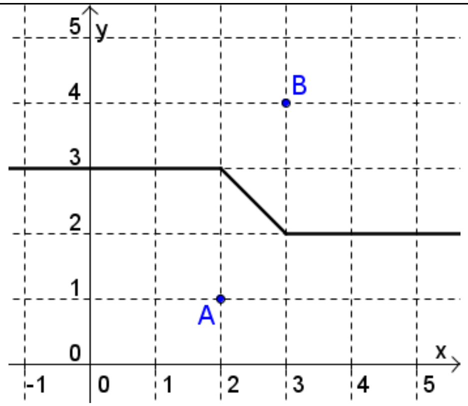
Question			Answer	Marks	Guidance
5			$2\sec^2\theta = 5\tan\theta$		
			$\Rightarrow 2(1+\tan^2\theta) = 5\tan\theta$	M1	$\sec^2\theta = 1+\tan^2\theta$ **used**
			$\Rightarrow 2\tan^2\theta - 5\tan\theta + 2 = 0$	A1	correct quadratic **oe**
			$\Rightarrow (2\tan\theta - 1)(\tan\theta - 2) = 0$	M1	solving their quadratic for $\tan\theta$ (follow rules for solving as in Question 1 [*,*]
			$\Rightarrow \tan\theta = ½$ or 2	A1	www
			$\Rightarrow \theta = 0.464$,	A1	first correct solution (or better)
			1.107	A1	second correct solution (or better) and no others in the range Ignore solutions outside the range. **SC** A1 for both 0.46 and 1.11 **SC** A1 for both 26.6° and 63.4° (or better) Do not award **SC**s if there are extra solutions in range.
			....		....
			OR		
			$2/\cos^2\theta = 5\sin\theta/\cos\theta$	M1	**using** both sec = 1/cos and tan = sin/cos
			$\Rightarrow 2\cos\theta = 5\sin\theta\cos^2\theta, \cos\theta \neq 0$	A1	correct one line equation $2 - 5\sin\theta\cos\theta = 0$ or $2\cos\theta = 5\sin\theta\cos^2\theta$ oe (or common denominator). Do not need $\cos\theta \neq 0$ at this stage.
			$\Rightarrow \cos\theta(2 - 5\sin\theta\cos\theta) = 0$		
			$\Rightarrow \cos\theta = 0, or \sin 2\theta = 0.8$	M1	**using** $\sin 2\theta = 2\sin\theta\cos\theta$ oe eg $2 = 5\sin\theta\sqrt{(1-\sin^2\theta)}$ and squaring
			$\Rightarrow \sin 2\theta = 0.8$	A1	$\sin 2\theta = 0.8$ or, say, $25\sin^4\theta - 25\sin^2\theta + 4 = 0$
			$\Rightarrow 2\theta = 0.9273 or 2.2143$		
			$\Rightarrow \theta = 0.464$,	A1	first correct solution (or better)
			1.107	A1	second correct solution (or better) and no others in range Ignore solutions outside the range SCs as above
				[6]	

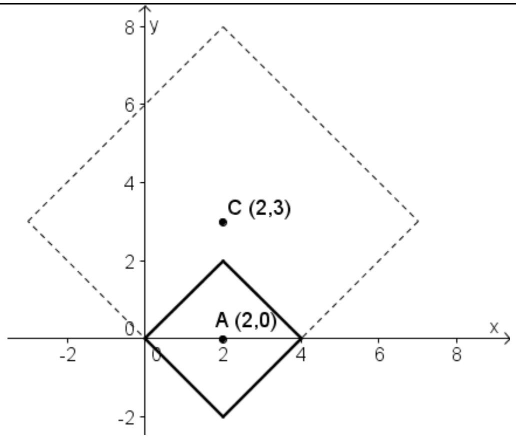
Question		Answer	Marks	Guidance
6	(i)	AC = cosec θ $\Rightarrow$ AD = cosecθ sec φ	M1 A1 **[2]**	or 1/sin θ oe but not if a fraction within a fraction
6	(ii)	DE = AD sin ($\theta + \varphi$) = cosec θ sec φ sin ($\theta + \varphi$) = cosecθ sec φ(sin θ cos φ + cos θ sin φ) $= \frac{\sin\theta\cos\varphi + \cos\theta\sin\varphi}{\sin\theta\cos\varphi}$ $= 1 + \frac{\cos\theta}{\sin\theta}\frac{\sin\varphi}{\cos\varphi}$ = 1 + tan φ / tan θ * …………………………………………………….. **OR** equivalent, eg from DE = CB + CD cos θ = 1 + CD cos θ = 1 + AD sin φ cos θ = 1 + cosec θ sec φ sin φ cos θ = 1 + tanφ /tanθ*	M1 M1 A1 M1 M1 A1 **[3]**	AD sin($\theta + \varphi$) with substitution for their AD correct compound angle formula used **Do not** award both M marks unless they are part of the same method. (They may appear in either order.) simplifying using tan = sin/cos. A0 if no intermediate step as **AG** …………………………………………………………………………………….. from triangle formed by using X on DE where CX is parallel to BE to get DX = CD cos θ and CB = 1 (oe trigonometry) substituting for both CD = AD sinφ and their AD oe to reach an expression for DE **in terms of θ and φ only** (M marks must be part of same method) **AG** simplifying to required form
7	(i)	DE = $\sqrt{[(-5)^2 + 0^2 + 1^2]} = \sqrt{26}$  cos θ = 5/√26 oe $\Rightarrow$ θ = 11.3°	M1 A1 M1 A1 **[4]**	oe oe using scalar products eg –5**i** + **k** with **i** oe or better (or 168.7°). Allow radians.

