MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-D

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$\tan 2\theta = 0\cdot 2$ $\Rightarrow 2\theta = 11\cdot 30...\text{or } 180 + 11\cdot 30...\text{or } ...$ $\Rightarrow \theta = 5\cdot 654...\text{or } 90 + 5\cdot 654...\text{or } ...$ $= 5\cdot 7 \text{ or } 95\cdot 7 \text{ or } 185\cdot 7 \text{ or } 275\cdot 7$ to 1dp. within the given range.	B1 B1 B2 **4**	B2 for all four answers; B1 for only two.
2		$x = 1 + \lambda,\ y = 2 + 3\lambda,\ z = 2\lambda$ (say) $\Rightarrow 0 = 2(1+\lambda) + 3(2+3\lambda) - 4(2\lambda) - 5$ $\Rightarrow \lambda = -1$ Required point is $(0, -1, -2)$.	B1 M1 A1 A1 **4**	
3		$\pi\int_1^5 y^2 \mathrm{d}x$ $= \pi\int_1^5 (x-1)\ \mathrm{d}x$ $= \pi\left[\frac{x^2}{2} - x\right]_1^5 = 8\pi$ $(= 25\cdot 13...)$ Volume is 8π units^3.	M1 A1 B1 A2 A1 **6**	Limits A1 for one bit
4	**(i)**	$\lvert\mathbf{a}\rvert = \sqrt{2^2 + 6^2 + 9^2} = 11$ A unit vector is either $\pm\frac{1}{11}\begin{pmatrix}2\\6\\9\end{pmatrix}$	M1 A1 **2**	
	(ii)	$\begin{pmatrix}2\\6\\9\end{pmatrix} + \lambda\begin{pmatrix}1\\3\\-1\end{pmatrix} = k\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\Rightarrow \begin{cases}0 = 2 + \lambda\\ 0 = 6 + 3\lambda\\ (k = 9 - \lambda)\end{cases}$ $\Rightarrow \lambda = -2$ since that satisfies both x and y component relations.	M1 A1 **2**	
	(iii)	Angle is θ where $\theta = \cos^{-1}\frac{\lvert\mathbf{a}\cdot\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert\cdot\lvert\mathbf{b}\rvert}$ $= \cos^{-1}\frac{2+18-9}{11\times\sqrt{1^2+3^2+1^2}} = \cos^{-1}\frac{1}{\sqrt{11}}$ $= 72\cdot 45...$ $= 72\cdot 5,$ to 1 dp.	B1 B1 B1 **3**	scalar product, B1 for $\lvert\mathbf{b}\rvert$

5	**(i)**	$\frac{x^3-x^2-3x-9}{x-3}$ $=\frac{(x-3)(x^2+2x+3)}{x-3}$ $=x^2+2x+3$	B1 B1 B1 **3**	For factor $(x-3)$
	(ii)	$x^3-x^2-3x-9=6(x-3)$ $\Rightarrow (x-3)(x^2+2x+3)-6(x-3)=0$ $\Rightarrow (x-3)(x^2+2x-3)=0$ $\Rightarrow (x-3)(x+3)(x-1)=0$ $\Rightarrow x=1, 3,\text{ or } -3$	M1 A1 A1 **3**	
6	**(i)**	LHS $=\frac{2\sin\theta\cos\theta}{2\sin\theta/\cos\theta}+\sin^2\theta$ $=\cos^2\theta+\sin^2\theta$ $=1$ $=$ RHS	M1 A1 E1 **3**	
	(ii)	$\sin(x+45^\circ)$ $=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$ $=\cos(x-45^\circ)$ **or** $\sin(x+45^\circ)$ $=\cos(90^\circ-[x+45^\circ])$ $=\cos(45^\circ-x)$ $=\cos(x-45^\circ)$ (even function)	M1 A1 B1 **3** M1 A1 E1	
7		$\frac{dy}{dx}=\frac{2x}{y}$ $\Rightarrow y\frac{dy}{dx}=2x$ $\Rightarrow \frac{y^2}{2}=x^2+c$ When $x=1,\ y=2,$ so $c=1.$ $\Rightarrow 2=y^2-2x^2$	B1 M1A1 B1 **4**	

