edexcel

Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Core Mathematics 4 (6666A/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x = \ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1. (a)	$\left\{\dfrac{1}{(4+3x)^3} =\right\} (4+3x)^{-3}$	Moving power to the top	M1
	$= \underline{(4)^{-3}}\left(1+\dfrac{3x}{4}\right)^{-3} = \underline{\dfrac{1}{64}}\left(1+\dfrac{3x}{4}\right)^{-3}$	$\underline{4^{-3}}$ or $\underline{\dfrac{1}{64}}$	B1
	$= \left\{\dfrac{1}{64}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+\dots\right]$	see notes	M1 A1
	$= \left\{\dfrac{1}{64}\right\}\left[1+(-3)\left(\dfrac{3x}{4}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{3x}{4}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{3x}{4}\right)^3+\dots\right]$		
	$= \dfrac{1}{64}\left[1-\dfrac{9}{4}x+\dfrac{27}{8}x^2-\dfrac{135}{32}x^3+\dots\right]$		
	$= \dfrac{1}{64}-\dfrac{9}{256}x; +\dfrac{27}{512}x^2-\dfrac{135}{2048}x^3+\dots$		A1; A1
			[6]
(b)	$\left\{\dfrac{1}{(4-9x)^3}\right\}$, so the coefficient of x^2 is $A=(9)\left(\dfrac{27}{512}\right)=\dfrac{243}{512}$	$9\times$(coeff x^2 in (a))	M1
		$\dfrac{243}{512}$	A1
			[2] **8**

Notes

(a)

M1: Writes down $(4+3x)^{-3}$ or uses power of -3.

This mark can be implied by a constant term of $(4)^{-3}$ or $\dfrac{1}{64}$.

<u>B1</u>: $\underline{4^{-3}}$ or $\underline{\dfrac{1}{64}}$ outside brackets or $\dfrac{1}{64}$ as candidate"s constant term in their binomial expansion.

M1: Expands $(\dots+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+(-3)(kx)$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\dots+\dfrac{(-3)(-4)}{2!}(kx)^2$

or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k\neq 1$ are fine for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate"s expansion. **Note** that $k\neq 1$.

You would award B1M1A0 for $\dfrac{1}{64}\left[1+(-3)\left(\dfrac{3x}{4}\right)+\dfrac{(-3)(-4)}{2!}(3x)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{3x}{4}\right)^3+\dots\right]$

because (kx) is not consistent.

Notes for Question 1 continued

1. (a) ctd

"Incorrect bracketing" $=\left\{\frac{1}{64}\right\}\left[1+(-3)\left(\frac{3x}{4}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{4}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{4}\right)+\ldots\right]$

is M1A0 unless recovered.

A1: For $\frac{1}{64}-\frac{9}{256}x$ **(simplified please)** or also allow $0.015625 - 0.03515625x$.

Allow Special Case A1A0 for either SC: $\frac{1}{64}\left[1-\frac{9}{4}x\,;\ldots\right]$ **or SC**: $\lambda\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]$

(where λ can be 1 or omitted), with each term in the [.........] either a simplified fraction or a decimal.

A1: Accept only $\frac{27}{512}x^2-\frac{135}{2048}x^3$ or $0.052734375x^2 - 0.06591796875x^3$

Candidates who write $=\frac{1}{64}\left[1+(-3)\left(-\frac{3x}{4}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{3x}{4}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{4}\right)^3+\ldots\right]$

where $k=-\frac{3}{4}$ and not $\frac{3}{4}$ and achieve $=\frac{1}{64}+\frac{9}{256}x;+\frac{27}{512}x^2+\frac{135}{2048}x^3+\ldots$ will get B1M1A1A0A0.

Note for final two marks:

$\frac{1}{64}\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]=\frac{1}{64}+\frac{9}{256}x+\frac{27}{512}x^2-\frac{135}{2048}x^3+\ldots$ scores final A0A1.

$\frac{1}{64}\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]=\frac{1}{64}-\frac{9}{256}+\frac{27}{512}x^2-\frac{135}{2048}x^3+\ldots$ scores final A0A1

Special case for the M1 mark

Award Special Case M1 for a correct simplified or un-simplified

$1+n(kx)+\frac{n(n-1)}{2!}(kx)^2+\frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n\neq-3$, $n\neq$ ***positive integer***

and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate"s expansion. **Note** that $k\neq1$.

