MEI

Mathematics in Education and Industry

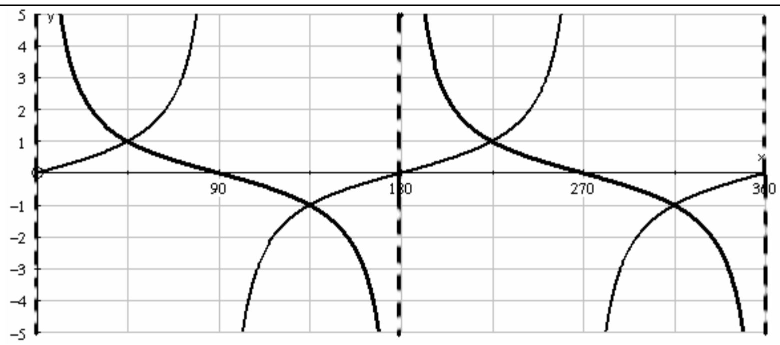
MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-B

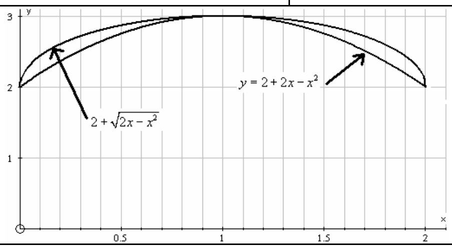
MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$2\sin 2\theta = \cos\theta$ $\Rightarrow 4\sin\theta\cos\theta = \cos\theta$ $\Rightarrow \cos\theta = 0 \text{ or } 4\sin\theta = 1$ $\Rightarrow \sin\theta = 0.25$ $\Rightarrow \theta = 90 \text{ or } 270 \text{ or } 14.5 \text{ or } 165.5$	M1 M1 A1 A1 **4**	Use of double angle formula Solve B1 for last two only
2		$x = 2\cos\theta + 3$ $\Rightarrow 2\cos\theta = x - 3$ $y = 2\sin\theta - 3$ $\Rightarrow 2\sin\theta = y + 3$ $\Rightarrow (x-3)^2 + (y+3)^2$ $= 4\cos^2\theta + 4\sin^2\theta$ $= 4$ i.e. circle, centre (3, –3) radius 2.	M1 A1 A1 A1 **4**	Getting cos and sin as subject
3		$\frac{1}{2-3x} = \frac{1}{2}\left(1 - \frac{3}{2}x\right)^{-1}$ $= \frac{1}{2}\left(1 + (-1)\left(-\frac{3}{2}x\right) + \frac{(-1)(-2)}{1\times 2}\left(-\frac{3}{2}x\right)^2 + \ldots\right)$ $= \frac{1}{2}\left(1 + \frac{3}{2}x + \frac{9}{4}x^2 + \ldots\right)$ $= \frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2$ Valid if $-1 < \frac{3}{2}x < 1 \Rightarrow -\frac{2}{3} < x < \frac{2}{3}$	M1 A1 M1 A1 B1 **5**	Extracting $\frac{1}{2}$
4	**(i)**	$\underline{m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$	B1 **1**	
	(ii)	$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$	M1 A1 **2**	
	(iii)	$\overrightarrow{BC} = \overrightarrow{AD}$ $\Rightarrow \underline{c} - \underline{b} = \underline{d} - \underline{a}$ $\Rightarrow \underline{d} = \underline{a} + \underline{c} - \underline{b}$ $= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	M1 A1 **2**	

	(iv)	$\overrightarrow{BD}$ (say) $= \underline{d} - \underline{b}$ $= \begin{pmatrix} 2 \\ -6 \end{pmatrix}$ $\overrightarrow{BM}$ (say) $= \underline{m} - \underline{b}$ $= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ so $\overrightarrow{BD} = 2\overrightarrow{BM}$ Hence B, D, M are collinear QED.	M1 A1 B1 **3**	
5	**(i)**	$t = \frac{x}{8}$ $y = 10\left(\frac{x}{8}\right) - 5\left(\frac{x}{8}\right)^2$ $= \frac{5x}{4} - \frac{5x^2}{64}$	M1 M1 A1 B1 **4**	t as subject Substitute for t
	(ii)	$y = \frac{5x}{4} - \frac{5x^2}{64}$ $= \frac{5}{64}\left(64 - (x-8)^2\right)$ So the maximum height is 5 metres (when $x = 8$).	M1 A1 B1 **3**	Or use calculus.
6			B1 B1 B1 B1 **4**	2 branches Zeros Asymptotes Crosses $y = \tan x$ at (45,1), (315,-1), (135,-1) and (225,1)
7		$\frac{dr}{dt} = \frac{k}{r}$ $5 = \frac{k}{200}$ $\Rightarrow k = 1000$ $\frac{dr}{dt} = \frac{1000}{r}$	B1 B1 B1 B1 **4**	