Question		Answer	Marks	Guidance
7	(ii)	$\overrightarrow{AE}=\begin{pmatrix}1\\4\\3\end{pmatrix}$, $\overrightarrow{ED}=\begin{pmatrix}5\\0\\-1\end{pmatrix}$	B1	**two** relevant direction vectors (or 6**i** + 4**j** + 2**k** oe)
		$\begin{pmatrix}1\\4\\3\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}=1-16+15=0$	B1	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms vector cross product with at least two correct terms in solution)
		$\begin{pmatrix}5\\0\\-1\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}=5+0-5=0$	B1	scalar product with second **direction** vector, with evaluation. (following **OR** above, A1 all correct ie a multiple of **i – 4j** + 5**k**) (NB finding only one direction vector and its scalar product is B1 only.)
		⇒ **i** – 4**j** + 5**k** is normal to AED ⇒ eqn of AED is $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}=\begin{pmatrix}0\\-4\\0\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}$	M1	for $x - 4y + 5z = c$ oe
		⇒ $x - 4y + 5z = 16$	A1	M1A0 for **i** – 4**j** + 5**k** = 16
		B lies in plane if $8 - 4(-a) + 5.0 = 16$	M1	allow M1 for subst in their plane or if found from say scalar product of normal with vector EB can also get M1A1
		⇒ $a = 2$	A1	**For first five marks above** **SC1**, if states, ‘if **i** – 4**j** + 5**k** is normal then of form $x - 4y + 5z$ = c’ and substitutes one coordinate gets M1A1, then substitutes other two coordinates A2 (not A1,A1).Then states so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal can get B1 provided that there is a clear argument ie M1A1A2B1. Without a clear argument this is B0. **SC2,** if finds two relevant vectors, B1 and then finds equation of the plane from vector form, $r = a + \mu b + \lambda c$ gets B1. Eliminating parameters B1cao. If then states ‘so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal’ can get B1 (4/5).
			[7]	

Question		Answer	Marks	Guidance
7	(iii)	D: 6 + 2 = 8 B: 8 + 0 = 8 C: 8 + 0 = 8 $\Rightarrow$ plane BCD is $x + z = 8$ Angle between **i** – 4**j** + 5**k** and **i** + **k** is θ $\Rightarrow \cos\theta = (1\times1+(-4)\times0 + 5\times 1)/\sqrt{42}\sqrt{2} = 6/\sqrt{84}$ $\Rightarrow \theta = 49.1°$	 B2,1,0 M1 M1 A1 A1 **[6]**	 or any valid method for finding $x + z = 8$ gets M1A1 between two correct relevant vectors complete method (including cosine) (**for M1** ft their normal(s) to their plane(s)) allow correct substitution or $\pm6/\sqrt{84}$, correct only or 0.857 radians (or better) acute only
8	(i)	h = 20, stops growing	B1 **[1]**	**AG** need interpretation
	(ii)	$h = 20 - 20e^{-t/10}$ $dh/dt = 2e^{-t/10}$ $20e^{-t/10} = 20 - 20(1 - e^{-t/10}) = 20 - h$ $=10dh/dt$ when $t = 0$, $h = 20(1 - 1) = 0$ …………………………………………………… **OR** verifying by integration $\int \frac{dh}{20-h} = \int \frac{dt}{10}$ $\Rightarrow -\ln(20-h) = 0.1t + c$ $h = 0, t = 0, \Rightarrow c = -\ln 20$ $\Rightarrow \ln(20-h) = -0.1t + \ln 20$ $\Rightarrow \ln(\frac{20-h}{20}) = -0.1t$ $\Rightarrow 20 - h = 20e^{-0.1t}$ $\Rightarrow h = 20(1-e^{-0.1t})$	 M1A1 M1 A1 B1 M1 A1 B1 M1 A1 **[5]**	 differentiation (for M1 need $ke^{-t/10}$, k const) oe eg $20 - h = 20 - 20(1 - e^{-t/10}) = 20e^{-t/10}$ $= 10dh/dt$ (showing sides equivalent) initial conditions …………………………………………………………………….. sep correctly and intending to integrate correct result (condone omission of c, although no further marks are possible) condone ln (h – 20) as part of the solution at this stage constant found from expression of correct form (at any stage) but B0 if say $c = \ln(-20)$ (found using $\ln(h - 20)$) combining logs and anti-logging (correct rules) correct form (do not award if B0 above)