(b)

M1: $9\times\left(\text{their } \frac{27}{512}\right)$ or $9\left(\text{their } \frac{27}{512}x^2\right)$

A1: For $\frac{243}{512}$. Note that $\frac{243}{512}x^2$ is A0.

Alternative method for part (b)

M1: for $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(-\frac{9}{4}\right)^2$ or $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(\frac{9}{4}\right)^2$ or $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(\frac{9x}{4}\right)^2$ or $\frac{1}{64}\left(\frac{243x^2}{8}\right)$

Also allow M1 for $\frac{1}{64}\left[\ldots\ldots+\frac{(-3)(-4)}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\ldots\right]$ or $\frac{1}{64}\left[\ldots\ldots+\frac{(-3)(-4)}{2!}\left(\frac{9x}{4}\right)^2+\ldots\ldots\right]$

Also allow M1 for $\lambda\left[\ldots\ldots+\frac{(-3)(-4)}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\ldots\right]$ or $\lambda\left[\ldots\ldots+\frac{(-3)(-4)}{2!}\left(\frac{9x}{4}\right)^2+\ldots\ldots\right]$

where λ is the multiplicative constant used by the candidate in part (a). **Note** that λ can be 1.

A1: For $\frac{243}{512}$. Note that $\frac{243}{512}x^2$ is A0.

Notes for Question 1 continued

Alternative Methods for part (a)

Alternative method 1: Candidates can apply an alternative form of the binomial expansion.

$$\left\{\frac{1}{(4+3x)^3} = \right\} (4+3x)^{-3} = (4)^{-3} + (-3)(4)^{-4}(3x) + \frac{(-3)(-4)}{2!}(4)^{-5}(3x)^2 + \frac{(-3)(-4)(-5)}{3!}(4)^{-6}(3x)^3$$

M1: Writes down $(4+3x)^{-3}$ or uses power of -3.

B1: 4^{-3} or $\frac{1}{64}$

M1: Any two of four (un-simplified or simplified) terms correct.

A1: All four (un-simplified or simplified) terms correct.

A1: $\frac{1}{64} - \frac{9}{256}x$

A1: $\frac{27}{512}x^2 - \frac{135}{2048}x^3$

Note: The terms in C need to be evaluated,

so ${}^{-3}C_0(4)^{-3} + {}^{-3}C_1(4)^{-4}(3x) + {}^{-3}C_2(4)^{-5}(3x)^2 + {}^{-3}C_3(4)^{-6}(3x)^3$ without further working is B0M0A0.

Alternative Method 2: Maclaurin Expansion

$(4+3x)^{-3}$	Moving power to the top	M1
$f''(x) = 108(4+3x)^{-5}$, $f'''(x) = -1620(4+3x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
$f'(x) = -3(4+3x)^{-4}(3)$	$\pm a(4+3x)^{-4}$; $a \neq \pm 1$	M1
	$-3(4+3x)^{-4}(3)$	A1 oe
$\left\{\therefore f(0) = \frac{1}{64}, f'(0) = -\frac{9}{256}, f''(0) = \frac{27}{256} \text{ and } f'''(0) = -\frac{405}{1024}\right\}$		
$f(x) = \frac{1}{64} - \frac{9}{256}x; + \frac{27}{512}x^2 - \frac{135}{2048}x^3 + \ldots$		A1; A1

Question Number	Scheme		Marks
2.	$\int x\cos\left(\frac{x}{2}\right)\mathrm{d}x$, $\left\{\begin{array}{l} u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos\left(\frac{x}{2}\right) \Rightarrow v = 2\sin\left(\frac{x}{2}\right)\end{array}\right\}$		
(i)	$= 2x\sin\left(\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$		M1 A1
	$= 2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)\{+c\}$		A1
			[3]
(ii)(a)	$\frac{1}{x^2(1-3x)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(1-3x)}$		
		At least one of "B" or "C" correct.	B1
	$B = 1,\ C = 9$	Breaks up their partial fraction correctly into ***three terms*** **and** both "B" $= 1$ and "C" $= 9$. **See notes below.**	B1 **cso**
	$1 \equiv Ax(1-3x) + B(1-3x) + Cx^2$ $x = 0,\quad 1 = B$ $x = \frac{1}{3},\quad 1 = \frac{1}{9}C \Rightarrow C = 9$ x^2 terms: $0 = -3A + C$ $0 = -3A + 9 \Rightarrow A = 3$ $x^2: 0 = -3A + C,\ x: 0 = A - 3B,$ constant: $1 = B$	Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	leading to $A = 3$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1
			[4]
(b)	$\int \frac{1}{x^2(1-3x)}\mathrm{d}x = \int \frac{3}{x} + \frac{1}{x^2} + \frac{9}{(1-3x)}\mathrm{d}x$		
		Either $\pm\frac{P}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\frac{Q}{x^2} \to \pm bx^{-1}$ or $\frac{R}{(1-3x)} \to \pm c\ln(1-3x)$	M1
	$= 3\ln x + \frac{x^{-1}}{(-1)} + \frac{9}{(-3)}\ln(1-3x)\{+c\}$	***At least*** two terms correctly integrated	A1ft
		All three terms correctly integrated. Ignore absence of „+ c'	A1ft
			[3] **10**

