Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Advanced Level

Tuesday 28 June 2005 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N20232A

1. Use the binomial theorem to expand

$$\sqrt{(4-9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)

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2. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(7)

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3. (*a*) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(*b*) Hence find the exact value of $\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm.

(5)

---

4. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx.$$

(7)

5.

Figure 1

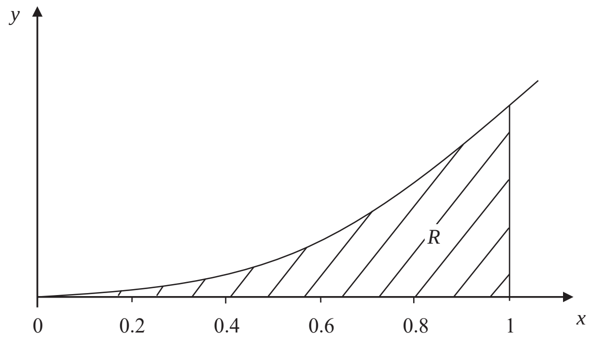

Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in Figure 1.

(*a*) Use integration to find the exact value of the area for R.

(5)

(*b*) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(*c*) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

6. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(4)

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(*c*) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined.

(4)

7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(*a*) Find the coordinates of B.

(4)

(*b*) Find the value of $\cos\theta$, giving your answer as a simplified fraction.

(4)

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(*c*) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(3)

(*d*) Find the position vector of the point D.

(2)

8. Liquid is pouring into a container at a constant rate of 20 $\text{cm}^3\ \text{s}^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(*a*) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(*b*) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(*c*) find the volume of liquid in the container at 10 s after the start.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666
Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23553A

1. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

---

2. (*a*) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(*b*) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \; dx$. Show all the steps of your working and give your answer to 4 decimal places.

(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \; dx$ is $\ln(1 + \sqrt{2})$.

(*c*) Calculate the % error in using the estimate you obtained in part (*b*).

(2)

---

3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}} \, dx.$$

(8)

4.

Figure 1

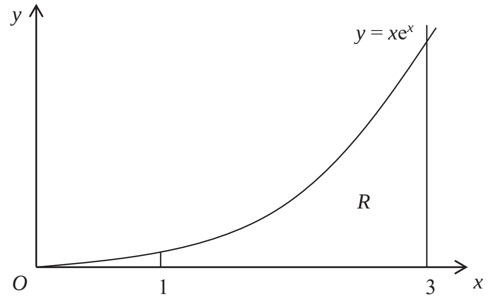

Figure 1 shows the finite region R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

5.

$$f(x) = \frac{3x^2 + 16}{(1-3x)(2+x)^2} = \frac{A}{(1-3x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}, \quad |x| < \tfrac{1}{3}.$$

(*a*) Find the values of A and C and show that $B = 0$.

(4)

(*b*) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3. Simplify each term.

(7)

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6. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(*a*) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(*b*) find the coordinates of P.

(5)

(*b*) Hence find the distance OP, giving your answer as a simplified surd.

(2)

---

7. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$.

(*a*) Find $\dfrac{dV}{dr}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(*b*) Using the chain rule, or otherwise, find an expression in terms of r and t for $\dfrac{dr}{dt}$.

(2)

(*c*) Given that $V = 0$ when $t = 0$, solve the differential equation $\dfrac{dV}{dt} = \dfrac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(*d*) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

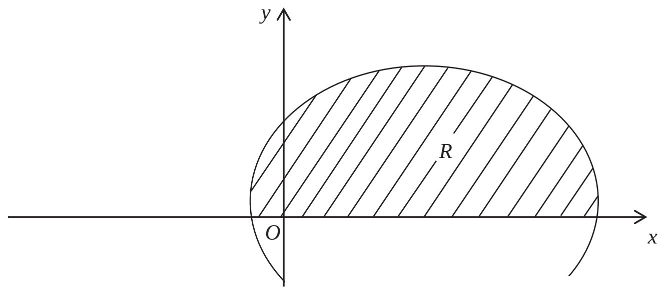
(3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}.

(2)

8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \leq t \leq 2\pi.$$

(*a*) Show that the curve crosses the x-axis where $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

(*b*) Show that the area R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(*c*) Use this integral to find the exact value of the shaded area.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Advanced Level

Thursday 15 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23563A

1. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

2.
$$\mathrm{f}(x) = \frac{3x-1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \tfrac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(*a*) find the values of A and B.

(3)

(*b*) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)

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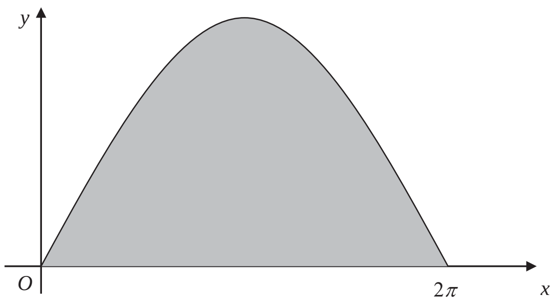
3. **Figure 3**



The curve with equation $y = 3 \sin \frac{x}{2}$, $0 \le x \le 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

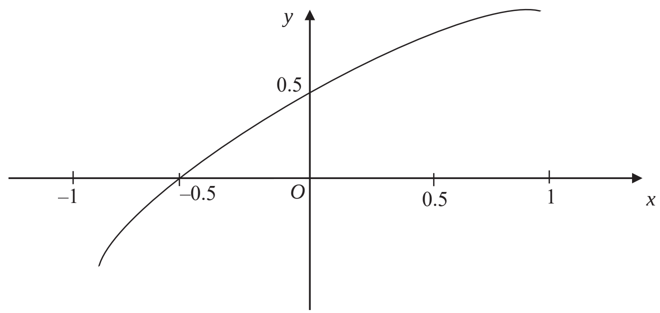
(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)

4. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

5. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(*a*) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(*b*) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(*c*) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

6.

Figure 3

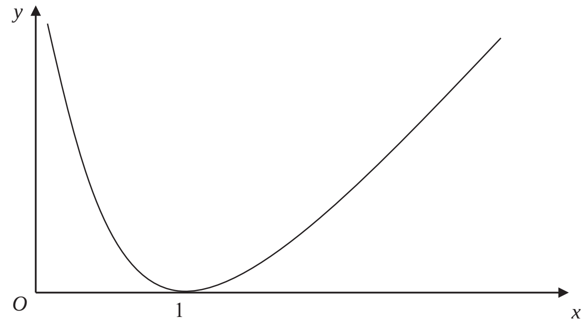


Figure 3 shows a sketch of the curve with equation $y = (x - 1) \ln x$, $x > 0$.

(*a*) Copy and complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2 \ln 3$

(1)

Given that $I = \int_1^3 (x-1) \ln x \ \mathrm{d}x$,

(*b*) use the trapezium rule

(i) with values at y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values at y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

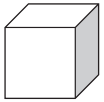
(*c*) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(*d*) Show, by integration, that the exact value of $\int_1^3 (x-1) \ln x \ \mathrm{d}x$ is $\frac{3}{2} \ln 3$.