Question			Answer	Marks	Guidance
8	(iii)		$\frac{200}{(20+h)(20-h)} = \frac{A}{20+h} + \frac{B}{20-h}$		
			$\Rightarrow \quad 200 = A(20 - h) + B(20 + h)$	M1	cover up, substitution or equating coeffs
			$h = 20 \Rightarrow 200 = 40B,\ B = 5$	A1	
			$h = -20 \Rightarrow 200 = 40A,\ A = 5$	A1	
			$200\, dh/dt = 400 - h^2$		
			$\Rightarrow \quad \int \frac{200}{400-h^2} dh = \int dt$	M1	separating variables and intending to integrate (condone sign error)
			$\Rightarrow \quad \int (\frac{5}{20+h} + \frac{5}{20-h}) dh = \int dt$		substituting partial fractions
			$\Rightarrow \quad 5\ln(20 + h) - 5\ln(20 - h) = t + c$	A1	ft their *A*,*B*, condone absence of *c*, Do not allow ln(h-20) for A1.
			When $t = 0,\ h = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$	B1	cao need to **show** this. *c* can be found at any stage. **NB *c* = ln(–1)** (from ln(*h* –20)) or similar **scores B0.**
			$\Rightarrow \quad 5\ln\frac{20+h}{20-h} = t$		
			$\Rightarrow \quad \frac{20+h}{20-h} = e^{t/5}$	M1	anti-logging an equation of the correct form . Allow if *c* = 0 clearly stated (provided that *c* = 0) even if B mark is not awarded, but do not allow if *c* omitted. Can ft their *c*.
			$\Rightarrow \quad 20 + h = (20 - h)e^{t/5} = 20e^{t/5} - he^{t/5}$		
			$\Rightarrow \quad h + he^{t/5} = 20e^{t/5} - 20$		
			$\Rightarrow \quad h(e^{t/5} + 1) = 20(e^{t/5} - 1)$	DM1	making *h* the subject, dependent on previous mark
			$\Rightarrow \quad h = \frac{20(e^{t/5}-1)}{e^{t/5}+1}$		**NB** method marks can be in either order, in which case the dependence is the other way around.(In which case, 20 + *h* is divided by 20 – *h* first to isolate *h*).
			$\Rightarrow \quad h = \frac{20(1-e^{-t/5})}{1+e^{-t/5}}$ *	A1	**AG must have obtained B1** (for *c*) in order to obtain final A1**.**
				[9]	
8	(iv)		As $t \to \infty$, $h \to 20$. So long-term height is 20m.	B1 **[1]**	www
8	(v)		1st model $h = 20(1 - e^{-0.1}) = 1.90..$ 2nd model $h = 20(e^{1/5} - 1)/(e^{1/5} + 1) = 1.99..$ so 2nd model fits data better	B1 B1 B1 dep **[3]**	Or 1st model $h = 2$ gives $t = 1.05..$ 2nd model $h = 2$ gives $t = 1.003..$ dep previous B1s correct