Section B				
8	**(i)**	$(1+t)^{-1} = 1 - t + t^2 ....$	B2 **2**	B1 for first two terms
	(ii)	$(2+t)^{-2} = 2^{-2}\left(1+\frac{t}{2}\right)^{-2}$ $= \frac{1}{4}\left(1 + \frac{-2}{1}\cdot\frac{t}{2} + \frac{-2\cdot -3}{1\cdot 2}\cdot\left(\frac{t}{2}\right)^2 ....\right)$ $= \frac{1}{4} - \frac{1}{4}t + \frac{3}{16}t^2 ...$	B1 M1 A1 **3**	
	(iii)	$5000(1+t)^{-1}(2+t)^{-2}$ $= 5000(1 - t + t^2 ...)(\frac{1}{4} - \frac{1}{4}t + \frac{3}{16}t^2 ...)$ $= \frac{5000}{4}(1 - t + t^2 ...)(1 - t + \frac{3}{4}t^2 ...)$ $= 1250\,(1 - t - t + t^2 + t^2 + \frac{3}{4}t^2 ...) = 1250\,(1 - 2t + \frac{11}{4}t^2 ...)$	B1 E1 **2**	
	(iv)	Displacement $= \int_0^{\frac{1}{2}} 1250\left(1 - 2t + \frac{11}{4}t^2\right)dt$ $= \left[1250\left(t - t^2 + \frac{11}{12}t^3\right)\right]_0^{\frac{1}{2}}$ $= 1250 \times \frac{35}{96} = 455\cdot 72...$ Displacement is 456 metres, to 3sf.	M1 A1 A1 **3**	
	(v)	$\frac{1}{(1+t)(2+t)^2} = \frac{A}{1+t} + \frac{B}{2+t} + \frac{C}{(2+t)^2}$ $\Rightarrow 1 = A(2+t)^2 + B(2+t) + C(1+t)$ Any method $\Rightarrow A = 1,\ B = -1,\ C = -1$ $\Rightarrow \frac{1}{(1+t)(2+t)^2} = \frac{1}{1+t} - \frac{1}{2+t} - \frac{1}{(2+t)^2}$	M1 A1 A2 **4**	Partial fractions 3 values, A1 for 2
	(vi)	Displacement $= \int_0^{\frac{1}{2}} \frac{5000}{(1+t)(2+t)^2}dt = \int_0^{\frac{1}{2}} 5000\left(\frac{1}{1+t} - \frac{1}{(2+t)^2} - \frac{1}{2+t}\right)dt$ $= 5000\left[\ln(1+t) + \frac{1}{2+t} - \ln(2+t)\right]_0^{\frac{1}{2}}$ $= 5000\left(\ln\frac{3}{2} - \frac{1}{10} - \ln\frac{5}{4}\right) = 5000\left(\ln\frac{6}{5} - \frac{1}{10}\right)$	M1 M1 A1 A1 **4**	Integral in PFs Integrating to lns
	(vii)	Series in (i) has region of validity only $-1 < t < 1$.	B1 **1**	

9	**(i)**	$2\cos\theta - \sin\theta \equiv R\cos(\theta+\alpha)$ $\equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\Rightarrow \begin{cases} 2 = R\cos\alpha \\ 1 = R\sin\alpha \end{cases}$ $\Rightarrow R^2 = 5$ So take $R=\sqrt{5}$ (if we take $R>0$) & $\alpha = \cos^{-1}\frac{2}{\sqrt{5}}$ (since both sin and cos +ve) $= 26\cdot 56...$ With these values, $R\sin(\theta+\alpha) \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $\equiv 2\sin\theta + \cos\theta$	M1 A1 A1 A1 **4**	
	(ii)	$\left.\begin{aligned} x &= R\cos(\theta+\alpha)+2 \\ y &= R\sin(\theta+\alpha)-1 \end{aligned}\right\}$ where R and α are as above $\Rightarrow (x-2)^2+(y+1)^2$ $= R^2$ $= 5$ This is a circle of centre (2,-1) and radius √5.	M1 A1 A1 **3**	circle radius and centre
	(iii)	When $\theta = 0$, $x = 2 - 0 + 2$ $= 4$, $y = 1 + 0 - 1$ $= 0$. Ie (4,0).	E1 **1**	
	(iv)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{-\sin\theta + 2\cos\theta}{-2\sin\theta - \cos\theta}$ $= -\frac{x-2}{y+1}$	B1 B1 B1 E1 **4**	Stated or implied. Numerator Denominator
	(v)	$\Rightarrow (y+1)\frac{dy}{dx} = -(x-2)$ $\Rightarrow \frac{1}{2}(y+1)^2$ $= -\frac{1}{2}(x-2)^2 + c$ $x=4, y=0,$ so $c = \frac{5}{2}$ Thus $(y+1)^2 + (x-2)^2 = 5$ [Other ways of doing the integrals are possible]	M1 A1 M1 A1 E1 **5**	Solving Finding c