(6)

7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\dfrac{dx}{dt} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(b) $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Advanced Level

Tuesday 23 January 2007 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
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N23562A

1. $$\mathrm{f}(x) = (2-5x)^{-2}, \quad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction.

(5)

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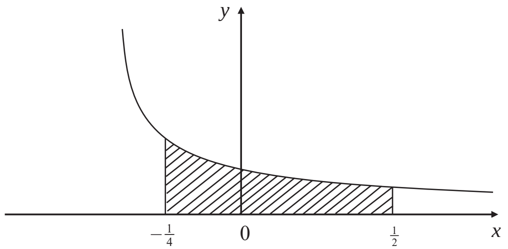
2. **Figure 1**



The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x-axis.

(*a*) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

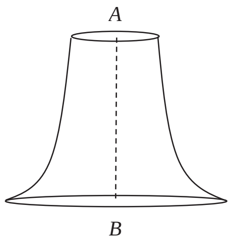


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (*a*).

(*b*) Find the volume of this paperweight.

(2)

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3. A curve has parametric equations

$$x = 7\cos t - \cos 7t, \quad y = 7\sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(*a*) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. You need not simplify your answer.

(3)

(*b*) Find an equation of the normal to the curve at the point where $t = \dfrac{\pi}{6}$.

Give your answer in its simplest exact form.

(6)

---

4. (*a*) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(*b*) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)y.$$

(5)

(*c*) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = \mathrm{f}(x)$.

(4)

---

5. A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(*a*) Use implicit differentiation to find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

(*b*) find the coordinates of the points where $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(5)

---

6. (*a*) Given that $y = 2^x$, and using the result $2^x = e^{x \ln 2}$, or otherwise, show that $\frac{dy}{dx} = 2^x \ln 2$.
(2)

(*b*) Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2, 16).
(4)

---

7. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O.

(*a*) Find the position vector of the point C, with position vector $\mathbf{c}$, given by $\mathbf{c} = \mathbf{a} + \mathbf{b}$.
(1)

(*b*) Show that $OACB$ is a rectangle, and find its exact area.
(6)

The diagonals of the rectangle, AB and OC, meet at the point D.

(*c*) Write down the position vector of the point D.
(1)

(*d*) Find the size of the angle ADC.
(6)

8.
$$I = \int_0^5 e^{\sqrt{(3x+1)}} \, dx .$$

(*a*) Given that $y = e^{\sqrt{(3x+1)}}$, copy and complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.
(3)

(*c*) Use the substitution $t = \sqrt{(3x+1)}$ to show that I may be expressed as $\int_a^b kte^t \, dt$, giving the values of a, b and k.
(5)

(*d*) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.
(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Advanced Subsidiary Level

Monday 18 June 2007 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26110A

1. $$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

(5)

---

2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2}\, dx.$$

(6)

---

3. (*a*) Find $\int x \cos 2x \;\; dx$.

(4)

(*b*) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \;\; dx$.

(3)

---

4. $$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\, dx$ is $2 + \ln k$, giving the value of the constant k.

(6)

---

5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(*a*) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(*b*) Find the cosine of the acute angle between AB and l_1.

(6)

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6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(*c*) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

7.

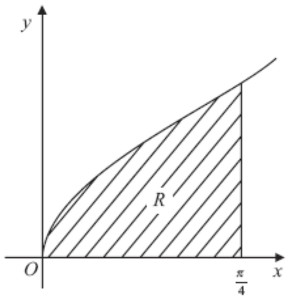


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(*a*) Given that $y = \sqrt{(\tan x)}$, copy and complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(*c*) Use integration to find an exact value for the volume of the solid generated.

(4)

8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(*a*) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(*b*) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(*c*) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(*d*) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Tuesday 22 January 2008 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26282A

1.

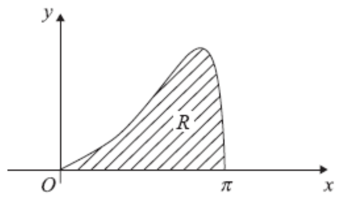


Figure 1

The curve shown in Figure 1 has equation $e^x\sqrt{(\sin x)}$, $0 \le x \le \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(*a*) Copy and complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(*b*) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

---

2. (*a*) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

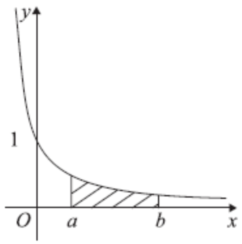
in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(*b*) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

(2)

3.



The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

---

4. (i) Find $\int \ln\left(\dfrac{x}{2}\right) \mathrm{d}x$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \;\, \mathrm{d}x$.

(5)

---

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

---

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

7.

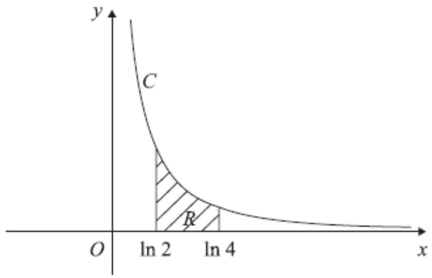


Figure 3

The curve C has parametric equations

$$x = \ln (t + 2), \quad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(*a*) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t\,.$$

(4)

(*b*) Hence find an exact value for this area.

(6)

(*c*) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(*d*) State the domain of values for x for this curve.

(1)

8. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $\mathrm{cm}^3\mathrm{s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(*a*) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - k\sqrt{h},$$

where k is a positive constant.

(3)

When $h = 25$, water is leaking out of the hole at 400 $\mathrm{cm}^3\mathrm{s}^{-1}$.

(*b*) Show that $k = 0.02$.

(1)

(*c*) Separate the variables of the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - 0.02\sqrt{h}$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h\,.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(*d*) find the exact value of $\displaystyle\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h$.

(6)

(*e*) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Thursday 12 June 2008 – Morning
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

H30427A

1.

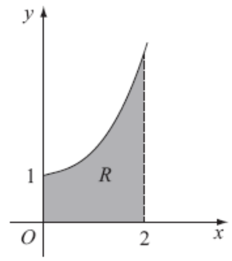


Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

---

2. (a) Use integration by parts to find $\int xe^x \, dx$.

(3)

(b) Hence find $\int x^2e^x \, dx$.

(3)

3.

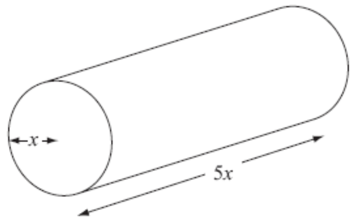


Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of $0.032\ \text{cm}^2\ \text{s}^{-1}$.