Question			Answer	Marks	Guidance	
1			(4,0) and (5,1) and (2,4)	B3 **[3]**	All three points correct (and no additional points) Accept a list of coordinates.	SC B1 any correct point SC B2 any two correct points (and no more than two incorrect additional points) B0 if points unclear
2				B3 **[3]**	B3: Exactly as shown (with line extended beyond (4,2) and before (0,3)) Award B1 for a locus including one partially correct line segment or at least two correct discrete points in that segment Award B2 for a locus that contains parts of all three correct line segments (or at least two discrete points within each of them) and no additional incorrect points.	
3	(i)		$t(P,A)=t(P,B)$	B1 **[1]**	(not t(P,A)=t(P,B)=3)	Accept words, such as, [distance] t(P,A) is equal to [distance] t(P,B).
3	(ii)		$t(P,A)=3$	B1 **[1]**		Accept words such as, all the points with [distance] t(P,A) equal to 3.
4			Evidence of 35 + *n*(3,4) 70	M1 A1 **[2]**	 70 www gets B2.	

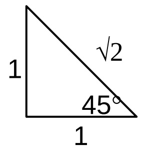
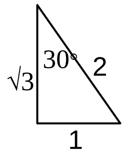
Question			Answer	Marks	Guidance	
5			The 10 routes shown from (0,0) to (3,2) each continue in one way via (4,2) to (4,3) and each continue in one way via (3,3) to (4,3) Hence 20	 B2 **[2]**	. www Need explanation SC B1 without explanation	Or, the 35 all pass through either 20(3,3) or 15 (4,2). 10 of the 20 at (3,3) come from (3,2) [and the rest from (2,3)] 10 of the 15 at (4,2) come from (3,2) [and the rest from (4,1)] So 10 + 10 = 20 are from (3,2) oe
6	**(i)**		$(-0.7, 5.3)$ $(-0.7, 0.7)$	B1 B1 **[2]**		SC B1 for (y =) 5.3 and 0.7.
6	**(ii)**			B1 **[1]**	A square with straight edges as shown with a vertex at (2,2). (and nothing more)	
7	**(i)**		B must lie on one of the two axes	B1 **[1]**	oe, say, horizontal and vertical lines from (0,0).	Do not accept a list of points unless an overall statement that includes **all** points is given (not just integers).

Question			Answer	Marks	Guidance	
7	(ii)		B lies on line $y = x$ or on line $y = -x$	B1 B1 **[2]**	oe	As **(i)**

Question			Answer	Marks	Guidance
1	(i)		$\frac{x}{(1+x)(1-2x)} = \frac{A}{1+x} + \frac{B}{1-2x}$		
			$\Rightarrow \quad x = A(1 - 2x) + B(1 + x)$	M1	expressing in partial fractions of correct form (at any stage) and attempting to use cover up, substitution or equating coefficients Condone a single sign error for M1 only.
			$x = ½ \Rightarrow ½ = B(1 + ½) \Rightarrow B = 1/3$	A1	www cao
			$x = -1 \Rightarrow -1 = 3A \Rightarrow A = -1/3$	A1	www cao
					(accept A/(1+x) +B/(1-2x) , A = −1/3, B = 1/3 as sufficient for full marks without needing to reassemble fractions with numerical numerators)
				[3]	

Question			Answer	Marks	Guidance
1	(ii)		$\frac{x}{(1+x)(1-2x)} = \frac{-1/3}{1+x} + \frac{1/3}{1-2x}$		
			$= \frac{1}{3}\left[(1-2x)^{-1} - (1+x)^{-1}\right]$	M1	correct binomial coefficients throughout for first three terms of
			$= \frac{1}{3}[1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+\ldots-(1+(-1)x+\frac{(-1)(-2)}{2}x^2+\ldots)]$		**either** $(1-2x)^{-1}$ or $(1+x)^{-1}$ oe ie 1,(–1),(–1)(–2)/2 , not nCr form. Or correct simplified coefficients seen.
			$= \frac{1}{3}[1+2x+4x^2+\ldots-(1-x+x^2+\ldots)]$	A1 A1	$1 + 2x + 4x^2$ $1 - x + x^2$ (or 1/3/-1/3 of each expression, ft their A/B) If $k(1-x+x^2)$ (A1) not clearly stated separately, condone absence of inner brackets (ie $1+2x+4x^2 \underline{-1-x+x^2)}$ **only if** subsequently it is clear that brackets were assumed, otherwise A1A0. [ie $-1-x+x^2$ is A0 unless it is followed by the correct answer] Ignore any subsequent incorrect terms
			$= \frac{1}{3}(3x+3x^2+\ldots) = x+x^2+\ldots$ so $a = 1$ and $b = 1$	A1	or from expansion of $x(1-2x)^{-1}(1+x)^{-1}$ www cao
			OR $x(1-x-2x^2) = x(1-(x+2x^2))$ $= x(1+x+2x^2+(-1)(-2)(x+2x^2)^2/2+\ldots\ldots\ldots)$	M1	correct binomial coefficients throughout for $(1-(x+2x^2))$ oe (ie 1,-1), at least as far as necessary terms $(1+x)$ (NB third term of expansion unnecessary and can be ignored)
			$= x(1+x+2x^2+x^2\ldots\ldots\ldots)$	A2	$x(1+x)$ www
			$= x + x^2\ldots\ldots$ so $a = 1$ and $b = 1$	A1	www cao
			Valid for $-½ < x < ½$ or $\|x\| < ½$	B1 **[5]**	independent of expansion. Must combine as one overall range. condone ≤ s (although incorrect) or a combination. Condone also, say $-½ < \|x\| < ½$ but not $x < ½$ or $-1 < 2x < 1$ or $-½ > x > ½$

