MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-B

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- There is an Insert booklet for use in Question **6.**
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Solve the equation $2\sin 2\theta = \cos\theta$ for $0° \le \theta \le 360°$. [4]

2 Show that the curve, given by the parametric equations given below, represents a circle.

$$x = 2\cos\theta + 3,\ y = 2\sin\theta - 3$$

State the radius and centre of this circle. [4]

3 Find the first three terms of the binomial expansion of $\dfrac{1}{2-3x}$.
Give the range of values of x for which the expansion is valid. [5]

4 The points A, B and C are given by the position vectors $\mathbf{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

M is the midpoint of AC.

(i) Find the position vector of M. [1]

(ii) Find the vector $\overrightarrow{BC}$. [2]

(iii) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. [2]

(iv) Show that D lies on BM. [3]

5 A ball is thrown towards a hedge. Its position relative to the point from which it was thrown is given by the parametric equations

$$x = 8t,\ y = 10t - 5t^2.$$

(i) Find the cartesian equation of the trajectory of the ball. [4]

(ii) The ball just clears the hedge. What can you say about the height of the hedge? [3]

6 **Use the Insert provided for this question.**

The graph of $y = \tan x$ is given on the Insert.

On this graph sketch the graph of $y = \cot x$.
Show clearly where your graph crosses the graph of $y = \tan x$ and indicate the asymptotes. [4]

7 When a stone is dropped into still water, ripples move outwards forming a circle of rippled water. At time t seconds after the stone hits the water the radius of the circle of ripples is increasing at a rate that is inversely proportional to the radius
When the radius is 200 cm the rate of increase of the radius is 5 cm per second.

Write down the differential equation that represents this situation. [4]

Section B (36 marks)

8 **(i)** Evaluate $A_0 = \int_0^2 \left(2 + 2x - x^2\right) \mathrm{d}x$. [2]

Fig 8.1 illustrates the cross-section of a proposed tunnel. Lengths are in metres. The equation of the curved section is $y = 2 + \sqrt{2x - x^2}$.

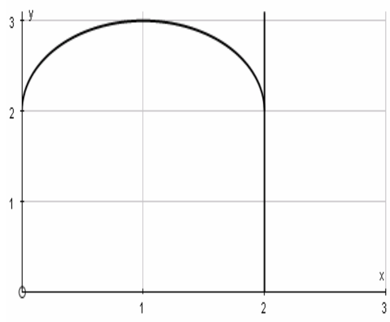

Fig. 8.1

The designers need to know the area of the cross-section, A m^2, so that they can work out the volume of the soil that will need to be removed when the tunnel is built.

(ii) An initial estimate, A_1, is given by the area of the 8 rectangles shown in Fig 8.2. Calculate A_1, and state whether it is an overestimate or underestimate for A. [4]

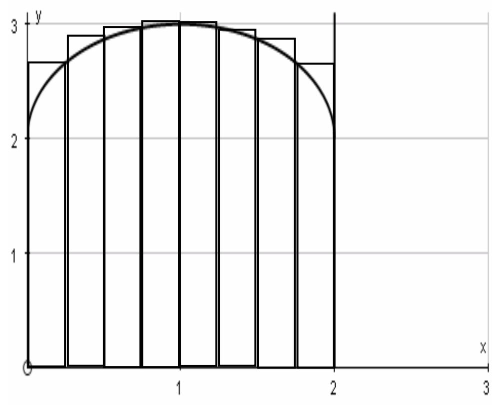

Fig. 8.2

(iii) On graph paper, draw the graphs of

$y = 2 + 2x - x^2$ and $y = 2 + \sqrt{2x - x^2}$ for $0 \le x \le 2$.

Make it clear which equation applies to which curve. [3]

(iv) State whether A_0 , your answer to part (i), is an underestimate for A or an overestimate. Give a reason for your answer. [2]

(v) The designers use the trapezium rule to estimate A. What values does this give when they take

(A) 2 strips, (B) 4 strips, (C) 8 strips?

What can you conclude about the value of A? [5]

(vi) The best estimate from the trapezium rule is denoted by A_2.
State, with a reason, whether the true value of A is nearer A_1 or A_2. [2]

9 A laser beam is aimed from a point (12, 10, 10) in the direction – 2**i** – 2**j** – 3**k** towards a plane surface.

(i) Give the equation of the path of the laser beam in vector form. [2]

The points A (1, 1, 1), B (1, 4, 2) and C (6, 1, 0) lie on the plane.

(ii) Show that the vector 3**i** – 5**j** + 15**k** is perpendicular to the plane and hence find the cartesian equation of the plane. [8]

(iii) Find the coordinate of the point where the laser beam hits the surface of the plane. [4]

(iv) Find the angle between the laser beam and the plane. [4]

Insert for question 6.

The graph of $y = \tan x$ is given below.

On this graph sketch the graph of $y = \cot x$.
Show clearly where your graph crosses the graph of $y = \tan x$ and indicate the asymptotes. [4]