

Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6666/01)

June 2008
6666 Core Mathematics C4
Mark Scheme

| Question | Scheme | Marks | | | | | | | | | | | | | | | | | | | | | |
|-------------------------------|--|---|------------|------------|------------|------------|-----|---|-----|-------|------------|------------|------------|------------|-------|--------|---|----------------|------------|------------|------------|------------|--|
| 1. (a) | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>0.4</td> <td>0.8</td> <td>1.2</td> <td>1.6</td> <td>2</td> </tr> <tr> <td>y</td> <td>e^0</td> <td>$e^{0.08}$</td> <td>$e^{0.32}$</td> <td>$e^{0.72}$</td> <td>$e^{1.28}$</td> <td>e^2</td> </tr> <tr> <td>or y</td> <td>1</td> <td>1.08329 ...</td> <td>1.37713...</td> <td>2.05443...</td> <td>3.59664...</td> <td>7.38906...</td> </tr> </table> | x | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 | y | e^0 | $e^{0.08}$ | $e^{0.32}$ | $e^{0.72}$ | $e^{1.28}$ | e^2 | or y | 1 | 1.08329 ... | 1.37713... | 2.05443... | 3.59664... | 7.38906... | |
| x | 0 | 0.4 | 0.8 | 1.2 | 1.6 | 2 | | | | | | | | | | | | | | | | | |
| y | e^0 | $e^{0.08}$ | $e^{0.32}$ | $e^{0.72}$ | $e^{1.28}$ | e^2 | | | | | | | | | | | | | | | | | |
| or y | 1 | 1.08329 ... | 1.37713... | 2.05443... | 3.59664... | 7.38906... | | | | | | | | | | | | | | | | | |
| | | <p>Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)</p> <p style="text-align: right;">B1 [1]</p> | | | | | | | | | | | | | | | | | | | | | |
| (b) Way 1 | $\text{Area} \approx \frac{1}{2} \times 0.4 \times \left[e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2 \right]$ $= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922} \text{ (4sf)}$ | <p>Outside brackets $\frac{1}{2} \times 0.4$ or 0.2</p> <p>For structure of trapezium rule [.....] ;</p> <p style="text-align: right;">B1; M1√ A1 cao [3]</p> | | | | | | | | | | | | | | | | | | | | | |
| <i>Aliter</i> (b) Way 2 | $\text{Area} \approx 0.4 \times \left[\frac{e^0 + e^{0.08}}{2} + \frac{e^{0.08} + e^{0.32}}{2} + \frac{e^{0.32} + e^{0.72}}{2} + \frac{e^{0.72} + e^{1.28}}{2} + \frac{e^{1.28} + e^2}{2} \right]$ <p>which is equivalent to:</p> $\text{Area} \approx \frac{1}{2} \times 0.4 \times \left[e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2 \right]$ $= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922} \text{ (4sf)}$ | <p>0.4 and a divisor of 2 on all terms inside brackets.</p> <p>One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.</p> <p style="text-align: right;">B1 M1√ A1 cao [3]</p> | | | | | | | | | | | | | | | | | | | | | |
| | | 4 marks | | | | | | | | | | | | | | | | | | | | | |

Note an expression like $\text{Area} \approx \frac{1}{2} \times 0.4 + e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the () brackets for

The M1 mark for structure is for the material found in the curly brackets ie
[first y ordinate + 2(intermediate ft y ordinate) + final y ordinate]

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|--------------------------|---|--|
| <p>2. (a)</p> <p>(b)</p> | $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ $\int x e^x dx = x e^x - \int e^x \cdot 1 dx$ $= x e^x - \int e^x dx$ $= x e^x - e^x (+ c)$ $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ $\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x dx$ $= x^2 e^x - 2 \int x e^x dx$ $= x^2 e^x - 2(x e^x - e^x) + c$ $\left\{ \begin{array}{l} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x (x^2 - 2x + 2) + c \end{array} \right\}$ | <p>Use of 'integration by parts' formula in the correct direction. (See note.)</p> <p>Correct expression. (Ignore dx)</p> <p>Correct integration with/without + c</p> <p>Use of 'integration by parts' formula in the correct direction.</p> <p>Correct expression. (Ignore dx)</p> <p>Correct expression including + c. (seen at any stage! in part (b))</p> <p>You can ignore subsequent working.</p> <p><i>Ignore subsequent working</i></p> <p>6 marks</p> |
| | <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 ISW</p> <p>[3]</p> <p>[3]</p> | |

Note integration by parts in the correct direction means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

+ c is not required in part (a).