(*a*) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures. **(4)**

(*b*) Find the rate of increase of the volume of the rod when $x = 2$. **(4)**

---

4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(*a*) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q. **(6)**

(*b*) Find the coordinates of P and Q. **(3)**

---

5. (*a*) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term. **(5)**

(*b*) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x. **(4)**

---

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection. **(6)**

(*b*) Show that l_1 and l_2 are perpendicular to each other. **(2)**

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(*c*) Show that A lies on l_1. **(1)**

The point B is the image of A after reflection in the line l_2.

(*d*) Find the position vector of B. **(3)**

---

7. (*a*) Express $\frac{2}{4-y^2}$ in partial fractions. **(3)**

(*b*) Hence obtain the solution of

$$2\cot x \frac{\mathrm{d}y}{\mathrm{d}x} = (4 - y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$. **(8)**

8.

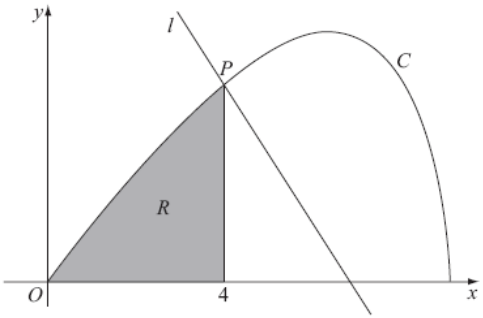


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \ \mathrm{d}t$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Subsidiary

Monday 19 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H31013A

1. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

2.

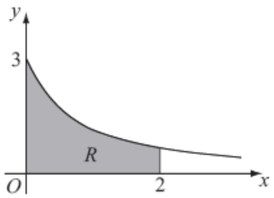


Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

---

3. $$\mathrm{f}(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}.$$

Given that f(x) can be expressed in the form

$$\mathrm{f}(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

(4)

---

4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

5.

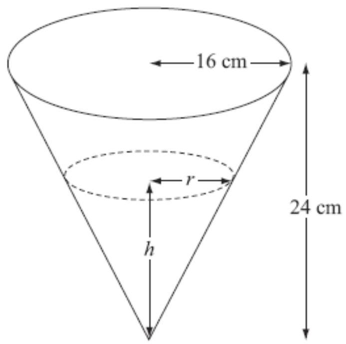


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

6. (a) Find $\int \tan^2 x \; dx$.

(2)

(b) Use integration by parts to find $\int \dfrac{1}{x^3} \ln x \; dx$.

(4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant.

(7)

7.

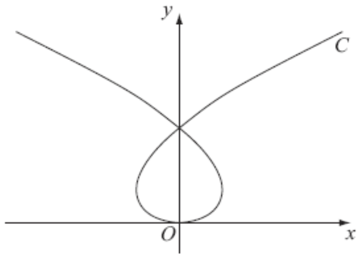


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 15 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H43265A

1. $$\mathrm{f}(x) = \frac{1}{\sqrt{(4+x)}}, \quad |x| < 4.$$

Find the binomial expansion of f (x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

2.

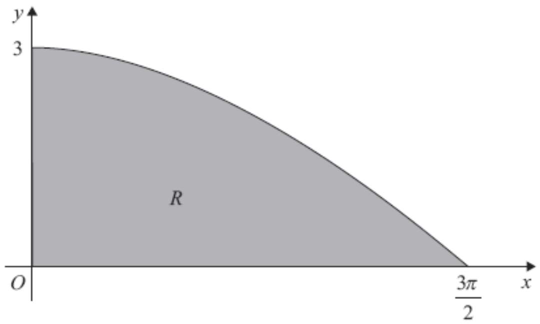


Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3\cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3\cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(*a*) Copy and complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(*c*) Use integration to find the exact area of R.

(3)

---

3. $$\mathrm{f}(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) (i) Hence find $\int \mathrm{f}(x)\ \mathrm{d}x$.

(3)

(ii) Find $\int_0^2 \mathrm{f}(x)\ \mathrm{d}x$ in the form $\ln k$, where k is a constant.

(3)

---

4. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The point P on C has coordinates $(0, 1)$.

(*b*) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

5.

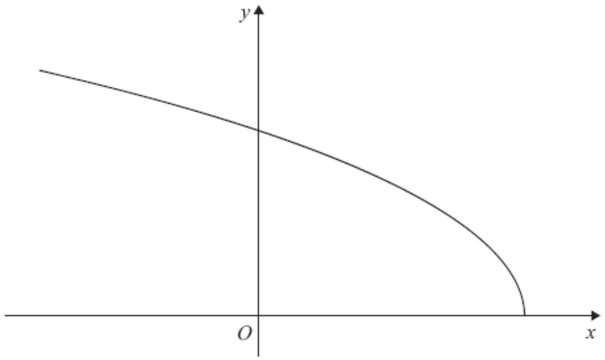


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

(*a*) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(*b*) Find a cartesian equation of the curve in the form

$$y = \text{f}(x), \quad -k \leq x \leq k,$$

stating the value of the constant k.

(4)

(*c*) Write down the range of f(x).

(2)

6. (*a*) Find $\int \sqrt{(5-x)}\ \text{d}x$.

(2)

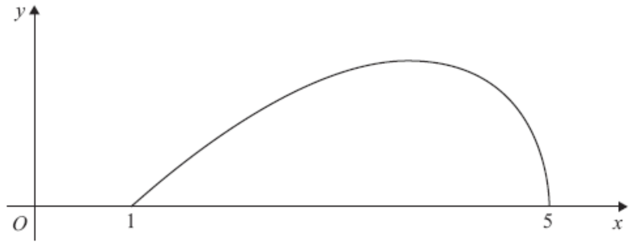


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \quad 1 \leq x \leq 5$$

(*b*) (i) Using integration by parts, or otherwise, find $\int (x-1)\sqrt{(5-x)}\ \text{d}x$.

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\ \text{d}x$. .

(2)

7. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find a vector equation for the line l.
(3)

(*b*) Find $\left|\overrightarrow{CB}\right|$.
(2)

(*c*) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.
(3)

(*d*) Find the shortest distance from the point C to the line l.
(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(*e*) find the area of the triangle CXB, giving your answer to 3 significant figures.
(3)

8. (*a*) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \; d\theta$.
(2)

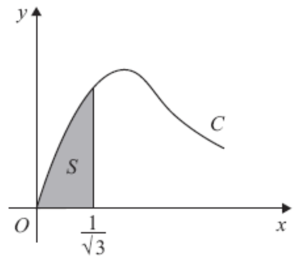


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \; d\theta,$$

where k is a constant.
(5)

(*c*) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.
(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 25 January 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35382A

1. (*a*) Find the binomial expansion of

$$\sqrt{(1-8x)}, \qquad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

2.

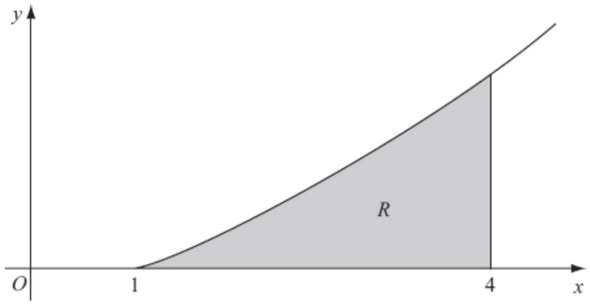


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) (i) Use integration by parts to find $\int x \ln x \; \mathrm{d}x$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

3. The curve C has equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(*b*) Find the value of y at P.

(3)

(*c*) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(*a*) Write down the coordinates of A.