Question			Answer	Marks	Guidance
2			cosec x + 5 cot x = 3 sin x		
			$\Rightarrow \quad \frac{1}{\sin x}+\frac{5\cos x}{\sin x}=3\sin x$	M1	**using** cosec x = 1/sin x and cot x = cos x / sin x
			$\Rightarrow \quad 1 + 5\cos x = 3\sin^2 x = 3(1 - \cos^2 x)$	M1	$\cos^2 x + \sin^2 x = 1$ **used** (both M marks must be part of same solution in order to score both marks)
			$\Rightarrow \quad 3\cos^2 x + 5\cos x - 2 = 0$ *	A1	**AG** (**Accept working backwards, with same stages needed**)
			$\Rightarrow \quad (3\cos x - 1)(\cos x + 2) = 0$	M1	use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and -2 when multiplied out) or comp square oe
			$\Rightarrow \quad \cos x = 1/3$,	A1	cos x = 1/3 www
			$x = 70.5°$,	A1	for 70.5° or first correct solution, condone over-specification (ie 70.5° or better eg 70.53°,70.5288° etc),
			289.5°	A1	for 289.5° or second correct solution (condone over-specification) and no others in the range Ignore solutions outside the range **SC**A1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48 **SC** Award A1A0 for 1.2,5.1 radians (or better) Do not award SC marks if there are extra solutions in the range
				[7]	

Question	Answer	Marks	Guidance
3	 tan 45° = 1/1 = 1*  tan 30° = 1/√3*		For both B marks **AG** so need to be convinced and need triangles but further explanation need not be on their diagram. Any given lengths must be consistent.
		B1	Need √2 or indication that triangle is isosceles oe
		B1	Need all three sides oe
	tan 75° = tan (45°+30°)	M1	use of **correct** compound angle formula with 45º,30º soi
	$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$	A1	substitution in terms of √3 in any **correct** form
	$= \frac{1+\sqrt{3}}{-1+\sqrt{3}}$	M1	eliminating fractions within a fraction (or rationalising, whichever comes first) provided compound angle formula is used as tan(*A*+*B*)= tan(*A*±*B*)/(1±tan*A*tan*B*).
	$= \frac{(1+\sqrt{3})^2}{3-1}$ (oe eg $\frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{9-3}$)	M1	rationalising denominator (or eliminating fractions whichever comes second)
	$= \frac{(3+2\sqrt{3}+1)}{3-1} = 2+\sqrt{3}$ *	A1	**correct** only, **AG** so need to see working
		[7]	

Question			Answer	Marks	Guidance
4	(i)		$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \ldots$	B1	need **r (**or another letter**)** = or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for first B1
			$\ldots + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	B1	
					NB answer is not unique eg $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$
					Accept **i/j/k** form and condone row vectors.
				[2]	
4	(ii)		$x + 3y + 2z = 4$		
			$\Rightarrow \quad -2\lambda + 3(1+\lambda) + 2(3+2\lambda) = 4$	M1	substituting their line in plane equation (condone a slip if intention clear)
			$\Rightarrow \quad 5\lambda = -5, \ \lambda = -1$	A1	www cao **NB λ is not unique** as depends on choice of line in (i)
			so point of intersection is (2, 0, 1)	A1	www cao
				[3]	
4	(iii)		Angle between $-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is θ where	M1	Angle between **i**+3**j**+2**k** and **their** direction from (i) ft condone a single sign slip if intention clear
			$\cos\theta = \frac{-2\times1+1\times3+2\times2}{\sqrt{9}\sqrt{14}} = \frac{5}{3\sqrt{14}}$	M1	correct formula (including cosine) , with substitution, for these vectors condone a single numerical or sign slip if intention is clear
			$\Rightarrow \quad \theta = 63.5°$	A1	www cao (63.5 in degrees (or better) or 1.109 radians or better)
				[3]	

