MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-D

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

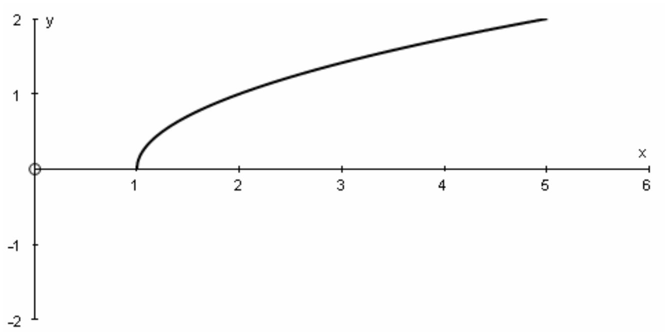
Section A (36 marks)

1 Solve the equation for values of θ in the range $0° < \theta < 360°$.

$$\cot 2\theta = 5 \qquad [4]$$

2 Find where the line $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ meets the plane $2x + 3y - 4z - 5 = 0$. [4]

3 The graph shows part of the curve $y^2 = (x - 1)$.



Find the volume when the area between this curve, the x-axis and the line $x = 5$ is rotated through 360° about the x-axis. [6]

4 You are given that $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

(i) Write down a unit vector parallel to **a**. [2]

(ii) Find the value of λ such that $\mathbf{a} + \lambda\mathbf{b}$ is parallel to **k**. [2]

(iii) Calculate the size of the angle between **a** and **b**. [3]

5 **(i)** Simplify $\dfrac{x^3 - x^2 - 3x - 9}{x - 3}$. [2]

(ii) Hence or otherwise solve the equation $x^3 - x^2 - 3x - 9 = 6(x - 3)$. [3]

6 Prove that

(i) $\dfrac{\sin 2\theta}{2\tan\theta}+\sin^2\theta=1,$ [3]

(ii) $\sin\left(x+45^0\right)=\cos\left(x-45^0\right).$ [3]

7 Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2x}{y}$ given that when $x = 1$, $y = 2$. [4]

Section B (36 marks)

8 Scientists predict the velocity (v kilometres per minute) for the new "outer explorer" spacecraft over the first minute of its entry to the atmosphere of the planet Titan to be modelled by the equation:

$$v=\frac{5000}{(1+t)(2+t)^2},\quad 0\le t\le 1 \text{ where } t \text{ represents time in minutes.}$$

(i) Use a binomial expansion to expand $(1 + t)^{-1}$ up to and including the term in t^2. [2]

(ii) Use a binomial expansion to expand $(2 + t)^{-2}$ up to and including the term in t^2. [3]

(iii) Hence, or otherwise, show that $v\approx 1250\left(1-2t+\dfrac{11t^2}{4}\right)$. [2]

(iv) The displacement of the spacecraft can be found by calculating the area under the velocity time graph. Use the approximation found in part (iii) to estimate the displacement of the spacecraft over the first half minute. [3]

(v) Write $\dfrac{1}{\left(1+t\right)\left(2+t\right)^2}$ in partial fractions. [4]

(vi) The displacement of the spacecraft in the first T minutes is given by $\int_0^T v\ \mathrm{d}t$

Calculate the exact value of the displacement of the spacecraft over the first half minute given by the model. [4]

(vii) On further investigation the scientists believe the original model may be valid for up to three minutes. Explain why the approximation in (iii) will be no longer be valid for this time interval. [1]

9 Two astronomers wish to model the path of motion of a particle in two dimensions. Experimental results show that the position of the particle can be found using the parametric equations

$$x = 2\cos\theta - \sin\theta + 2 \qquad y = \cos\theta + 2\sin\theta - 1 \qquad (0 \le \theta \le 360^0)$$

One astronomer uses trigonometry.

(i) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants to be determined.

Show also that, for the same values of R and α,

$$\cos\theta + 2\sin\theta = R\sin(\theta + \alpha).$$ [4]

(ii) Hence, or otherwise, show that the path of particle may be written in the form

$$(x-2)^2 + (y+1)^2 = 5$$

Describe the path of the particle. [3]

The second astronomer sets up a first order differential equation with the condition that $x = 4$ when $y = 0$.

(iii) Verify that the point with parameter $\theta = 0$ has coordinates (4, 0). [1]

(iv) Find $\frac{dy}{dx}$ in terms of θ. Deduce that x and y satisfy the differential equation

$$\frac{dy}{dx} = -\frac{x-2}{y+1}.$$ [4]

(v) Solve this differential equation, using the condition that $y = 0$ when $x = 4$.

Hence show that the two solutions give the same cartesian equation for the path of particle. [5]