(1)

(*b*) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(*c*) Find the coordinates of X.

(1)

(*d*) Find the vector $\overrightarrow{AX}$.

(2)

(*e*) Hence, or otherwise, show that $\left|\overrightarrow{AX}\right| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(*f*) find the length of AY, giving your answer to 3 significant figures.

(3)

5. (*a*) Find $\int \frac{9x+6}{x} \, dx, \quad x > 0$.

(2)

(*b*) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)

6. The area A of a circle is increasing at a constant rate of 1.5 cm^2 s^{-1}. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

(5)

7.

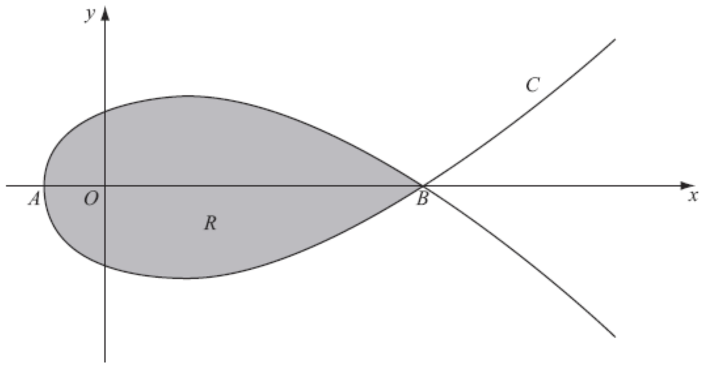


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

8. (a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4 - x^2)}}\, dx \,.$$

(7)

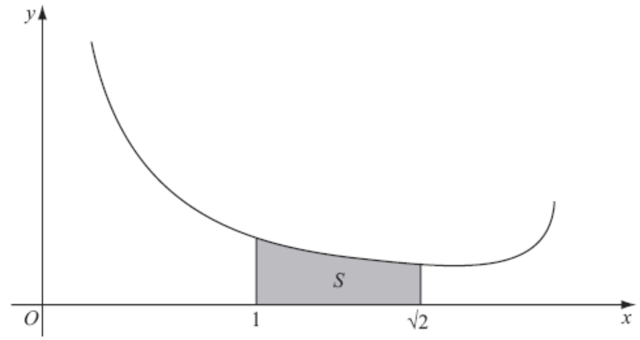


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4 - x^2)^{\frac{1}{4}}}$, $\quad 0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Friday 18 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
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There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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1.

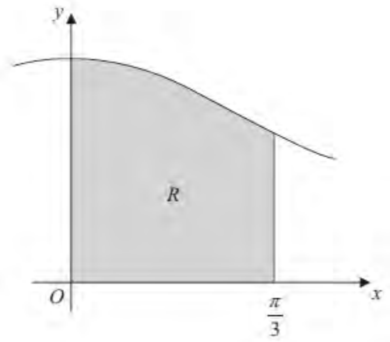

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(*a*) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(*b*) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R.

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1).$$

(6)

---

3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)

---

4. A curve C has parametric equations

$$x = \sin^2 t, \; y = 2 \tan t, \; 0 \leq t < \frac{\pi}{2}.$$

(*a*) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(*b*) Find the x-coordinate of P.

(6)

5.

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2.

Give each coefficient as a simplified fraction.

(7)

6.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(*a*) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(*b*) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta \, f(\theta) \, d\theta$.

(7)

---

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(*a*) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(*b*) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(*c*) Hence, or otherwise, find the area of the triangle ABC.

(5)

8.

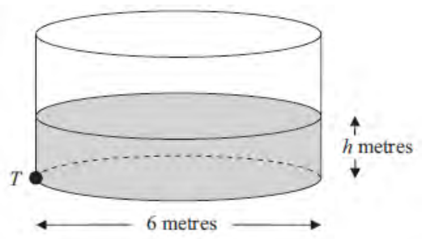

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that, t minutes after the tap has been opened,

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h).$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Wednesday 26 January 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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Answers without working may not gain full credit.

H35405A

1. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$.

(6)

---

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

---

3. (*a*) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(*b*) Hence find $\int \frac{5}{(x-1)(3x+2)} \, dx$, where $x > 1$.

(3)

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$.

(6)

---

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(*a*) Find $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(*c*) the value of p,

(4)

(*d*) the distance AC.

(2)

---

5. (*a*) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \qquad |x| < \frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a + bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$.

Find

(*b*) the value of a and the value of b,

(5)

(*c*) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

---

6. The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0.$$

Find

(a) an equation of the normal to C at the point where $t = 3$,
(6)

(b) a cartesian equation of C.
(3)

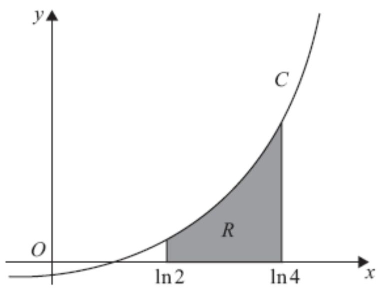

Figure 1

The finite area R, shown in Figure 1, is bounded by C, the x-axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x-axis.

(c) Use calculus to find the exact volume of the solid generated.
(6)

7.
$$I = \int_2^5 \frac{1}{4 + \sqrt{(x-1)}}\,dx.$$

(a) Given that $y = \dfrac{1}{4 + \sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.
(4)

(c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.
(8)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 20 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38160A

1.
$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C.

(4)

2.
$$\mathrm{f}(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}.$$

Find the first three non-zero terms of the binomial expansion of $\mathrm{f}(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

3.

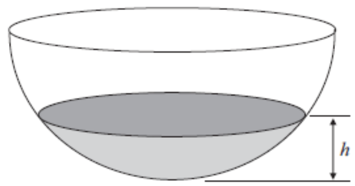

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \le h \le 0.25.$$

(a) Find, in terms of π, $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3 s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

4.

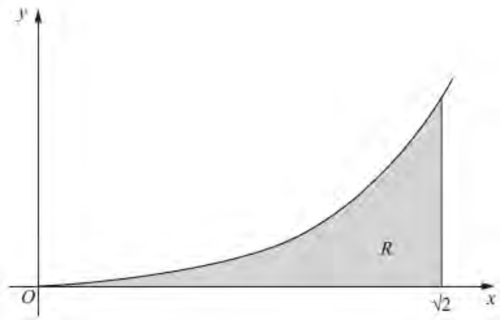

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.
(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \; \mathrm{d}u \,.$$

(4)

(d) Hence, or otherwise, find the exact area of R.
(6)

5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \qquad x > 0, \quad y > 0,$$

at the point on the curve where $x = 2$. Give your answer as an exact value.
(7)

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where μ and λ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.
(6)

(b) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.
(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.
(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.
(4)

7.

