

1. $f(x) = \frac{1}{\sqrt{4+x}}, |x| < 4$

Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(6)

Lined area for writing the binomial expansion.



2.

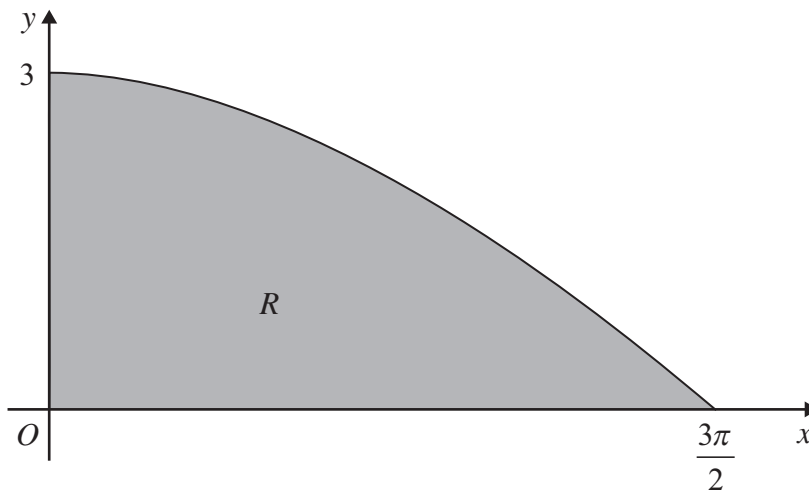


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. (4)
- (c) Use integration to find the exact area of R . (3)



$$3. \quad f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants A , B and C . **(4)**

(b) (i) Hence find $\int f(x) dx$. **(3)**

(ii) Find $\int_0^2 f(x) dx$ in the form $\ln k$, where k is a constant. **(3)**



Question 4 continued

Handwritten response area for Question 4 continued. The page contains 28 horizontal lines for writing.



5.

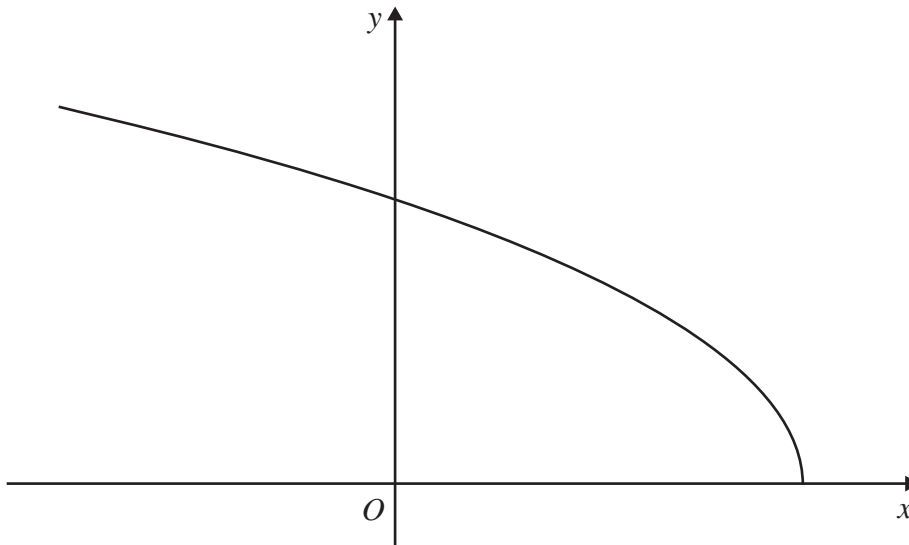


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2 \cos 2t, \quad y = 6 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$. (4)

(b) Find a cartesian equation of the curve in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant k . (4)

(c) Write down the range of $f(x)$. (2)



6. (a) Find $\int \sqrt{5-x} dx$. (2)

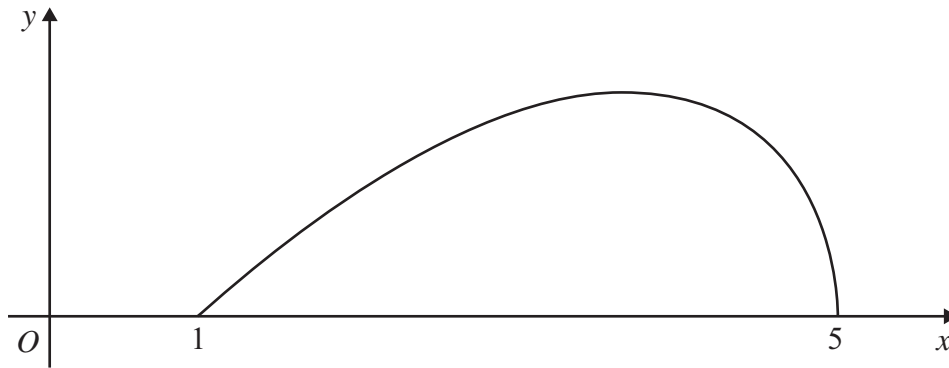


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1) \sqrt{5 - x}, \quad 1 \leq x \leq 5$$

- (b) (i) Using integration by parts, or otherwise, find

$$\int (x - 1) \sqrt{5 - x} dx \quad (4)$$

- (ii) Hence find $\int_1^5 (x - 1) \sqrt{5 - x} dx$. (2)



7. Relative to a fixed origin O , the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B .

(a) Find a vector equation for the line l . (3)

(b) Find $|\vec{CB}|$. (2)

(c) Find the size of the acute angle between the line segment CB and the line l , giving your answer in degrees to 1 decimal place. (3)

(d) Find the shortest distance from the point C to the line l . (3)

The point X lies on l . Given that the vector \vec{CX} is perpendicular to l ,

(e) find the area of the triangle CXB , giving your answer to 3 significant figures. (3)



8. (a) Using the identity $\cos 2\theta = 1 - 2 \sin^2 \theta$, find $\int \sin^2 \theta d\theta$. (2)

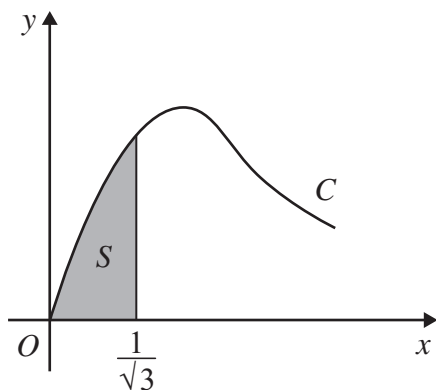


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = 2 \sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C , the line $x = \frac{1}{\sqrt{3}}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_0^{\frac{\pi}{6}} \sin^2 \theta d\theta$$

where k is a constant.

(5)

- (c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)