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|-----------------|---|--|
| 3. (a) | <p>From question, $\frac{dA}{dt} = 0.032$</p> <p>$\left\{ A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$</p> <p>$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032) \frac{1}{2\pi x}; \left\{ = \frac{0.016}{\pi x} \right\}$</p> <p>When $x = 2\text{ cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$</p> <p>Hence, $\frac{dx}{dt} = 0.002546479\dots \text{ (cm s}^{-1}\text{)}$</p> | <p>$\frac{dA}{dt} = 0.032$ seen or implied from working. B1</p> <p>$2\pi x$ by itself seen or implied from working B1</p> <p>$0.032 \div \text{Candidate's } \frac{dA}{dx};$ M1;</p> <p>awrt 0.00255 A1 cso [4]</p> |
| (b) | <p>$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$</p> <p>$\frac{dV}{dx} = 15\pi x^2$</p> <p>$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 \cdot \left(\frac{0.016}{\pi x} \right); \{ = 0.24x \}$</p> <p>When $x = 2\text{ cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48} \text{ (cm}^3 \text{ s}^{-1}\text{)}$</p> | <p>$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$ B1</p> <p>$\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V in one variable B1 $\sqrt{\quad}$</p> <p>Candidate's $\frac{dV}{dx} \times \frac{dx}{dt};$ M1 $\sqrt{\quad}$</p> <p>$\underline{0.48}$ or awrt 0.48 A1 cso [4]</p> |
| | | 8 marks |

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| 4. (a) | <p style="text-align: center;">$3x^2 - y^2 + xy = 4$ (eqn *)</p> <p style="text-align: center;">$\frac{dy}{dx}$ \times $\left\{ \frac{6x-2y}{dx} \frac{dy}{dx} + \left(y + x \frac{dy}{dx} \right) = 0 \right.$</p> <p style="text-align: center;">$\left\{ \frac{dy}{dx} = \frac{-6x-y}{x-2y} \right\}$ or $\left\{ \frac{dy}{dx} = \frac{6x+y}{2y-x} \right\}$</p> <p>$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x-y}{x-2y} = \frac{8}{3}$</p> <p>giving $-18x - 3y = 8x - 16y$</p> <p>giving $13y = 26x$</p> <p>Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$</p> | <p>Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $x \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$) M1</p> <p>Correct application $\left(\underline{\quad} \right)$ of product rule B1</p> <p>$(3x^2 - y^2) \rightarrow \left(\underline{6x-2y} \frac{dy}{dx} \right)$ and $(4 \rightarrow \underline{0})$ A1</p> <p><i>not necessarily required.</i></p> <p>Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation. M1 *</p> <p>Attempt to combine either terms in x or terms in y together to give either ax or by. dM1 *</p> <p>simplifying to give $\underline{y - 2x = 0}$ AG A1 cso</p> |
| (b) | <p>At P & Q, $y = 2x$. Substituting into eqn *</p> <p>gives $3x^2 - (2x)^2 + x(2x) = 4$</p> <p>Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$</p> <p>$y = 2x \Rightarrow y = \pm 4$</p> <p>Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$</p> | <p>Attempt replacing y by $2x$ in at least one of the y terms in eqn* M1</p> <p>Either $x = 2$ or $x = -2$ A1</p> <p style="border: 1px solid black; display: inline-block; padding: 2px;">Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$ A1</p> <p style="text-align: right;">[3]</p> |
| | | 9 marks |

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| 5. (a) | <p>** represents a constant (which must be consistent for first accuracy mark)</p> $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)}^{-\frac{1}{2}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{1} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}}$ <p style="text-align: right;">(4)^{-1/2} or 1/2 outside brackets</p> <p>Expands (1 + ** x)^{-1/2} to give a simplified or an un-simplified 1 + (-1/2)(** x);</p> <p>A correct simplified or an un-simplified [.....] expansion with candidate's followed through (** x)</p> $= \frac{1}{2} \left[1 + (-\frac{1}{2})(** x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (** x)^2 + \dots \right]$ <p>with ** ≠ 1</p> $= \frac{1}{2} \left[1 + (-\frac{1}{2})(-\frac{3x}{4}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (-\frac{3x}{4})^2 + \dots \right]$ $= \frac{1}{2} \left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots \right]$ $\left\{ = \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right\}$ <p>(b)</p> $(x+8) \left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots \right)$ $= \frac{1}{2}x + \frac{3}{16}x^2 + \dots$ $+ 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \dots$ $= 4 + 2x + \frac{33}{32}x^2 + \dots$ | <p><u>B1</u></p> <p>M1;</p> <p>A1 √</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Award SC M1 if you see $(-\frac{1}{2})(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2$</p> </div> <p>$\frac{1}{2} [1 + \frac{3}{8}x; \dots]$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>SC: $K [1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots]$</p> </div> <p>$\frac{1}{2} [\dots; \frac{27}{128}x^2]$</p> <p><i>Ignore subsequent working</i></p> <p>[5]</p> <p>Writing (x+8) multiplied by candidate's part (a) expansion. M1</p> <p>Multiply out brackets to find a constant term, two x terms and two x² terms.</p> <p>Anything that cancels to $4 + 2x; \frac{33}{32}x^2$</p> <p style="text-align: center;"> ↓ M1 ↓ A1; A1 </p> <p>[4]</p> <p>9 marks</p> |

