MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-C

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$\frac{8}{x}-\frac{9}{x+1}=1$ $\Rightarrow 8(x+1)-9x=x(x+1)$ $\Rightarrow 0=x^2+2x-8$ $\Rightarrow 0=(x+4)(x-2)$ $\Rightarrow x=-4$ or 2	M1 A1 A1 **3**	
2		$3\operatorname{cosec}^2 x=2\cot^2 x+3$ $\Rightarrow 3(1+\cot^2 x)=2\cot^2 x+3$ $\Rightarrow \cot^2 x=0$ $\Rightarrow \cot x=0$ $\Rightarrow x=90^\circ$ *or* 270°	M1 A1 A1 A1 **4**	Use identity One for each angle
3		$V=\pi\int_0^2 y^2\,dx$ $=\pi\int_0^2 x^4\,dx=\pi\left[\frac{x^5}{5}\right]_0^2$ $=\frac{32}{5}\pi$	M1 A1 A1 A1 **4**	
4		$\frac{dx}{dt}=2t,\ \ \frac{dy}{dt}=3$ $\Rightarrow \frac{dy}{dx}=\frac{3}{2t}$ When $t=-2,\ \ x=4,\ y=-6,\ \frac{dy}{dx}=-\frac{3}{4}$ Equation of tangent: $y+6=-\frac{3}{4}(x-4)$ $\Rightarrow 4y+3x+12=0$	M1 A1 B1 M1 A1 **5**	
5	**(i)**	$\frac{-2}{1-x}+\frac{3}{1-2x}$ (by cover-up rule - or any valid method	B1 M1 A1 A1 **4**	for correct canonical form A1 for each of -2 and 3

	(ii)	$\int_2^3 \frac{1+x}{(1-x)(1-2x)}\,dx$ $= \int_2^3 \left(\frac{-2}{1-x} + \frac{3}{1-2x}\right) dx$ $= \left[2\ln\lvert 1-x\rvert - \frac{3}{2}\ln\lvert 1-2x\rvert\right]_2^3$ $= 2\ln\frac{2}{1} - \frac{3}{2}\ln\frac{5}{3}$ $(= 0.620...)$	M1 A1 B1 A1 **4**	B1 for essential modulus signs; or equivalent answer
6	**(i)**	$r\sin(\theta+\alpha) \equiv r\sin\theta\cos\alpha + r\cos\theta\sin\alpha$ $\Rightarrow \begin{cases} r\cos\alpha = 3 \\ r\sin\alpha = 4 \end{cases}$ $\Rightarrow \begin{cases} r = 5 \\ \text{and} \\ \alpha = \arctan\frac{4}{3} \\ \quad = 53.13..... \end{cases}$	M1 A1 A1 **3**	
	(ii)	$-5 \le f(\theta) \le 5$	B1 **1**	
	(iii)	$1 = 5\sin(\theta + 53.1...)$ $\Rightarrow \theta + 53.1... = \arcsin 0.2$ or $= 180 - \arcsin 0.2$ (or....) $\Rightarrow \theta = 11.53... - 53.1...$ or $= 180 - 11.53... - 53.1...$ or.... $\Rightarrow \theta = 115.33...$ $= 115.3$, to 1dp.	M1 A1 **2**	
7	**(i)**	$\frac{1}{\sqrt{25-x}}$ $= (25-x)^{-\frac{1}{2}}$ $= 25^{-\frac{1}{2}}\left(1-\frac{x}{25}\right)^{-\frac{1}{2}}$ $= \frac{1}{5}\left(1-\frac{x}{25}\right)^{-\frac{1}{2}}$	B1 B1 **2**	For index. For correct factorisation and $25^{-\frac{1}{2}}$
	(ii)	$= \frac{1}{5}\left(1 + \frac{\frac{-1}{2}}{1}.\left[-\frac{x}{25}\right] + \frac{\frac{-1}{2}\cdot\frac{-3}{2}}{1.2}.\left[-\frac{x}{25}\right]^2 + \frac{\frac{-1}{2}\cdot\frac{-3}{2}\cdot\frac{-5}{2}}{1.2.3}.\left[-\frac{x}{25}\right]^3 + ...\right)$ $= \frac{1}{5} + \frac{x}{250} + \frac{3x^2}{25000} + \frac{x^3}{250000} + ...$	M1 A2 **3**	A1 for first three terms and A1 for the fourth.
	(iii)	$-25 < x < 25$	B1 **1**	

