

Worked Solutions

Edexcel C4 Paper B

1. (a) $xy - 2y + 5x - 10 = 12$

$$x \frac{dy}{dx} + y \cdot 1 - 2 \frac{dy}{dx} + 5 = 0$$

$$\frac{dy}{dx}(x - 2) = -(y + 5)$$

$$\frac{dy}{dx} = \frac{y + 5}{2 - x} \quad (2)$$

(b) at (4, 1), $\frac{dy}{dx} = \frac{1 + 5}{2 - 4} = -3$

$$\therefore \text{gradient of normal} = \frac{1}{3}$$

equation of normal is $y - 1 = \frac{1}{3}(x - 4)$

$$3y = x - 1 \quad (3)$$

2. (a) $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r \quad (1)$$

(b) given $\frac{dr}{dt} = \frac{1}{4}$.

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$r = 3, \quad \frac{dA}{dt} = (2\pi \times 3) \times \left(\frac{1}{4}\right) = \frac{3\pi}{2} \text{ cm}^2 \text{ s}^{-1} \quad (4)$$

3. (a) $1 + \frac{1}{2}(8x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(8x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3.2}(8x)^3 + \dots$

$$= 1 + 4x - 8x^2 + 32x^3 \quad (3)$$

(b) $-\frac{1}{8} < x < \frac{1}{8} \quad (1)$

(c) $(1 + ax)(1 + 4x - 8x^2 + 32x^3) = 1 + 4x - 8x^2 + 32x^3 + ax + 4ax^2 - 8ax^3$

$$4 + a = -8 + 4a$$

$$12 = 3a, \quad a = 4$$

$$\text{coef. of } x^3 = 32 - 8a = 32 - 32 = 0 \quad (6)$$

4. (a) $t = 10, \quad m = 500e^{-0.2} = 409.4 \text{ grams} \quad (1)$

(b) $m = 300, \quad 300 = 500e^{-0.02t}$

$$e^{-0.02t} = \frac{3}{5}$$

$$\ln e^{-0.02t} = \ln\left(\frac{3}{5}\right)$$

$$-0.02t = \ln\frac{3}{5}$$

$$t = 25.5 \text{ years} \quad (2)$$

(c) $\frac{dm}{dt} = 500 \times (-0.02)e^{-0.02t}$

when $t = 1, \quad \frac{dm}{dt} = 500 \times (-0.02) \times e^{-0.02} = -9.8 \text{ g/year}$

mass is decreasing at 9.8 g/year (3)

$$5. (a) (i) \int (1 + e^x + e^{-x} + 1) dx = 2x + e^x - e^{-x} + c \quad (2)$$

$$(ii) \int (6x - 1)^{-\frac{1}{2}} dx = 2 \times \frac{1}{6} (6x - 1)^{\frac{1}{2}} + c = \frac{1}{3} \sqrt{6x - 1} + c \quad (2)$$

$$(b) \int_0^{\frac{\pi}{6}} x \cos x \, dx = \int_0^{\frac{\pi}{6}} x \frac{d}{dx} (\sin x) \, dx$$

$$= [x \sin x]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x \, dx$$

$$= \frac{\pi}{6} \cdot \frac{1}{2} - 0 + [\cos x]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \quad (5)$$

$$6. (a) \frac{dy}{d\theta} = 2.2 \cos \theta (-\sin \theta)$$

$$\frac{dx}{d\theta} = \cos \theta$$

$$\frac{dy}{dx} = \frac{-4 \sin \theta \cos \theta}{\cos \theta} = -4 \sin \theta$$

$$\text{at } \theta = \frac{\pi}{6}, \text{ gradient of tangent} = -4 \cdot \frac{1}{2} = -2$$

$$\text{at } \theta = \frac{\pi}{6}, x = \frac{1}{2}, y = 2 \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{3}{2}$$

$$\text{equation of tangent is } y - \frac{3}{2} = -2 \left(x - \frac{1}{2} \right)$$

$$2y + 4x = 5 \quad (5)$$

$$(b) y = 2 \cos^2 \theta = 2(1 - \sin^2 \theta)$$

$$y = 2(1 - x^2) \quad (2)$$

$$7. (a) \frac{1}{(y-1)y} = \frac{1}{y-1} - \frac{1}{y} \quad (3)$$

$$(b) \int \left(\frac{1}{y-1} - \frac{1}{y} \right) dy = \int \cos x \, dx$$

$$\ln(y-1) - \ln y = \sin x + c$$

$$y = 5, \quad x = 0, \quad \ln 4 - \ln 5 = 0 + c$$

$$\ln \frac{4}{5} = c$$

$$\therefore \ln \frac{y-1}{y} = \sin x + \ln \frac{4}{5}$$

$$\ln \frac{y-1}{y} - \ln \frac{4}{5} = \sin x$$

$$\ln \frac{5(y-1)}{4y} = \sin x$$

$$\frac{5y-5}{4y} = e^{\sin x}$$

$$5y - 5 = 4y e^{\sin x}$$

$$y(5 - 4e^{\sin x}) = 5$$

$$y = \frac{5}{5 - 4e^{\sin x}} \quad (7)$$

8. (a) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

equation of AB is $r = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (3)

(b) $\vec{AC} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ -7 \\ -3 \end{pmatrix} = 12 - 12 = 0$

so $12\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ is perpendicular to AC. (2)

(c) let $\angle BAC = \theta$

$$\vec{AB} \cdot \vec{AC} = |\vec{AB}| \times |\vec{AC}| \cos \theta$$

$$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \sqrt{2^2 + 3^2 + 1^2} \times \sqrt{1^2 + 4^2} \cos \theta$$

$$6 = \sqrt{14}\sqrt{17} \cos \theta$$

$$\cos \theta = \frac{6}{\sqrt{14}\sqrt{17}} \quad \theta = 67^\circ$$
 (4)

9. (a) At A $x \ln x = 0$

$$\therefore \ln x = 0 \quad (x \neq 0) \quad x = 1$$
 (1)

(b) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x = 1 + \ln x$

at P $\frac{dy}{dx} = 0$, $\ln x = -1$

$$x = e^{-1} = \frac{1}{e} \quad \text{and} \quad y = \frac{1}{e} \ln e^{-1} = -\frac{1}{e}$$

(c) area = $\int_{\frac{1}{e}}^1 x \ln x \, dx = \int_{\frac{1}{e}}^1 \ln x \frac{d}{dx} \left(\frac{x^2}{2} \right) dx$ (By parts)

$$= \left[\frac{x^2}{2} \ln x \right]_{\frac{1}{e}}^1 - \int_{\frac{1}{e}}^1 \frac{x^2}{2} \cdot \frac{1}{x} \, dx$$

$$= 0 - \frac{1}{2e^2} \ln e^{-1} - \left[\frac{x^2}{4} \right]_{\frac{1}{e}}^1 = \frac{1}{2e^2} - \left[\frac{1}{4} - \frac{1}{4e^2} \right] = \frac{3}{4e^2} - \frac{1}{4}$$
 (6)