MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4

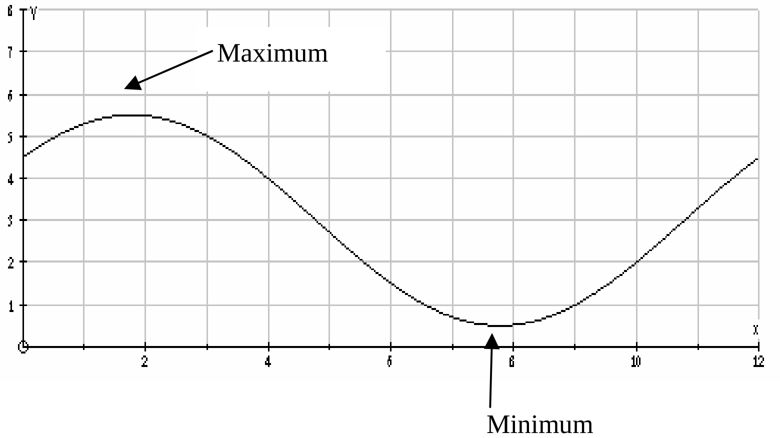
Practice Paper C4-A

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$(2+3x)^{-2} = 2^{-2}\left(1+\frac{3x}{2}\right)^{-2}$	M1 A1	Extract 2 remaining bracket
		$= \frac{1}{4}\left(1+\frac{(-2)}{1}\left(\frac{3x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{3x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{3x}{2}\right)^3+...\right)$	M1	For sight of numerator and denominator and power
		$\Rightarrow$ coefficient is $\frac{1}{4}\times\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{3}{2}\right)^3 = \frac{1}{4}\times(-4)\times\frac{27}{8} = -\frac{27}{8}$	A1 A1 **5**	Sign
2		$V = \pi\int_0^2 (x^2+1)^2\,dx$	M1	Integral
		$= \pi\int_0^2 (x^4+2x^2+1)\,dx$	M1	Multiply out
		$= \pi\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_0^2 = \pi\left(\frac{2^5}{5}+\frac{2.2^3}{3}+2\right)-0$	A1	
		$= \frac{206\pi}{15}$	A1 **4**	
3		$\sec^2\theta = 2\tan\theta + 4 \Rightarrow 1+\tan^2\theta = 2\tan\theta+4$ $\Rightarrow \tan^2\theta - 2\tan\theta - 3 = 0$	M1 A1	Use of identity
		$\Rightarrow (\tan\theta-3)(\tan\theta+1)=0$	M1	Factorising
		$\Rightarrow \tan\theta = 3 \Rightarrow \theta = 71.6,\ 251.6$ $\tan\theta = -1 \Rightarrow \theta = 135,\ 315$	A1 A1 **5**	both both
4	**(i)**	$\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}3\\-1\\2\end{pmatrix} = 3-2-2=-1$	M1	
		$n_1.n_2 = \sqrt{6}\sqrt{14}\cos\theta \Rightarrow \cos\theta = \frac{-1}{\sqrt{84}} \approx -0.1091$ $\Rightarrow \theta = 96.3^0$	M1 A1 A1 **4**	
	(ii)	**a.b.** = 0 $\Rightarrow 3-2-k=0 \Rightarrow k=1$	M1 A1 **2**	

5	**(i)**	$x = at^2 \Rightarrow \dot{x} = 2at$ $y = 2at \Rightarrow \dot{y} = 2a \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$; at P Grad = $\frac{1}{p}$ Tangent: $(y - 2ap) = \frac{1}{p}(x - ap^2)$ $\Rightarrow py - x = ap^2$ When $y = 0$, $x = -ap^2$ Normal: $(y - 2ap) = -p(x - ap^2)$ $\Rightarrow y + px = ap^3 + 2ap$ When $y = 0, x = ap^2 + 2a$	M1 A1 M1 A1 M1 A1 **6**	
	(ii)	$\lvert \mathrm{TN} \rvert = ap^2 + 2a + ap^2 = 2a(p^2 + 1)$ $\Rightarrow Area = \frac{1}{2} \times height \times 2a(p^2 + 1)$ $= \frac{1}{2} \times 2ap \times 2a(p^2 + 1) = 2a^2 p(p^2 + 1)$	M1 A1 **2**	
6	**(i)**	The values for this question are: x y 1 0.5 1.25 0.3902 1.5 0.3077 1.75 0.2462 2 0.2 $T_2 = \frac{1}{2} \times 0.5(y_1 + 2y_{1.5} + y_2) \approx 0.3288$	M1 A1 A1 **3**	
	(ii)	$T_4 = \frac{1}{2} \times 0.25(y_1 + 2(y_{1.25} + y_{1.5} + y_{1.75}) + y_2) \approx 0.3235$	M1 A1 A1 **3**	
		The true value will be less that 0.3235 One could be reasonably confident that it is accurate to 2 d.p.	B1 B1 **2**	