Section B				
8	**(i)**	$\frac{dV}{dt} = -k \ (k > 0)$ $\Rightarrow V = -kt + c$ When $t = 0, V = 10000$ $\Rightarrow V = 10000 - kt$ When $t = 5, V = 5000$ $\Rightarrow 5k = 5000$ $\Rightarrow k = 1000$ Thus $V = 10000 - 1000t$ When $t = 7$, $V = 3000$ Value is £3000 after 7 years. The model breaks down since when $t > 10$ it predicts V to be negative.	B1 M1 A1 A1 B1 **5**	D.E solving and substituting For c and k
	(ii)	$\frac{dV}{dt} = \frac{-k}{t^{\frac{1}{2}}} \ (k > 0)$ $= -kt^{-\frac{1}{2}}$ $\Rightarrow V = -2kt^{\frac{1}{2}} + c$ When $t = 0, V = 10000$ $\Rightarrow c = 10000$ $\Rightarrow V = -2kt^{\frac{1}{2}} + 10000$ When $t = 5, V = 5000$, so $5000 = -2\sqrt{5}k + 10000$ $\Rightarrow k = 500\sqrt{5}$ Thus $V = 10000 - 1000\sqrt{5}t^{\frac{1}{2}}$ When $t = 7$, $V = 10000 - 1000\sqrt{35}$ $= 4083.9....$ Its value this time is £4080 to 3sf. It breaks down this time after $t = (2\sqrt{5})^2$ $= 20$ for the same reason.	B1 M1 A1 A1 B1 **6**	D.E solving and substituting for c, and k.

	(iii)	$\frac{dV}{dt} = kV$ $\Rightarrow \int \frac{1}{V} = k\int dt$ $\Rightarrow \ln V = kt + c$ $\Rightarrow V = Ae^{kt}$ When $t = 0, V = 10000$ $\Rightarrow A = 10000$ $\Rightarrow V = 10000e^{kt}$ When $t = 5,\ V = 5000$ $\Rightarrow 5000 = 10000e^{5k}$ $\Rightarrow 5k = \ln\frac{1}{2}$ $\Rightarrow k = -\frac{1}{5}\ln 2$ $\Rightarrow V = 10000\left(\frac{1}{2}\right)^{\frac{t}{5}}$ The value after 7 years is £3790 (to 3sf). This does not break down (- it just tends to zero).	M1 A1 A1 A1 A1 B1 **6**	Solving and substituting For A For k
	(iv)	The predicted values are (approximately) model 1 £2000 model 2 £3680 (3675.4…) model 3 £3300 (3298.7…) Model 3 works best.	B1 B1 **2**	All 3

9	**(i)**	M(0,-4,0)	B1 **1**	
	(ii)	$\overrightarrow{AM} = \begin{pmatrix} 0 \\ -4 \\ -6 \end{pmatrix}$ $\overrightarrow{AN} = \frac{9}{13}\begin{pmatrix} 0 \\ -4 \\ -6 \end{pmatrix}$ $\Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + \frac{9}{13}\begin{pmatrix} 0 \\ -4 \\ -6 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ -\frac{36}{13} \\ \frac{24}{13} \end{pmatrix} = \frac{12}{13}\begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$	B1 M1 A1 M1 A1 **5**	
	(iii)	$\vec{ON} = \frac{12}{13}\begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} \Rightarrow \vec{ON}.\vec{AM} = \frac{12}{13}\begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}.\begin{pmatrix} 0 \\ -4 \\ -6 \end{pmatrix}$ $= \frac{12}{13}(0+12-12) = 0$ $\vec{ON}.\vec{BC} = \frac{12}{13}\begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}.\begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} = \frac{12}{13}(0+0+0) = 0$	M1 B1 A1 **3**	Dot product For BC Both = 0
	(iv)	Normal has direction [0, –3, 2] So equation is $-3y + 2z = d$ Substitute any point gives $d = 12$ i.e. $-3y + 2z = 12$	B1 M1 A1 A1 **4**	
	(vi)	Angle is AMO AMO = $\tan^{-1}\frac{6}{4} = 56.3^0$ OR: Angle between **n** and $\vec{OA} = \arccos\left(\frac{0+0+2}{\sqrt{13}\sqrt{1}}\right)$ $= \arccos\frac{2}{\sqrt{13}}$	B1 M1 A1 A1 **4**	

