MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-C

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Solve the equation.

$$\frac{8}{x}-\frac{9}{x+1}=1$$ [3]

2 Solve the equation $3\text{cosec}^2x = 2\cot^2x + 3$ for values of x in the range $0^o < x < 360^o$. [4]

3 The curve $y^2 = x - 1$ for $1 \le x \le 3$ is rotated through 360^o about the x-axis.
Find the volume of the solid formed. [4]

4 A curve is given by the parametric equations $x = t^2$, $y = 3t$ for all values of t.
Find the equation of the tangent to the curve at the point where $t = -2$. [5]

5 **(i)** Express $\dfrac{1+x}{(1-x)(1-2x)}$ in partial fractions. [4]

(ii) Hence find $\displaystyle\int_2^3 \frac{1+x}{(1-x)(1-2x)}\,\mathrm{d}x$. [4]

6 The function $f(\theta) = 3\sin\theta + 4\cos\theta$ is to be expressed in the form $r\sin(\theta + \alpha)$ where $r > 0$ and $0^o < \alpha < 90^o$.

(i) Find the values of r and α. [3]

(ii) Write down the maximum and minimum value of $f(\theta)$. [1]

(iii) Solve the equation $f(\theta) = 1$ for $0^o \le \theta \le 180^o$. [2]

7 **(i)** Show that $\dfrac{1}{\sqrt{25-x}} = \dfrac{1}{5}\left(1-\dfrac{x}{25}\right)^{-\frac{1}{2}}$. [2]

(ii) Hence expand $\dfrac{1}{\sqrt{25-x}}$ in ascending powers of x up to and including the term in x^3. [3]

(iii) Write down the range of values of x for which the expansion is valid. [1]

Section B (36 marks)

8 The new price of a particular make of car is £10 000. When its age is t years, the list price is £V. When $t = 5$, $V = 5000$.

Aloke, Ben and Charlie all run outlets for used cars. Each of them has a different model for the depreciation.

(i) Aloke claims that the rate of depreciation is constant. Write this claim as a differential equation.
Solve the differential equation and hence find the value of a car that is 7 years old according to this model.
Explain why this model breaks down for large t. [5]

(ii) Ben believes that the rate of depreciation is inversely proportional to the square root of the age of the car. Express this claim as a differential equation and hence find the value of a car that is 7 years old according to this model.
Does this model ever break down? [6]

(iii) Charlie believes that a better model is given by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = kV\,.$$

Solve this differential equation and find the value of the car after 7 years according to this model.
Does this model ever break down? [6]

(iv) Further investigation reveals that the average value of this particular type of car when 8 years old is £3000.

Find the value of V when $t = 8$ for the three models above. Which of the three models best predicts the value of V at this time? [2]

9 Beside a major route into a county town the authorities decide to build a large pyramid. Fig. 9.1 shows this pyramid, ABCDE O is the centre point of the horizontal base BCDE. A coordinate system is defined with O as the origin. The *x*-axis and *y*-axis are horizontal and the *z*-axis is vertical, as shown in Fig. 9.1 The vertices of the pyramid are

A(0, 0, 6), B(−4, −4, 0), C (4,−4, 0), D (4, 4, 0) and E(−4, 4, 0).

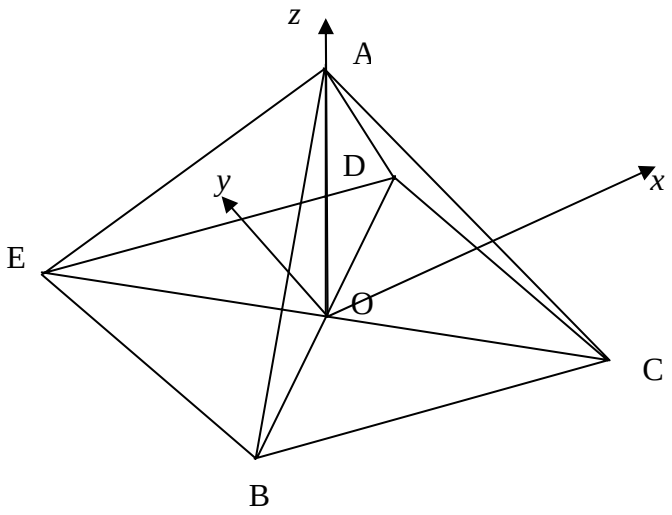

Fig.9.1

The pyramid is supported by a vertical pole OA and there are also support rods from O to points on the triangular faces ABC, ACD, ADE and AEB. One of the rods, ON, is shown in fig.9.2 which shows one quarter of the pyramid.

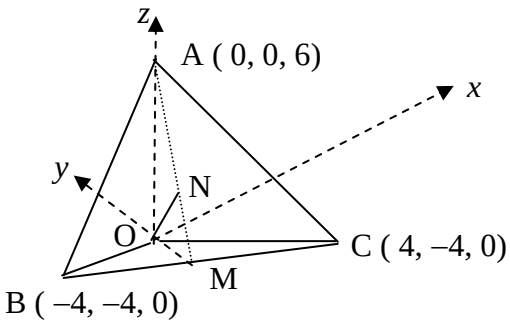

Fig. 9.2

M is the mid-point of the line BC.

(i) Write down the coordinates of M. [1]

(ii) Write down the vector $\vec{AM}$ and hence the coordinates of the point N which divides $\vec{AM}$ so that the ratio AN : NM = 2:1. [5]

(iii) Show that ON is perpendicular to both AM and BC. [3]

(iv) Hence write down the equation of the plane ABC in its simplest form. [4]

(v) Find the angle between the face ABC and the ground. [4]