Question			Answer	Marks	Guidance
5			$\begin{pmatrix}10\\5\\-5\end{pmatrix}=\lambda\begin{pmatrix}3\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\2\end{pmatrix}$	M1	required form, can be soi from two or more correct equations
			$\Rightarrow \quad 3\lambda - \mu = 10$	M1	forming at least two equations and attempting to solve oe
			$2\lambda + \mu = 5 \Rightarrow 5\lambda = 15, \quad \lambda = 3$	A1	www
			$\Rightarrow \quad 9 - \mu = 10, \ \mu = -1$	A1	www
			$-5 = -\lambda + 2\mu, \quad -5 = -3 + 2 \times -1$ true	A1	verifying third equation, **do not** give BOD accept a statement such as $\begin{pmatrix}10\\5\\-5\end{pmatrix}=3\begin{pmatrix}3\\2\\-1\end{pmatrix}+-1\begin{pmatrix}-1\\1\\2\end{pmatrix}$ as verification Must **clearly** show that the solutions satisfy all the equations.
			coplanar	B1 **[6]**	oe **independent** of all above marks

Question			Answer	Marks	Guidance
6	(i)		$v\mathrm{d}v/\mathrm{d}x + 4x = 0$		
			$\int v\mathrm{d}v = -\int 4x\,\mathrm{d}x$	M1	separating variables and intending to integrate
			$\frac{1}{2}v^2 = -2x^2 + c$	A1	oe condone absence of c. [Not immediate $v^2 = -4x^2$ (+c)]
			When $x = 1$, $v = 4$, so $c = 10$	B1	finding c, must be convinced as AG, need to see at least the statement given here oe (condone change of c)
			so $v^2 = 20 - 4x^2$ *	A1	**AG** following finding c convincingly
					Alternatively, **SC** v^2=20-4x^2, by differentiation, $2v\,\mathrm{d}v/\mathrm{d}x = -8x$ $v\mathrm{d}v/\mathrm{d}x + 4x = 0$ scores B2 if , in addition, they check the initial conditions a further B1 is scored (ie 16=20-4). Total possible 3/4.
				[4]	
6	(ii)		$x = \cos 2t + 2\sin 2t$		
			when $t = 0$, $x = \cos 0 + 2\sin 0 = 1$*	B1	**AG** need some justification
			$v = \mathrm{d}x/\mathrm{d}t = -2\sin 2t + 4\cos 2t$	M1	differentiating, accept ±2,±4 as coefficients but not ±1,±2 and not ±1/2,±1 from integrating
				A1	cao
			$v = 4\cos 0 - 2\sin 0 = 4$*	A1	www AG
				[4]	

Question			Answer	Marks	Guidance
6	(iii)		$\cos 2t + 2\sin 2t = R\cos(2t - \alpha) = R(\cos 2t\cos\alpha + \sin 2t\sin\alpha)$		**SEE APPENDIX 1 for further guidance**
			$R = \sqrt{5}$	B1	or 2.24 or better (not ± unless negative rejected)
			$R\cos\alpha = 1$, $R\sin\alpha = 2$	M1	correct pairs soi
			$\tan\alpha = 2$,	M1	correct method
			$\alpha = 1.107$	A1	cao radians only, 1.11 or better (or multiples of π that round to 1.11)
			$x = \sqrt{5}\cos(2t - 1.107)$ $v = -2\sqrt{5}\sin(2t - 1.107)$	A1	differentiating or otherwise, ft their numerical R, α (not degrees) required form **SC** B1 for $v = \sqrt{20}\cos(2t + 0.464)$ oe
			EITHER $v^2 = 20\sin^2(2t - \alpha)$ $20 - 4x^2 = 20 - 20\cos^2(2t - \alpha)$	M1	squaring their v (if of required form with same α as x), and x, and attempting to show $v^2 = 20 - 4x^2$ ft their R, α (incl. degrees) [α may not be specified].
			$= 20(1 - \cos^2(2t - \alpha)) = 20\sin^2(2t - \alpha)$ so $v^2 = 20 - 4x^2$	A1	cao www (condone the use of over-rounded α (radians) or degrees)
			OR multiplying out $v^2 = (-2\sin 2t + 4\cos 2t)^2$ $= 4\sin^2 2t - 16\sin 2t\cos 2t + 16\cos^2 2t$ and $4x^2 = 4(\cos^2 2t + 4\sin 2t\cos 2t + 4\sin^2 2t)$ $= 4\cos^2 2t + 16\sin 2t\cos 2t + 16\sin^2 2t$ (need middle term) and attempting to show that $v^2 + 4x^2 = 4(\sin^2 2t + \cos^2 2t) + 16(\cos^2 2t + \sin^2 2t)$ $= 4 + 16 = 20$ (or $20 - 4x^2 = v^2$) oe	M1	differentiating to find v (condone coefficient errors), squaring v and x and multiplying out (need attempt at middle terms) and attempting to show $v^2 = 20 - 4x^2$
			so $v^2 = 20 - 4x^2$	A1	cao www
				[7]	