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| 6. (a) | <p>Lines meet where:</p> $\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p> i: $-9 + 2\lambda = 3 + 3\mu$ (1) Any two of j: $\lambda = 1 - \mu$ (2) k: $10 - \lambda = 17 + 5\mu$ (3) </p> <p>(1) - 2(2) gives: $-9 = 1 + 5\mu \Rightarrow \mu = -2$</p> <p>(2) gives: $\lambda = 1 - (-2) = 3$</p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ <p>Intersect at $\mathbf{r} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$ or $\mathbf{r} = \underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$</p> <p>Either check k: $\lambda = 3$: LHS = $10 - \lambda = 10 - 3 = 7$ $\mu = -2$: RHS = $17 + 5\mu = 17 - 10 = 7$</p> <p>(As LHS = RHS then the lines intersect.)</p> | <p>Need any two of these correct equations seen anywhere in part (a). M1</p> <p>Attempts to solve simultaneous equations to find one of either λ or μ dM1</p> <p>Both $\underline{\lambda = 3}$ & $\underline{\mu = -2}$ A1</p> <p>Substitutes their value of either λ or μ into the line l_1 or l_2 respectively. This mark can be implied by any two correct components of $(-3, 3, 7)$. ddM1</p> <p>$\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$ or $\underline{-3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}}$ A1</p> <p>or $(-3, 3, 7)$</p> <p>Either check that $\lambda = 3, \mu = -2$ in a third equation or check that $\lambda = 3, \mu = -2$ give the same coordinates on the other line. B1</p> <p>Conclusion not needed.</p> <p>[6]</p> |
| (b) | <p>$\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$</p> $\text{As } \mathbf{d}_1 \cdot \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)} = 0$ <p>Then l_1 is perpendicular to l_2.</p> | <p>Dot product calculation between the two direction vectors: $\underline{(2 \times 3) + (1 \times -1) + (-1 \times 5)}$ or $\underline{6 - 1 - 5}$ M1</p> <p>Result '$=0$' and appropriate conclusion A1</p> <p>[2]</p> |

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| 6. (c) | <p>Equating \mathbf{i} ; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$</p> $\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ <p>(= \overline{OA}. Hence the point A lies on l_1.)</p> | <p>Substitutes candidate's $\lambda = 7$ into the line l_1 and finds $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$. The conclusion on this occasion is not needed.</p> <p>B1</p> <p>[1]</p> |
| (d) | <p>Let $\overline{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection</p> $\overline{AX} = \overline{OX} - \overline{OA} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ $\overline{OB} = \overline{OA} + \overline{AB} = \overline{OA} + 2\overline{AX}$ $\overline{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ <p>Hence, $\overline{OB} = \begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}$ or $\overline{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$</p> | <p>Finding the difference between their \overline{OX} (can be implied) and \overline{OA}.</p> $\overline{AX} = \pm \left(\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} \right)$ <p>M1 $\sqrt{\pm}$</p> $\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \left(\text{their } \overline{AX} \right)$ <p>dM1 $\sqrt{\pm}$</p> $\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix} \text{ or } \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ <p>or $\underline{(-11, -1, 11)}$</p> <p>A1</p> <p>[3]</p> |
| | | 12 marks |

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| 7. (a) | $\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ $2 \equiv A(2+y) + B(2-y)$ <p>Let $y = -2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$</p> <p>Let $y = 2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$</p> <p>giving $\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$</p> <p>(If no working seen, but candidate writes down correct partial fraction then award all three marks. If no working is seen but one of A or B is incorrect then M0A0A0.)</p> | <p>Forming this identity. NB: A & B are not assigned in this question</p> <p>M1</p> <p>Either one of $A = \frac{1}{2}$ or $B = \frac{1}{2}$</p> <p>A1</p> <p>$\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$, aef</p> <p>A1 cao</p> <p>[3]</p> |