Notes

2. (i)	**M1:** Integration by parts is applied in the form $\pm\lambda x\sin\left(\frac{x}{2}\right) \pm \int \mu\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$ (where $\lambda \neq 0,\ \mu \neq 0$) **A1:** $2x\sin\left(\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$ or equivalent. Can be un-simplified.
	A1: $2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$ or $2x\sin\left(\frac{x}{2}\right) - -\frac{2}{\left(\frac{1}{2}\right)}\cos\left(\frac{x}{2}\right)$ or equivalent with/without $+c$ Can be un-simplified.

	Notes for Question 2 continued
(ii)(a)	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \cos\left(\frac{x}{2}\right)$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. **BE CAREFUL!** Candidates will assign ***their own*** "*A*, *B* and *C*" for this question. **B1:** At least one of "*B*" or "*C*" are correct. **B1:** Breaks up their partial fraction correctly into three terms **and** both $"B" = 1$ and $"C" = 9$. **Note:** If a candidate does not give partial fraction decomposition then: • the 2nd B1 mark can follow from a correct identity. **M1:** Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "*A*" or "*B*" or "*C*". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously. **A1:** Correct value for "*A*" which is found using a correct identity and follows from their partial fraction decomposition. **Note:** If a candidate does not give partial fraction decomposition then: • the final A1 mark can be awarded for a correct "*A*" if a candidate writes out their partial fractions at the end. **Note:** The correct partial fraction from no working scores B1B1M1A1. **Note:** A number of candidates will start this problem by writing out the correct identity and then attempt to find "*A*" or " *B*" or "*C*". Therefore the B1 marks can be awarded from this method. **Note:** $\frac{1}{x^2(1-3x)} \equiv \frac{B}{x^2} + \frac{C}{(1-3x)}$ leading to $"B" = 1$ or $"C" = 9$ will only score a maximum of B1B0M0A0.
(ii)(b)	**M1:** Either $\pm\frac{P}{x} \to \pm a\ln x$ or $\pm\frac{Q}{x^2} \to \pm bx^{-1}$ or $\frac{R}{(1-3x)} \to \pm c\ln(1-3x)$, from their constants *P*, *Q*, *R*. **A1ft:** At least two terms from any of $\pm\frac{P}{x}$ or $\pm\frac{Q}{x^2}$ or $\frac{R}{(1-3x)}$ correctly integrated. Can be un-simplified. **A1ft:** All 3 terms from $\pm\frac{P}{x}$, $\pm\frac{Q}{x^2}$ and $\frac{R}{(1-3x)}$ correctly integrated. Can be un-simplified with/without $+c$. **NOTE:** Ignore subsequent working for applying limits after integration. **NOTE:** Integrating $\frac{9}{(1-3x)}$ to give $-3\ln\lvert 3x-1\rvert$ is correct but $-3\ln(3x-1)$ is incorrect. **NOTE:** The final two marks in (ii)(b) are both follow through accuracy marks. **NOTE:** Some candidates are applying limits of $x = 0$ and $x = \frac{1}{3}$ to their integrated expression. You can award up to all three marks in (ii)(b) for the integrated expression and ignore the application of limits. **NOTE:** A candidate who achieves full marks in (ii)(a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of M1A1A0 in (ii)(b).