Section B				
7	**(i)**	$\dfrac{dr}{dt} = \dfrac{k}{(t+1)}$	B1 **1**	
	(ii)	$t = 3, \dfrac{dr}{dt} = 0.5 \Rightarrow \dfrac{k}{(4)} = 0.5 \Rightarrow k = 2$	B1 **1**	
	(iii)	$\dfrac{dr}{dt} = \dfrac{2}{(t+1)} \Rightarrow r = 2\ln(t+1) + c$ $t = 0, r = 0 \Rightarrow c = 0$ When $t = 10$, $r = 2\ln 11 \approx 4.796$ cm	M1 A1 A1 A1 **4**	
	(iv)	$\dfrac{dr}{dt} = \dfrac{k}{(t+1)(t+2)}$ $\dfrac{dr}{dt} = 0.5, t = 3 \Rightarrow 0.5 = \dfrac{k}{4 \times 5} \Rightarrow k = 10$	B1 M1 A1 **3**	
	(v)	$\dfrac{1}{(t+1)(t+2)} = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ $\Rightarrow A(t+2) + B(t+1) \equiv 1$ $\Rightarrow A + B = 0,\ 2A + B = 1$ $\Rightarrow A = 1, B = -1$ $\Rightarrow \dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ $\dfrac{dr}{dt} = \dfrac{10}{(t+1)(t+2)} = 10\left(\dfrac{1}{t+1} - \dfrac{1}{t+2}\right)$ $\Rightarrow r = 10\ln\left(\dfrac{t+1}{t+2}\right) + c$ $r = 0, t = 0 \Rightarrow c = -10\ln\dfrac{1}{2}$ $\Rightarrow r = 10\ln 2\left(\dfrac{t+1}{t+2}\right)$	M1 A1 M1 A1 M1 A1 A1 **7**	Using partial fractions both Integrate to log functions condone omission of c Sub to find c
	(vi)	$r = 10\ln 2\left(\dfrac{t+1}{t+2}\right)$ $t = 10 \Rightarrow r = 10\ln\dfrac{22}{12} \approx 6.06$ metres	M1 A1 **2**	Substituting

8	**(i)**	$R\left(\frac{2}{R}\sin 30t+\frac{1.5}{R}\cos 30t\right)=R\left(\sin 30t\cos\alpha+\cos 30t\sin\alpha\right)$ Where $R=\sqrt{2^2+1.5^2}=2.5,\ \cos\alpha=\frac{2}{2.5}\Rightarrow\alpha=36.9^0$ $\Rightarrow 2.5\sin(30t+36.9)$	M1 A1 A1 **3**	 R α
	(ii)	Maximum is $2.5+3=5.5$ Occurs when $\sin\left(30t+\alpha\right)=1$ $\Rightarrow 30t+\alpha=90\Rightarrow 30t=53.1\Rightarrow t\approx 1.77$ $\Rightarrow$ time is approx. 0146	B1 M1 A1 A1 **4**	 Follow through
	(iii)	High water = 3 + 2.5 = 5.5 Low water = 3 – 2.5 = 0.5 Range = 5 metres	B1 **1**	
	(iv)		B1 B1 **2**	Shape Maximum and minimum
	(v)	$3+2.5\sin(30t+36.9)=2$ $\Rightarrow\sin(30t+36.9)=-0.4$ $\Rightarrow 30t+36.9=203.6$ or 336.4 $\Rightarrow t=5.56$ or 9.98 i.e. Midnight to 0533 and 0959 to midday.	M1 A1 A1 A1 A1 **5**	 f.t. Both Both f.t.
	(vi)	The time when the water level is falling fastest 3 hrs after high water - i.e. at 0446. From the graph an estimate of the rate is 5metres per 4 hours or approx 1.25 metres per hour or approx 2 cm per minute. (Calculation gives 1.31 metres per hour, so allow anything from 1.25 - 1.35 metres per hour.)	B1 M1 A1 **3**	