MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

Practice Paper C4-A

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

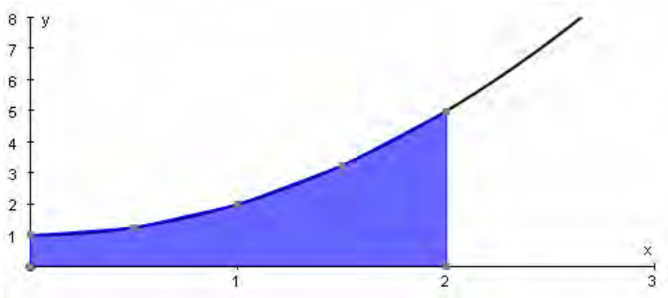
INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Find the coefficient of the term in x^3 in the expansion of $\dfrac{1}{(2+3x)^2}$. [5]

2 The graph shows part of the curve $y = x^2 + 1$.



Find the volume when the area between this curve, the axes and the line $x = 2$ is rotated through 360^0 about the x-axis. [4]

3 Solve the equation $\sec^2\theta = 2\tan\theta + 4$ for $0° < \theta < 360°$. [5]

4 You are given that $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ k \end{pmatrix}$.

(i) Find the angle between **a** and **b** when $k = 2$. [4]

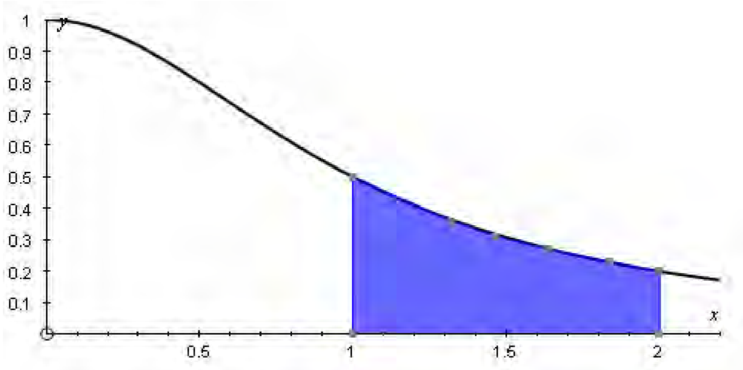
(ii) Find the value of k such that **a** and **b** are perpendicular. [2]

5 A curve is given by the parametric equations $x = at^2$, $y = 2at$ (where a is a constant).
A point P on the curve has coordinates $(ap^2, 2ap)$.

(i) Find the coordinates of the point, T, where the tangent to the curve at P meets the x-axis and the coordinates of the point N where the normal to the curve at P meets the x-axis. [6]

(ii) Hence show that the area of the triangle PTN is $2a^2p(p^2 + 1)$ square units. [2]

6 The graph shows part of the curve $y = \dfrac{1}{1+x^2}$.



Use the trapezium rule to estimate the area between the curve, the x-axis and the lines $x = 1$ and $x = 2$ using

(i) 2 strips, [3]

(ii) 4 strips. [3]

What can you conclude about the true value of the area? [2]

Section B (36 marks)

7 A quantity of oil is dropped into water. When the oil hits the water it spreads out as a circle. The radius of the circle is r cm after t seconds and when $t = 3$ the radius of the circle is increasing at the rate of 0.5 centimetres per second.

One observer believes that the radius increases at a rate which is proportional to $\dfrac{1}{(t+1)}$.

(i) Write down a differential equation for this situation, using k as a constant of proportionality. [1]

(ii) Show that $k = 2$. [1]

(iii) Calculate the radius of the circle after 10 seconds according to this model. [4]

Another observer believes that the rate of increase of the radius of the circle is proportional to $\dfrac{1}{(t+1)(t+2)}$.

(iv) Write down a new differential equation for this new situation. Using the same initial conditions as before, find the value of the new constant of proportionality. [3]

(v) Hence solve the differential equation. [7]

(vi) Calculate the radius of the circle after 10 seconds according to this model. [2]

8 The height of tide at the entrance to a harbour on a particular day may be modelled by the function $h = 3 + 2\sin 30t + 1.5\cos 30t$ where h is measured in metres, t in hours after midnight and $30t$ is in degrees.
[The values 2 and 1.5 represent the relative effects of the moon and sun respectively.]

(i) Show that $2\sin 30t + 1.5\cos 30t$ can be written in the form $2.5\sin(30t + \alpha)$, where α is to be determined. [3]

(ii) Find the height of tide at high water and the first time that this occurs after midnight. [4]

(iii) Find the range of tide during the day. [1]

(iv) Sketch the graph of h against t for $0 \le t \le 12$, indicating the maximum and minimum points. [2]

(v) A sailing boat may enter the harbour only if there is at least 2 metres of water. Find the times during this morning when it may enter the harbour. [5]

(vi) From your graph estimate the time at which the water falling fastest and the rate at which it is falling. [3]