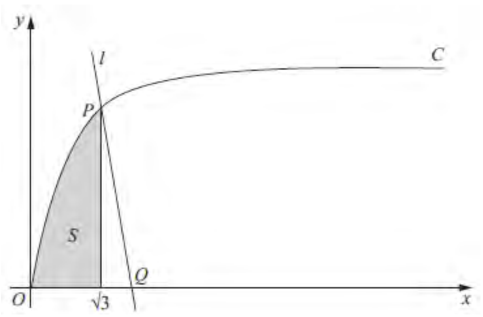

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(*a*) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(*b*) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*c*) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

8. (*a*) Find $\int (4y+3)^{-\frac{1}{2}}\, \mathrm{d}y$.

(2)

(*b*) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{(4y+3)}}{x^2},$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Wednesday 25 January 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Instructions to Candidates

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Information for Candidates

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Advice to Candidates

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P40085A

1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(*a*) Find the gradient of the curve at P.

(5)

(*b*) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

---

2. (*a*) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(*b*) Using your answer to part (*a*), find $\int x^2 \cos 3x \, dx$.

(3)

---

3. (*a*) Expand

$$\frac{1}{(2-5x)^2}, \qquad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots,$$

(*b*) find the value of the constant k,

(2)

(*c*) find the value of the constant A.

(2)

4.

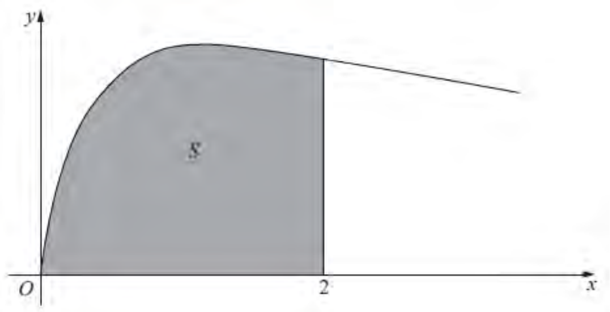

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2+4}\right)}, \quad x \geq 0.$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

5.

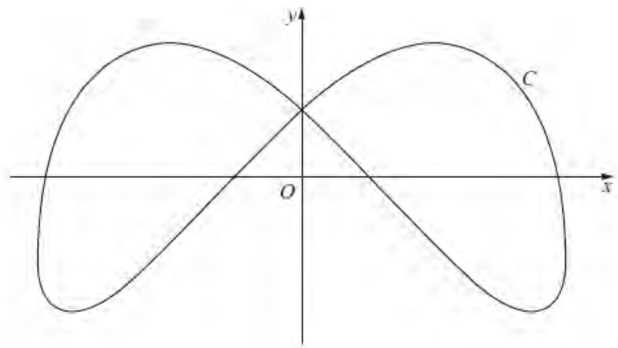

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leq t < 2\pi.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(*b*) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$.

(5)

6.

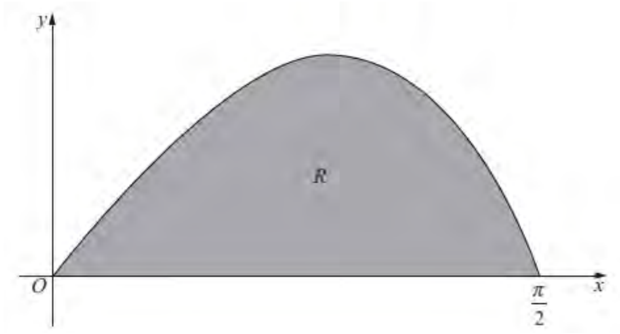

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places.
(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant.
(5)

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures.
(3)

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.
(2)

(*b*) Find a vector equation for the line l.
(2)

(*c*) Show that the size of the angle BAD is 109°, to the nearest degree.
(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(*d*) Find the position vector of C.
(2)

(*e*) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.
(3)

(*f*) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.
(2)

8. (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(*c*) Hence show that the population cannot exceed 5000.

(1)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Thursday 21 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41484A

1. $$\mathrm{f}(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(*a*) Find the values of the constants *A*, *B* and *C*.

(4)

(*b*) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(ii) Find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

2.

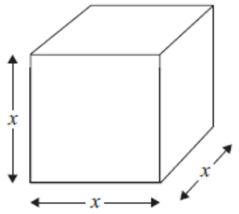

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

(*a*) Show that $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$.

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1},

(*b*) find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 8$,

(2)

(*c*) find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

(3)

---

3. $$\mathrm{f}(x) = \frac{6}{\sqrt{(9-4x)}}, \qquad |x| < \frac{9}{4}.$$

(*a*) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (*a*) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(*b*) $\mathrm{g}(x) = \frac{6}{\sqrt{(9+4x)}}, \qquad |x| < \frac{9}{4},$

(1)

(*c*) $\mathrm{h}(x) = \frac{6}{\sqrt{(9-8x)}}, \qquad |x| < \frac{9}{8}.$

(2)

---

4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{y\cos^2 x}.$$

(5)

---

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

(*b*) Find the coordinates of the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(7)

6.

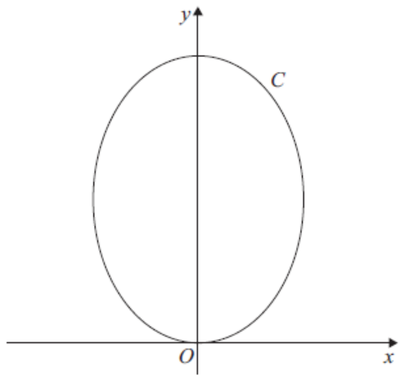

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t, \qquad y = 4\cos^2 t, \qquad 0 \le t \le \pi.$$

(*a*) Show that $\dfrac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be determined.

(5)

(*b*) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(*c*) Find a cartesian equation of C.

(3)

7.

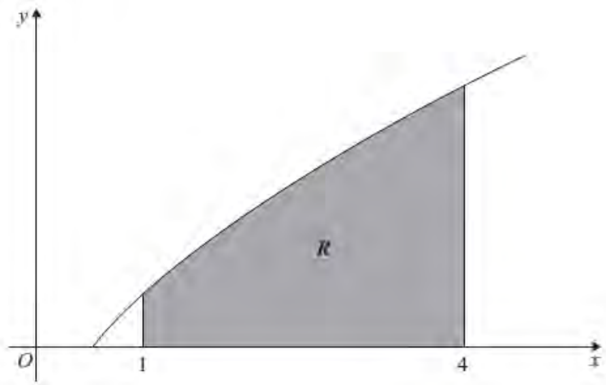

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(*a*) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*b*) Find $\displaystyle\int x^{\frac{1}{2}} \ln 2x \; dx$.

(4)

(*c*) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

---

8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41860A

1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3},$$

find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

---

2. (*a*) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx .$$

(5)

(*b*) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx .$$

(2)

---

3. Express $\dfrac{9x^2 + 20x - 10}{(x+2)(3x-1)}$ in partial fractions.

(4)

4.