Question Number	Scheme		Marks
3.	$N = 5000(1.04)^t, \quad t \in \mathbb{R}, t \geqslant 0.$		
(a)	$\{t = 0 \Rightarrow\}$ $N = 5000$ (bacteria)	5000	B1 **cao**
			[1]
(b)	$\pm\left(\frac{5000(1.04)^2 - 5000}{5000}\right)100$ or $\pm\left(\frac{5408 - 5000}{5000}\right)100$		M1
	$= 8.16$ (%)	8 or 8.2 or 8.16	A1
			[2]
(c)	$\frac{dN}{dt} = 5000(1.04)^t \ln(1.04)$ or $\frac{dN}{dt} = 5000\left(e^{t\ln(1.04)}\right)\ln(1.04)$ or $\frac{dN}{dt} = N\ln(1.04)$ or $\frac{1}{N}\frac{dN}{dt} = \ln(1.04)$		M1 A1
	At $t = T$, $15000 = 5000(1.04)^T \Rightarrow 3 = (1.04)^T \Rightarrow T = \frac{\ln 3}{\ln 1.04} = 28.01...$		
	$\{\text{At } t = T,\}$ $\frac{dN}{dt} = 5000(3)\ln(1.04)$ or $\frac{dN}{dt} = 5000(1.04)^{28.01...}\ln(1.04)$	Substitutes their found $(1.04)^T$ or their found T into $\frac{dN}{dt}$ or N into $\frac{dN}{dt} = N\ln(1.04)$	dM1
	$= 588.3106973... \left(\frac{\text{bacteria}}{\text{hour}}\right)$	590 or awrt 588	A1
			[4]
			7

Notes

(a)

B1: 5000 cao.

(b)

M1: A **full method** for finding a percentage increase.

A1: 8 or 8.1 or 8.16

Note: $(1.04)^2$ or 1.0816 or 0.0816 by itself is M0; but followed by either 8 or 8.2 or 8.16 is M1A1.

Note: Applying $\left(\frac{5000(1.04)^2 - 5000}{5408}\right)100$ or equivalent (answer of 7.54(%)) is M0A0.

(c)

M1: Award M1 for $\frac{dN}{dt} = \pm\lambda(1.04)^t$ or $\frac{dN}{dt} = \pm\lambda N$ or $\frac{dN}{dt} = \pm\lambda e^{t\ln 1.04}$ or $\frac{1}{N}\frac{dN}{dt} = \pm\lambda$ where $\lambda \neq 0$ is a constant.

EXCEPTION: Award M0, however, for $\frac{dN}{dt} = ...(1.04)^{t-1}$ or $\frac{dN}{dt} = ...(1.04)^{t+1}$ or equivalent.

Note: Award M0 for expressions such as $\frac{dN}{dt} = 5000(1.04)^{t-1}$ or $\frac{dN}{dt} = 5000t(1.04)^{t-1}$

Note: You can award M1 for $\frac{dN}{dt} = 5000(1.04)^t$

Be careful: $\frac{dN}{dt} = 5000(1.04)^t \ln(1.04)^t$ is M0.

Notes for Question 3 continued	
3. (c) contd.	**A1:** $\frac{dN}{dt} = 5000(1.04)^t \ln(1.04)$ or $\frac{dN}{dt} = 5000\left(e^{t\ln(1.04)}\right)\ln(1.04)$ or $\frac{dN}{dt} = N\ln(1.04)$ or $\frac{1}{N}\frac{dN}{dt} = \ln(1.04)$ or equivalent. **dM1: (dependent on the first M mark)** For substituting their found $(1.04)^T$ (or $(1.04)^t$) or their found T (or t) into their $\frac{dN}{dt} = f(t)$; or their found N or $N = 15000$ into their $\frac{dN}{dt} = f(N)$. **A1:** 590 or anything that rounds to 588

Question Number	Scheme		Marks
4. (a)	1.4792	1.4792	B1 **cao**
			[1]
(b)	Area $\approx \frac{1}{2}\times \ln 2\,;\times\left[\underline{2.1333+2(\text{their } 1.4792+1.0079)+0.6667}\right]$		B1 M1
	$=\frac{\ln 2}{2}\times 7.7742... = 2.694332406... = 2.69$ (2 dp)	awrt 2.69	A1
			[3]
(c)(i)	$\{u=1+3e^{-x}\}\Rightarrow \underline{\frac{du}{dx}=-3e^{-x}}$ or $\underline{\frac{dx}{du}=\frac{-1}{(u-1)}}$		B1 oe
	$\left\{\int \frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,dx=\right\}-\frac{4}{9}\int\frac{1}{\sqrt{u}}\,du$	$\pm\lambda\int\frac{1}{\sqrt{u}}\,du$	M1
		$-\frac{4}{9}\int\frac{1}{\sqrt{u}}\,du$	A1 oe
	$=-\frac{4}{9}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\{+c\}$	giving $\pm\beta u^{\frac{1}{2}}$	dM1
	$=-\frac{8}{9}u^{\frac{1}{2}}\{+c\}$		
	$=-\frac{8}{9}\sqrt{(1+3e^{-x})}\{+c\}$		A1
			[5]
(ii)	$=-\frac{8}{9}\left(\sqrt{(1+3e^{-0})}-\sqrt{(1+3e^{--3\ln 2})}\right)$	Applying limits of $x=-3\ln 2$ and $x=0$ to an expression of the form $\pm A\sqrt{(1+3e^{-x})}$ and subtracts either way round. **See notes.**	M1
	$=-\frac{8}{9}\left(\sqrt{4}-\sqrt{25}\right)$		
	$=\frac{8}{3}$	$\frac{8}{3}$ or awrt 2.67	A1
			[2]
			11