Question			Answer	Marks	Guidance
6	(iv)		$x = \sqrt{5}\cos(2t - \alpha)$ or otherwise x max = $\sqrt{5}$ when $\cos(2t - \alpha) = 1$, $2t - 1.107 = 0$, $2t = 1.107$ $t = 0.55$	B1 M1 A1 **[3]**	ft their R oe (say by differentiation) ft their α in radians or degrees for method only cao (or answers that round to 0.554)
7	(i)		$u = 10$, $x = 5 \ln 10 = 11.5$ so OA = $5\ln 10$ when $u = 1$, $y = 1 + 1 = 2$ so OB = 2 When $u = 10$, $y = 10 + 1/10 = 10.1$ So AC = 10.1	M1 A1 M1 A1 A1 **[5]**	Using $u = 10$ to find OA accept 11.5 or better Using $u = 1$ to find OB or $u = 10$ to find AC In the case where values are given in coordinates instead of OA=,OB=,AC=, then give A0 on the first occasion this happens but allow subsequent As. Where coordinates are followed by length eg B(0, 2), length=2 then allow A1.

Question			Answer	Marks	Guidance
7	(ii)		$\frac{dy}{dx}=\frac{dy/du}{dx/du}=\frac{1-1/u^2}{5/u}$	M1 A1	their dy/du /dx/du
			$\left[=\frac{u^2-1}{5u}\right]$		Award A1 if **any** correct form is seen at any stage including unsimplified (can isw)
			EITHER		
			When u = 10, dy/dx = 99/50 = 1.98	M1	**substituting u =10 in their expression**
			tan (90 – θ)= 1.98 $\Rightarrow$ θ = 90 – 63.2	M1	**or by geometry**, say using a triangle and the gradient of the line
			= 26.8°	A2	26.8°, or 0.468 radians (or better) cao **SC** M1M0A1A0 for 63.2° (or better) or 1.103 radians(or better)
			OR		
			When u = 10, dy/dx = 99/50 = 1.98	M1	
			tan(90– θ) = 99/50 $\Rightarrow$ tanθ = 50/99	M1	**allow use of their expression for M marks**
			θ = 26.8°	A2	26.8°, or 0.468 radians (or better) cao
				[6]	
7	(iii)		$x = 5 \ln u \Rightarrow x/5 = \ln u$, $u = e^{x/5}$	M1	Need some working
			$\Rightarrow$ $y = u + 1/u = e^{x/5} + e^{-x/5}$	A1	Need some working as AG
				[2]	

Question		Answer	Marks	Guidance
7	(iv)	Vol of rev = $\int_0^{5\ln 10} \pi y^2 \mathrm{d}x = \int_0^{5\ln 10} \pi (e^{x/5} + e^{-x/5})^2 \mathrm{d}x$	M1	need $\pi\ (e^{x/5} + e^{-x/5})^2$ and dx soi. Condone wrong limits or omission of limits for M1. Allow M1 if y prematurely squared as eg $(e^{2x/5} + e^{-2x/5})$
		$= \int_0^{5\ln 10} \pi (\mathrm{e}^{2x/5} + 2 + \mathrm{e}^{-2x/5}) \mathrm{d}x$	A1	including **correct** limits at some stage (condone 11.5 for this mark)
		$= \pi \left[(\frac{5}{2}\mathrm{e}^{2x/5} + 2x - \frac{5}{2}\mathrm{e}^{-2x/5}) \right]_0^{5\ln 10}$	B1	$[\frac{5}{2}e^{2x/5} + 2x - \frac{5}{2}e^{-2x/5}]$ allow if no π and/or no limits or incorrect limits
		= π(250 + 10ln10 – 0.025 – 0)	M1	substituting both limits (their OA and 0) in an expression of correct form ie $ae^{2x/5} + be^{-2x/5} + cx\ ,\quad a,b,c \neq 0$ and subtracting in correct order (- 0 is sufficient for lower limit) Condone absence of π for M1
		= 858	A1	accept 273π and answers rounding to 273π or 858
			[5]	NB The integral can be evaluated using a change of variable to u. This involves changing dx to (dx/du)x du. For completely correct work from this method award full marks. Partially correct solutions must include the change in dx. If in doubt consult your TL. **Remember to indicate second box has been seen even if it has not been used.**