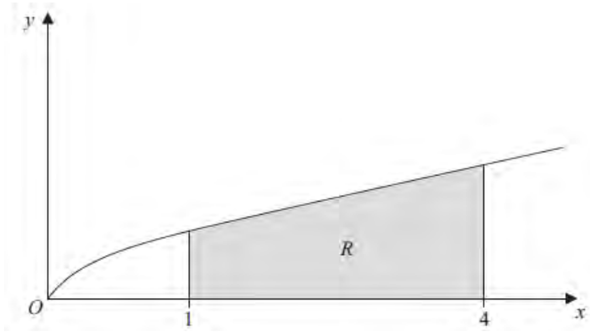

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(*a*) Copy and complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(*c*) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

5.

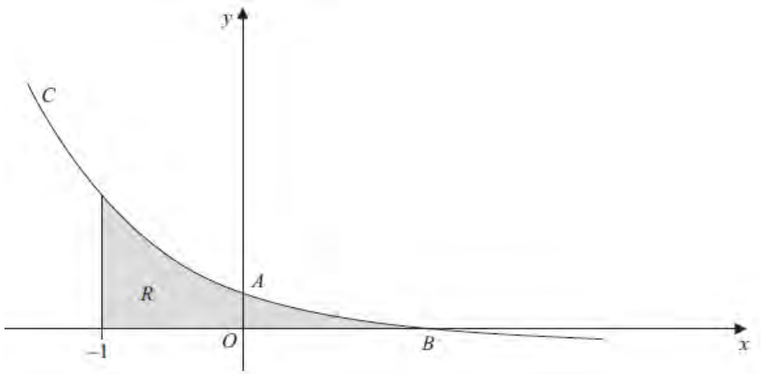

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1.$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).
(2)

(b) Find the x-coordinate of the point B.
(2)

(c) Find an equation of the normal to C at the point A.
(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.
(6)

6.

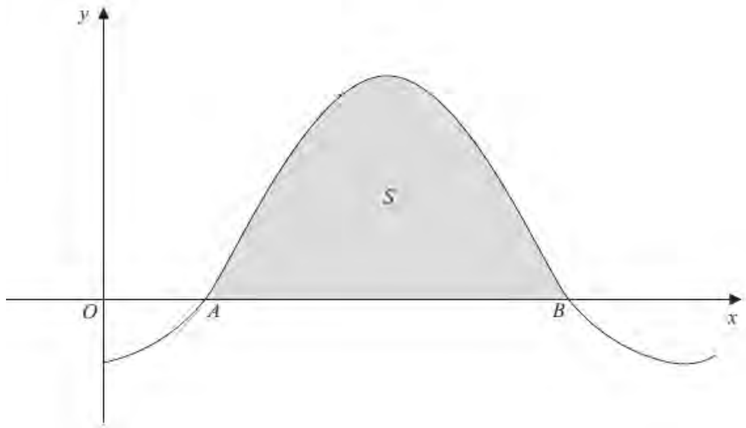

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.
(3)

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.
(6)

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Given that l_1 and l_2 meet, find the position vector of their point of intersection. **(5)**

(*b*) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place. **(3)**

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(*c*) find the exact coordinates of P. **(5)**

---

8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{(3-\theta)}{125}.$$

(*a*) By solving the differential equation, show that

$$\theta = A\mathrm{e}^{-0.008t} + 3,$$

where A is a constant. **(4)**

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(*b*) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute. **(5)**

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01R

Edexcel GCE

Core Mathematics C4 (R)

Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42954A

1. Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

(4)

---

2. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

---

3. Using the substitution $u = 2 + \sqrt{(2x+1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

---

4. (*a*) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(*b*) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(3)

5.

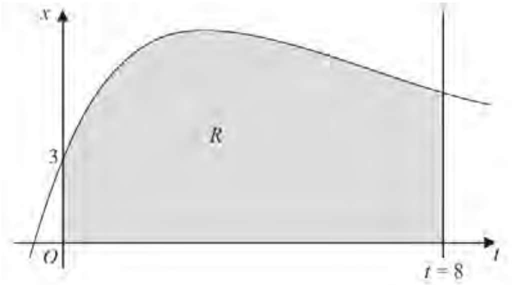

Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(*a*) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218		5.223

(1)

(*b*) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places.

(3)

(*c*) Use calculus to find the exact value for the area of R.

(6)

(*d*) Find the difference between the values obtained in part (*b*) and part (*c*), giving your answer to 2 decimal places.

(1)

---

6. Relative to a fixed origin O, the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a, b and c are constants and λ is a parameter.

Given that the point A lies on the line l,

(a) find the value of a.
(3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the values of b and c,
(5)

(c) find the distance AB.
(2)

The image of the point B after reflection in the line l is the point B'.

(d) Find the position vector of the point B'.
(2)

7.

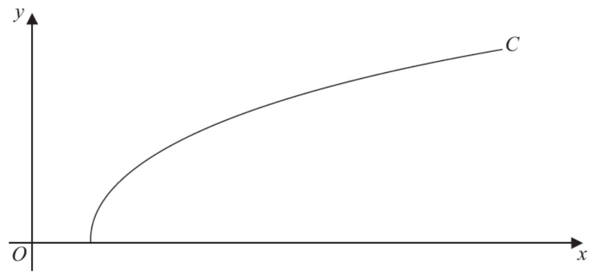

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$.
(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad a \leq x \leq b$$

stating values of a and b.
(3)

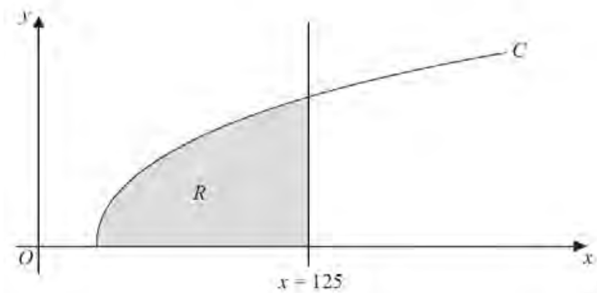

Figure 3

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.
(5)

8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x)\text{, where } M \text{ is a constant.}$$

(*a*) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent.
(2)

Given that initially the mass of waste products is zero,

(*b*) solve the differential equation, expressing x in terms of k, M and t.
(6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(*c*) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.
(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Subsidiary

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43137A

1. (*a*) Find $\int x^2 e^x \, dx$.

(5)

(*b*) Hence find the exact value of $\int_0^1 x^2 e^x \, dx$.

(2)

---

2. (*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \qquad |x| < 1$$

(6)

(*b*) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

3.

