

Worked Solutions

Edexcel C4 Paper L

1. $\int \frac{1}{y} dy = \int 2x \, dx$

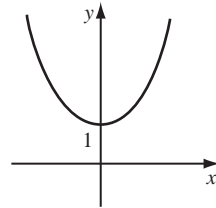
$\ln y = x^2 + c$

$y = e, \quad x = 1: \quad \ln e = 1 + c$

$c = 0$

$\therefore \ln y = x^2$

$y = e^{x^2}$



(4)

2. (a) $V = \pi \int y^2 \, dx = \pi \int_{-2}^0 (t^2 + 1)^2 \frac{dx}{dt} \, dt \quad \frac{dx}{dt} = 1$ (5)

(b) $\pi \int_{-2}^0 (t^2 + 1)^2 \, dt = \pi \int_{-2}^0 (t^4 + 2t^2 + 1) \, dt$

$= \pi \left[\frac{t^5}{5} + \frac{2}{3}t^3 + t \right]_{-2}^0$

$= \pi \left[0 - \left(\frac{1}{5}(-32) + \frac{2}{3}(-8) + (-2) \right) \right] = \frac{206}{15}\pi$ (2)

3. (a) $(\pi, 0)$ (1)

(b) $\frac{dy}{dx} = x \cos x + \sin x$

when $x = 2.02, \quad \frac{dy}{dx} \approx 0.02$

when $x = 2.04, \quad \frac{dy}{dx} \approx -0.03$ (4)

(c) Area $= \int_0^{\pi} x \sin x \, dx = \int_0^{\pi} x \frac{d}{dx}(-\cos x) dx = [-x \cos x]_0^{\pi} - \int_0^{\pi} (-\cos x) \, dx$
 $= [-x \cos x + \sin x]_0^{\pi} = \pi$ (4)

4. (a) $17 \left(\frac{17^2 - 1}{17^2} \right)^{\frac{1}{2}} = \frac{17}{17} [(17 - 1)(17 + 1)]^{\frac{1}{2}} = \sqrt{16} \sqrt{18}$
 $= 4\sqrt{18} = 4\sqrt{9 \times 2} = 12\sqrt{2}$ (3)

(b) $(1 - x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2} \left(-\frac{1}{2} \right)}{2} (-x)^2 = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \dots$ (2)

(c) put $x = \frac{1}{17^2}, \left(1 - \frac{1}{17^2} \right)^{\frac{1}{2}} \approx 1 - \frac{1}{2} \cdot \frac{1}{17^2} \approx \frac{577}{578}$

$\therefore 17 \left(\frac{577}{578} \right) \approx 12\sqrt{2}$

$\sqrt{2} \approx \frac{17}{12} \times \frac{577}{578}$

$\sqrt{2} \approx \frac{577}{408}$ (5)

$$5. (a) \int x \frac{d}{dx} \left(\frac{1}{k} \sin kx \right) dx = \frac{x}{k} \sin kx - \frac{1}{k} \int \sin kx \, dx$$

$$= \frac{x}{k} \sin kx + \frac{1}{k^2} \cos kx + c \quad (4)$$

$$(b) \left[\frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x \right]_0^{\frac{\pi}{4}} = \frac{\pi}{8} \sin \frac{\pi}{2} + 0 - \left(0 + \frac{1}{4} \right) = \frac{1}{8}(\pi - 2) \quad (4)$$

$$(c) \cos 2x = 2 \cos^2 x - 1$$

$$\therefore 2 \cos^2 x = \cos 2x + 1$$

$$\int_0^{\frac{\pi}{4}} 2x \cos^2 x \, dx = \int_0^{\frac{\pi}{4}} (x \cos 2x + x) dx$$

$$= \frac{1}{8}(\pi - 2) + \left[\frac{x^2}{2} \right]_0^{\frac{\pi}{4}} = \frac{1}{8}(\pi - 2) + \frac{\pi^2}{32} \quad (4)$$

$$6. (a) \vec{AB} = \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix}, \text{ line through } A \text{ and } B \text{ is } \mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix} \quad (2)$$

$$(b) \text{ If } P \text{ lies on } AB \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix}$$

The equation is satisfied for x, y, z with $\lambda = \frac{1}{3}$ (2)

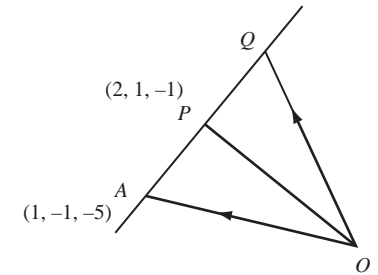
$$(c) \vec{OP} \cdot \vec{AB} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix} = 6 + 6 - 12 = 0$$

$\therefore OP$ is perpendicular to AB (2)

$$(d) \vec{AP} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

As $|\vec{OQ}| = |\vec{OA}|$ we have $\vec{AP} = \vec{PQ}$

$$\therefore \text{ position vector of } Q \text{ is } \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$



$$= \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad (4)$$

$$7. (a) \frac{dy}{dx} - 2x + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(1+x) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{1+x} \quad (6)$$

$$(b) \text{ Gradient at } \left(1, \frac{9}{2} \right) = \frac{2 - \frac{9}{2}}{1+1} = -\frac{5}{4} \quad (1)$$

$$(c) \frac{dy}{dx} = 0 \text{ where } y = 2x \text{ (from (a))}$$

substitute $y = 2x$ into $y - x^2 + xy = 8$

$$2x - x^2 + x(2x) = 8$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4, 2$$

stationary points are $(-4, -8)$ and $(2, 4)$ (4)

$$8. (a) \frac{dy}{dx} = \frac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$\text{at } x = 1, \frac{dy}{dx} = \frac{1}{4}$$

\therefore gradient of normal is -4

$$\text{equation of normal is } y - \frac{1}{2} = -4(x - 1)$$

$$y = -4x + 4\frac{1}{2}$$

(3)

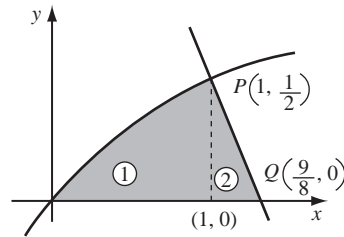
(b) normal cuts x -axis where $y = 0$

$$\text{i.e. } 4x = \frac{9}{2}$$

$$x = \frac{9}{8}$$

shaded area = area (1) + area (2)

$$\text{area (1)} = \int_0^1 \frac{x}{x+1} dx$$



let $u = x + 1$

$$\frac{du}{dx} = 1$$

when $x = 1, u = 2$

$$x = 0, u = 1$$

$$\begin{aligned} \therefore \text{area (1)} &= \int_1^2 \left(\frac{u-1}{u} \right) du = \int_1^2 \left(1 - \frac{1}{u} \right) du \\ &= \left[u - \ln u \right]_1^2 = 2 - \ln 2 - (1 - \ln 1) \\ &= 1 - \ln 2 \end{aligned}$$

area (2) = $\frac{1}{2}$ base \times height of Δ

$$= \frac{1}{2} \times \frac{1}{8} \times \frac{1}{2} = \frac{1}{32}$$

$$\begin{aligned} \therefore \text{shaded area} &= 1 - \ln 2 + \frac{1}{32} \\ &= \approx 0.338 \text{ units}^2 \end{aligned}$$

(8)