Question			Answer	Marks	Guidance	
1			Point R marked correctly at (-60, 45) Point M marked correctly at (0, 20)	B1 B1 **[2]**	half way up the relevant square Generously applied if above half way If unclear, B0B0.	Need labelling with letters or co-ordinates oe (condone one not labelled if other is labelled (and no others marked)) **SC B1** BOD both marked (and no others) and no labelling.
2			For 30° S, 10 hours daylight For 60° S, 5.5 hours daylight. Difference 4.5 hours	B1 B1 B1ft **[3]**	cao soi allow 5< *t* <6 soi 10- *t* dependent on both previous B marks soi	10 only not 5 or 6 **SC(1)** allow **B3** for 4<difference in length<5 (**without wrong working**) **SC(2)**allow **B2** if uses t=5 or t=6 eg, 10,6,4 www **SC(3)** If calculates (using the equation (4)) can obtain all three marks. Approximate values are 10.07, 5.51, 4.56. (May also work with 5 and 2.5<*t*<3 if doubled at end eg 5 B1 dependent on later doubling 2.6 B1dependent on later doubling (5-2.6) x2=4.8 B1)
			IT IS ESSENTIAL TO CHECK & ANNOTATE PAGES ATTACHED TO QUESTIONS 101, 102, 103			

Question			Answer	Marks	Guidance	
3			10^o north is B 1^o north is A 5^o south is D 15^o south is C	B2 **[2]**	All four answers correct **SC B1** Any two answers correct	
4			$\tan y = -\frac{1}{\tan\alpha}\cos(15t)$ $\alpha = 23.44°, y = 60°$, t is to be found			
			$\cos(15t) = -0.7509...$ $15t = 138.6737...$	M1	substitute in formula and attempt to solve (as far as 15t=invcos…….)oe	
			$t = 9.2449...$	A1	accept 9.2 or better	
			Daylight hours are $2 \times 9.2449... = 18.4898...$	DM1	doubling, dependent on first M1	
			So 18.5 hours (to 3s.f.)	A1	or approx 18 hours www (accept 18.4898,18.489,18.49,18.5 or 18.4 (from 2x9.2),18.48)	any reasonable accuracy or stating error is approx 0.49 oe
			OR Using t=9	M1	t=18/2=9 and substituted in formula	any reasonable accuracy
			α=23.44, t=9, y is to be found tan y =1.6309……..	DM1	and attempt to solve (as far as y= inv tan constant) oe	
			y=58.485° so approx 60°	A2 **[4]**	or 58.49°/58.5°/approx 60° www	

Question			Answer	Marks	Guidance	
5	(i)		$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times (n+10)\right)$			
			On February 2nd, $n = 31 + 2 = 33$	B1	calculate n= 31+2 =33 (days in Jan + Feb) **soi**	SC B1 condone 30+2
			$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times 43\right)$	M1	substitution of their n+10 in equation (3) and attempt to evaluate	Where n=31,32,33,34 only
			$a = -17.31$	A1	or -17.306 or rounds to -17.3	
						NB n= 32+10 gives -17.576 gaining B1M1A0
				[3]		
5	(ii)		$\tan y = -\frac{1}{\tan \alpha} \cos(15t)$			
			$\tan 53 = -\frac{1}{\tan(-17.306)} \times \cos(15t)$	M1	use of **their** α in equation (4)	
			$t = \frac{1}{15} \arccos(-\tan 53 \times \tan(-17.306))$	DM1	making t the subject	
			$t = 4.3717$	A1	4.37 or better	**SC ft** from **-**17.576 Obtains A1ft for 4.343 (or 4.34) And then A1ft for 16:21 (or 16:20)
			Sunset is at 12 hrs + 4 hours 22 minutes, and so 16:22 hrs	A1	cao	
				[4]		