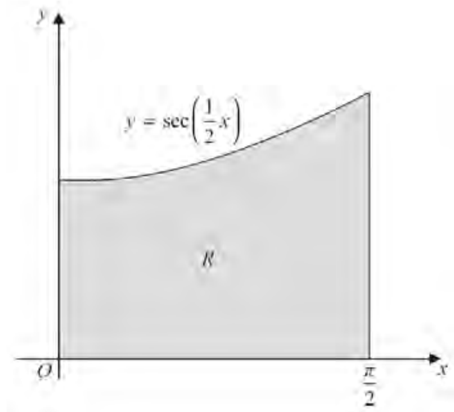

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \qquad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(*a*) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(*b*) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(*c*) Use calculus to find the exact volume of the solid formed.

(4)

---

4. A curve C has parametric equations

$$x = 2\sin t, \qquad y = 1 - \cos 2t, \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(*a*) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$.

(4)

(*b*) Find a cartesian equation for C in the form

$$y = f(x), \qquad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(3)

(*c*) Write down the range of $f(x)$.

(2)

---

5. (*a*) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,dx = \int \frac{2}{u(2u-1)}\,du$$

(3)

(*b*) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

---

6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ°C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(*a*) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100°C, the kettle switches off.

(*b*) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

---

7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x-coordinate of Q is negative,

(*b*) use your answer to part (*a*) to find the coordinates of Q.

(7)

---

8. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(b) find the coordinates of the two possible positions of B.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666A/01

Pearson Edexcel International Advanced Level

Core Mathematics C4

Advanced

Monday 27 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P43018A

1. (*a*) Find the binomial expansion of

$$\frac{1}{(4+3x)^3}, \qquad |x| < \frac{4}{3}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

In the binomial expansion of

$$\frac{1}{(4-9x)^3}, \qquad |x| < \frac{4}{9}$$

the coefficient of x^2 is A.

(*b*) Using your answer to part (*a*), or otherwise, find the value of A.
Give your answer as a simplified fraction.

(2)

---

2. (i) Find

$$\int x \cos\left(\frac{x}{2}\right) \mathrm{d}x$$

(3)

(ii) (*a*) Express $\dfrac{1}{x^2(1-3x)}$ in partial fractions.

(4)

(*b*) Hence find, for $0 < x < \dfrac{1}{3}$

$$\int \frac{1}{x^2(1-3x)} \mathrm{d}x$$

(3)

---

3. The number of bacteria, N, present in a liquid culture at time t hours after the start of a scientific study is modelled by the equation

$$N = 5000(1.04)^t, \qquad t \geq 0$$

where N is a continuous function of t.

(*a*) Find the number of bacteria present at the start of the scientific study.

(1)

(*b*) Find the percentage increase in the number of bacteria present from $t = 0$ to $t = 2$.

(2)

Given that $N = 15\,000$ when $t = T$,

(*c*) find the value of $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ when $t = T$, giving your answer to 3 significant figures.

(4)

4.

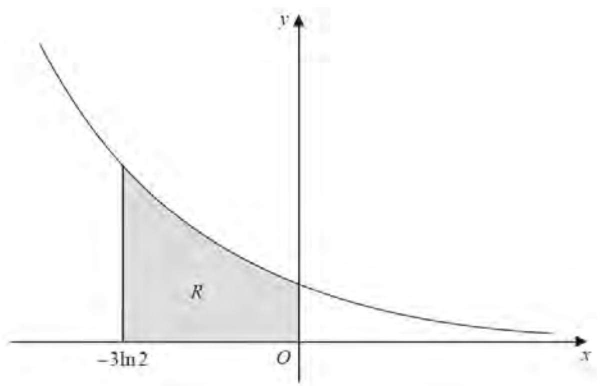

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line $x = -3\ln 2$ and the y-axis.

The table below shows corresponding values of x and y for $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$.

x	$-3\ln2$	$-2\ln2$	$-\ln2$	0
y	2.1333		1.0079	0.6667

(*a*) Complete the table above by giving the missing value of y to 4 decimal places.
(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

(*c*) (i) Using the substitution $u = 1 + 3e^{-x}$, or otherwise, find

$$\int \frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,\mathrm{d}x$$

(5)

(ii) Hence find the value of the area of R.
(2)

5. Given that $y = 2$ at $x = \dfrac{\pi}{8}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3y^2}{2\sin^2 2x}$$

giving your answer in the form $y = \mathrm{f}(x)$.
(6)

6. Oil is leaking from a storage container onto a flat section of concrete at a rate of 0.48 $\mathrm{cm}^3\ \mathrm{s}^{-1}$. The leaking oil spreads to form a pool with an increasing circular cross-section. The pool has a constant uniform thickness of 3 mm.

Find the rate at which the radius r of the pool of oil is increasing at the instant when $r = 5$ cm. Give your answer, in $\mathrm{cm\ s}^{-1}$, to 3 significant figures.
(5)

7. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \le t \le \pi$$

where t is a parameter.

(*a*) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.
(2)

The point P lies on C where $t = \dfrac{2\pi}{3}$.

The line l is a normal to C at P.

(*b*) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(*c*) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.
(6)

8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$$

where λ and μ are scalar parameters.

(*a*) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.

(3)

The point A has position vector $\begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$.

(*b*) Show that A lies on l_1.

(1)

The lines l_1 and l_2 intersect at the point X.

(*c*) Write down the coordinates of X.

(1)

(*d*) Find the exact value of the distance AX.

(2)

The distinct points B_1 and B_2 both lie on the line l_2.

Given that $AX = XB_1 = XB_2$,

(*e*) find the area of the triangle AB_1B_2 giving your answer to 3 significant figures.

(3)

Given that the x coordinate of B_1 is positive,

(*f*) find the exact coordinates of B_1 and the exact coordinates of B_2.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01R
Edexcel GCE

Core Mathematics C4 (R)

Advanced Subsidiary

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43166A

1. (*a*) Find the binomial expansion of

$$\frac{1}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(*b*) Hence, or otherwise, find the expansion of

$$\frac{3+x}{\sqrt{(9-10x)}}, \qquad |x| < \frac{9}{10}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(3)

2.

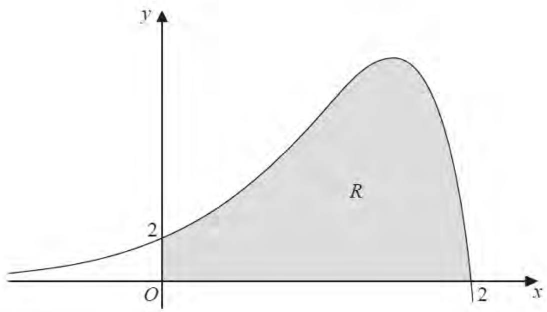

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$.

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(*a*) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.

(3)

(*b*) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.

(1)

(*c*) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.

(5)

3. $$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer.

(5)

(*b*) Find the values of y for which $\frac{dy}{dx} = 0$.

(5)

---

4. (*a*) Express $\frac{25}{x^2(2x+1)}$ in partial fractions.