<table>
<tr><td colspan="5">Section B</td></tr>
<tr><td>8</td><td>(i)</td><td>$\int_0^2 (2+2x-x^2)dx$
$=\left[2x+x^2-\frac{x^3}{3}\right]_0^2$
$=\left(4+4-\frac{8}{3}\right)$
$=5\frac{1}{3}$
Area is 5.33 m^2, to 3sf.</td><td>M1

A1
2</td><td></td></tr>
<tr><td></td><td>(ii)</td><td><table><tr><td>x</td><td>y</td></tr><tr><td>0.25</td><td>2.661438</td></tr><tr><td>0.5</td><td>2.866025</td></tr><tr><td>0.75</td><td>2.968246</td></tr><tr><td>1</td><td>3</td></tr><tr><td>total</td><td>11.49571</td></tr></table>Area estimate
$=2\times 0.25\times 11.49571$
$=5.747855$
= 5.75 to 3 dp.
This is an overestimate.</td><td>M1
A1

A1
B1
4</td><td>A1 for 11.4… or equivalent, evident or implied.</td></tr>
<tr><td></td><td>(iii)</td><td>
</td><td>B1
B1

B1

3</td><td>B1 for each curve,

for labels</td></tr>
<tr><td></td><td>(iv)</td><td>It is an underestimate, since the curve is everywhere below the correct one (except at x = 1).</td><td>B1
B1
2</td><td></td></tr>
<tr><td></td><td>(v)</td><td><table><tr><td>x</td><td>y</td><td></td><td></td><td></td></tr><tr><td>0</td><td>2</td><td></td><td></td><td></td></tr><tr><td>0.25</td><td></td><td></td><td></td><td>2.661438</td></tr><tr><td>0.5</td><td></td><td></td><td>2.866025</td><td></td></tr><tr><td>0.75</td><td></td><td></td><td></td><td>2.968246</td></tr><tr><td>1</td><td></td><td>3</td><td></td><td></td></tr><tr><td>1.25</td><td></td><td></td><td></td><td>2.968246</td></tr><tr><td>1.5</td><td></td><td></td><td>2.866025</td><td></td></tr><tr><td>1.75</td><td></td><td></td><td></td><td>2.661438</td></tr><tr><td>2</td><td>2</td><td></td><td></td><td></td></tr><tr><td>total</td><td>4</td><td>3</td><td>5.732051</td><td>11.25937</td></tr></table>For 2 strips, estimate is
$(4+2\times 3)\times\frac{1}{2}=5$;
for 4 strips, estimate is
$(4+2\times(3+5.732051))\times\frac{0.5}{2}=5.3660255$;
for 8 strips, estimate is
$(4+2\times(3+5.732051+11.25937))\times\frac{0.5}{2}=5.49785525$
A is probably about 5.5 from the rate of convergence which is evident in these three estimates.</td><td>M1

A1

A1

A1

B1
5</td><td>For the values for the trapezium method.</td></tr>
<tr><td></td><td>(vi)</td><td>The trapezium rule uses an approximation which follows the curve much more closely so it will be the better.</td><td>B1
B1
2</td><td></td></tr>
</table>

9	**(i)**	$\underline{r} = \begin{pmatrix} 12 \\ 10 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix}$	M1 A1 **2**	
	(ii)	$\overrightarrow{AB}$ (say) $= \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$, $\overrightarrow{AC}$ (say) $= \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$ $\overrightarrow{AB}.\begin{pmatrix} 3 \\ -5 \\ 15 \end{pmatrix} = 0,\quad \overrightarrow{AC}.\begin{pmatrix} 3 \\ -5 \\ 15 \end{pmatrix} = 0.$ Since $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are non-parallel vectors in the plane and perpendicular to the vector given, it must be perpendicular to the plane. Cartesian equation is $(\underline{r} - \underline{a}).\begin{pmatrix} 3 \\ -5 \\ 15 \end{pmatrix} = 0$ $\Rightarrow 3x - 5y + 15z = 13$	M1 A1 M1 A1 E1 M1 A1 A1 **8**	Find vectors on surface Scalar product for explanation of result = 0 Or any other valid procedure.
	(iii)	Substitute the parametric equation of the line into the plane to get $3(12 - 2\lambda) - 5(10 - 2\lambda) + 15(10 - 3\lambda) = 13$ $\Rightarrow \lambda = 3$ Hence the point is (6,4,1).	M1 A1 A1 A1 **5**	Substitution correct λ. Point
	(iv)	The angle required is the complement of the angle between the direction of the laser beam and the normal. The latter angle is θ where $\cos\theta = \dfrac{\left\| \begin{pmatrix} 3 \\ -5 \\ 15 \end{pmatrix} . \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 3 \\ -5 \\ 15 \end{pmatrix} \right\| . \left\| \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix} \right\|}$ $= \left\| \dfrac{-6 + 10 - 45}{\sqrt{9 + 25 + 225}.\sqrt{4 + 4 + 9}} \right\|$ so $90 - \theta = \arcsin\left(\dfrac{41}{\sqrt{259}\sqrt{17}} \right)$ $= 38.16.....$ Required angle is 38.2^{o}, to 1dp.	M1 M1 A1 A1 **4**	Scalar product Substitute in formula