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| 7. (b) | <p> $\int \frac{2}{4-y^2} dy = \int \frac{1}{\cot x} dx$ $\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)} dy = \int \tan x dx$ $\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) + (c)$ $y=0, x=\frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right) + c$ $\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$ $-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$ $\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$ $\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$ $\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$ $\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$ <p>Hence, $\underline{\underline{\sec^2 x = \frac{8+4y}{2-y}}}$</p> </p> | <p>Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.</p> <p>B1</p> <p>$\ln(\sec x)$ or $-\ln(\cos x)$ B1 Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$ M1; their $\int \frac{1}{\cot x} dx = \text{LHS}$ correct with ft for their A and B and no error with the "2" with or without $+c$ A1 \sqrt</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c; </div> <p>M1*</p> <p>Using either the quotient (or product) or power laws for logarithms CORRECTLY. M1</p> <p>Using the log laws correctly to obtain a single log term on both sides of the equation. dM1*</p> <p>A1 aef</p> <p style="text-align: right;">[8]</p> <p>11 marks</p> |

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| 8. (a) | <p>At $P(4, 2\sqrt{3})$ either $4 = 8\cos t$ or $2\sqrt{3} = 4\sin 2t$</p> <p>\Rightarrow only solution is $t = \frac{\pi}{3}$ where $0 \leq t \leq \frac{\pi}{2}$</p> | <p>M1</p> <p>A1</p> <p>[2]</p> |
| 8. (b) | <p>$x = 8\cos t, \quad y = 4\sin 2t$</p> <p>$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$</p> <p>At $P, \quad \frac{dy}{dx} = \frac{8\cos(\frac{2\pi}{3})}{-8\sin(\frac{\pi}{3})}$</p> <p>$\left\{ = \frac{8(-\frac{1}{2})}{(-8)(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$</p> <p>Hence $m(N) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$</p> <p>N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$</p> <p>N: $y = -\sqrt{3}x + 6\sqrt{3}$ AG</p> <p>or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$</p> <p>so N: $\boxed{y = -\sqrt{3}x + 6\sqrt{3}}$</p> | <p>Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.</p> <p>M1</p> <p>A1</p> <p>M1*</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>You may need to check candidate's substitutions for M1*</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Note the next two method marks are dependent on M1*</p> </div> <p>Uses $m(N) = -\frac{1}{\text{their } m(T)}$.</p> <p>Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their } m_N)x + "c"$.</p> <p>$y = -\sqrt{3}x + 6\sqrt{3}$</p> <p>dM1*</p> <p>dM1*</p> <p>A1 cs0 AG</p> <p>[6]</p> |

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|----------|--|---|
| 8. (c) | $A = \int_0^4 y \, dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4 \sin 2t \cdot (-8 \sin t) \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32 \sin 2t \cdot \sin t \, dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2 \sin t \cos t) \cdot \sin t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64 \cdot \sin^2 t \cos t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 64 \cdot \sin^2 t \cos t \, dt$ | <p>attempt at $A = \int \frac{y}{\frac{dx}{dt}} \, dt$ correct expression (ignore limits and dt)</p> <p>Seeing $\sin 2t = 2 \sin t \cos t$ anywhere in PART (c).</p> <p>Correct proof. Appreciation of how the negative sign affects the limits. Note that the answer is given in the question.</p> <p>M1 A1 M1 A1 AG [4]</p> |
| (d) | <p>{Using substitution $u = \sin t \Rightarrow \frac{du}{dt} = \cos t$ } {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$ }</p> $A = 64 \left[\frac{\sin^3 t}{3} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad \text{or} \quad A = 64 \left[\frac{u^3}{3} \right]_{\frac{\sqrt{3}}{2}}^1$ $A = 64 \left[\frac{1}{3} - \left(\frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right) \right]$ $A = 64 \left(\frac{1}{3} - \frac{1}{8} \sqrt{3} \right) = \frac{64}{3} - 8\sqrt{3}$ <p>(Note that $a = \frac{64}{3}$, $b = -8$)</p> | <p>$k \sin^3 t$ or ku^3 with $u = \sin t$ Correct integration ignoring limits.</p> <p>Substitutes limits of either ($t = \frac{\pi}{2}$ and $t = \frac{\pi}{3}$) or ($u = 1$ and $u = \frac{\sqrt{3}}{2}$) and subtracts the correct way round.</p> <p>$\frac{64}{3} - 8\sqrt{3}$</p> <p>Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.</p> <p>M1 A1 dM1 A1 aef isw [4]</p> |
| | | 16 marks |