(4)

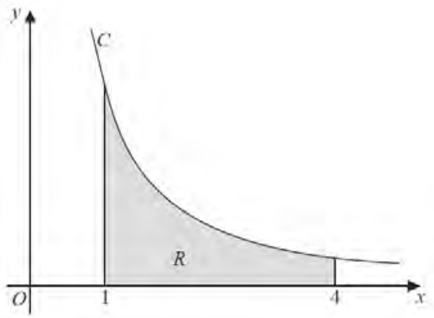

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = \frac{5}{x\sqrt{(2x+1)}}$, $x > 0$.

The finite region R is bounded by the curve C, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

This region is shown shaded in Figure 2.

The region R is rotated through 360° about the x-axis.

(*b*) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form $a + b\ln c$, where a, b and c are constants.

(6)

---

5. At time t seconds the radius of a sphere is r cm, its volume is V cm^3 and its surface area is S cm^2.

[*You are given that* $V = \frac{4}{3}\pi r^3$ *and that* $S = 4\pi r^2$]

The volume of the sphere is increasing uniformly at a constant rate of 3 cm^3 s^{-1}.

(*a*) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

(4)

(*b*) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

(2)

---

6. With respect to a fixed origin, the point A with position vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ lies on the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix}, \qquad \text{where } \lambda \text{ is a scalar parameter,}$$

and the point B with position vector $4\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p is a constant, lies on the line l_2 with equation

$$\mathbf{r} = \begin{pmatrix}7\\0\\7\end{pmatrix} + \mu\begin{pmatrix}3\\-5\\4\end{pmatrix}, \qquad \text{where } \mu \text{ is a scalar parameter.}$$

(*a*) Find the value of the constant p.

(1)

(*b*) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C.

(4)

(*c*) Find the size of the angle ACB, giving your answer in degrees to 3 significant figures.

(3)

(*d*) Find the area of the triangle ABC, giving your answer to 3 significant figures.

(2)

---

7. The rate of increase of the number, N, of fish in a lake is modelled by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{(kt-1)(5000-N)}{t}, \qquad t > 0,\ 0 < N < 5000$$

In the given equation, the time t is measured in years from the start of January 2000 and k is a positive constant.

(*a*) By solving the differential equation, show that

$$N = 5000 - Ate^{-kt}$$

where A is a positive constant.

(5)

After one year, at the start of January 2001, there are 1200 fish in the lake.

After two years, at the start of January 2002, there are 1800 fish in the lake.

(*b*) Find the exact value of the constant A and the exact value of the constant k.

(4)

(*c*) Hence find the number of fish in the lake after five years. Give your answer to the nearest hundred fish.

(1)

8.

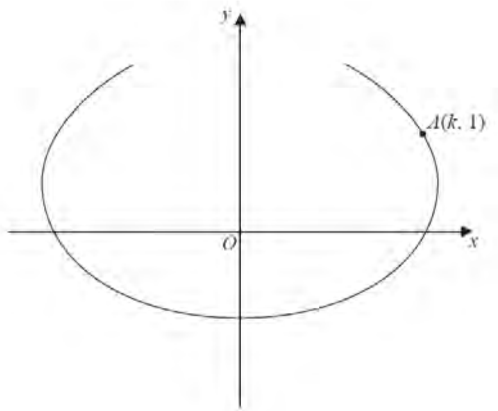

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4\sin t,\ \ y = 1 - 2\cos t, \qquad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(*a*) find the exact value of k,

(2)

(*b*) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$.

(*c*) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43165A

1. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

(*b*) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

---

2. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(*a*) find the value of the constant k,

(2)

(*b*) find the value of the constant A, giving your answer in its simplest form.

(3)

3.

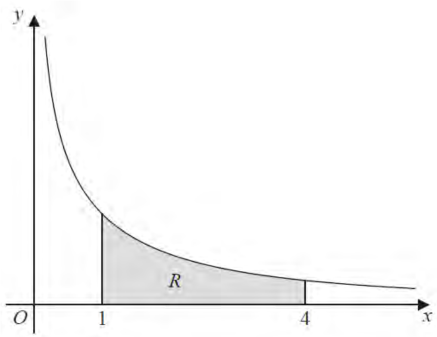

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.
(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(*c*) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (*b*) is an overestimate or an underestimate for the area of R.
(1)

(*d*) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

(6)

4.

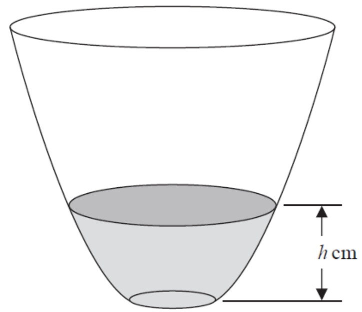

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \qquad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm^3 s^{-1}.

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$.
(5)

5.

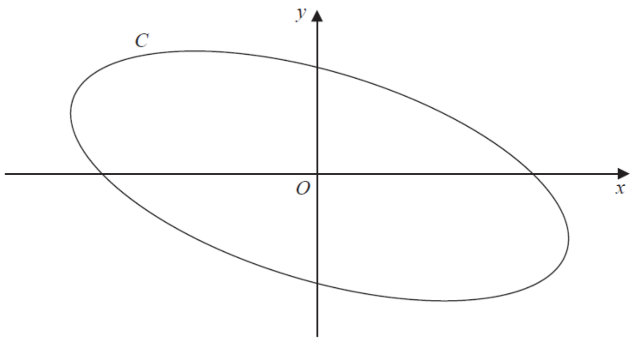

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \qquad y = 2\sin t, \qquad 0 \leq t \leq 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

---

6. (i) Find

$$\int xe^{4x}\,dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \qquad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

7.

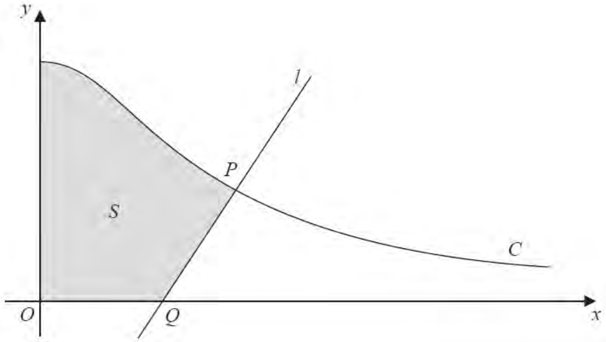

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

(*a*) Find the x coordinate of the point Q.

(6)

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis, the y-axis and the line l. This shaded region is rotated 2π radians about the x-axis to form a solid of revolution.

(*b*) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[*You may use the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(9)

8. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$.

The line l_1 passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.

(2)

(*b*) Hence find a vector equation for the line l_1.

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

Given that angle PBA is θ,

(*c*) show that $\cos\theta = \frac{1}{3}$.

(3)

The line l_2 passes through the point P and is parallel to the line l_1.

(*d*) Find a vector equation for the line l_2.

(2)

The points C and D both lie on the line l_2.

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(*e*) find the coordinates of C and the coordinates of D.

(3)

(*f*) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)

TOTAL FOR PAPER: 75 MARKS

END