Notes

(a) **B1:** 1.4792 correct answer only. Look for this on the table or in the candidate"s working.

(b) **B1**: Outside brackets $\frac{1}{2}\times\ln 2$ or $\frac{\ln 2}{2}$ or awrt 0.35 or $\frac{\text{awrt } 0.69}{2}$.

Also allow $-\frac{1}{2}\times\ln 2$ or $-\frac{\ln 2}{2}$ or awrt -0.35 or $-\frac{\text{awrt } 0.69}{2}$.

M1: <u>For structure of trapezium rule</u>[.............]

A1: anything that rounds to 2.69

<u>Note:</u> It can be possible to award : (a) B0 (b) B1M1A1 (awrt 2.69)

<u>Note:</u> Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 2.66666…

<u>Note:</u> Award B1M1A1 for $\frac{\ln 2}{2}(2.1333+0.6667)+\ln 2(\text{their } 1.4792+1.0079)= 2.694332406...$

Notes for Question 4 continued

4. (b) contd.

Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly,**

Award B1M0A0 for $\frac{1}{2}\times\ln 2 + 2.1333 + 2(\text{their } 1.4792 + 1.0079) + 0.6667$ (nb: answer of 8.12077...).

Award B1M0A0 for $\frac{1}{2}\times\ln 2\ (2.1333 + 0.6667) + 2(\text{their } 1.4792 + 1.0079)$ (nb: answer of 5.94461...).

Alternative method for part (b): Adding individual trapezia

$$\text{Area} \approx \ln 2\times\left[\frac{2.1333+1.4792}{2} + \frac{1.4792+1.0079}{2} + \frac{1.0079+0.6667}{2}\right] = 2.694332406...$$

B1: $\ln 2$ and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.69

(c)(i)

NOTE: YOU CAN MARK (c)(i) AND (c)(ii) TOGETHER.

B1: For $\frac{du}{dx} = -3e^{-x}$ or $du = -3e^{-x}\,dx$ or $\frac{dx}{du} = \frac{1}{-3e^{-x}}$ or $\frac{dx}{du} = -\frac{e^x}{3}$ or equivalent.

Award B1 for $\frac{dx}{du} = \frac{-\frac{1}{3}}{\frac{u-1}{3}}$ (which can be obtained from differentiating $x = -\ln\left(\frac{u-1}{3}\right)$).

M1: Applying the substitution and achieving $\pm\lambda\int\frac{1}{\sqrt{u}}\,(du)$ or $\pm\lambda\int u^{-\frac{1}{2}}\,(du)$, $\lambda\neq 0$

Note: Any $(u-1)$ terms need to be cancelled out for this M1 mark.

A1: $\int\frac{4}{3u^{\frac{1}{2}}}\frac{(du)}{(-3)}$ or $-\frac{4}{9}\int\frac{1}{\sqrt{u}}(du)$ or $-\frac{4}{9}\int u^{-\frac{1}{2}}(du)$ or $\int\frac{-\frac{4}{3}}{3\sqrt{u}}(du)$ or equivalent.

Ignore the presence of limits, but note that $\int_{-3\ln 2}^{0}\frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,dx = \int_{4}^{25}\frac{4}{9\sqrt{u}}\,du$

dM1: (dependent on the first M mark) Integrates $\pm\lambda\int\frac{1}{\sqrt{u}}\,du$ to give $\pm\beta u^{\frac{1}{2}}$, $\lambda\neq 0$, $\beta\neq 0$

A1: $-\frac{8}{9}\sqrt{(1+3e^{-x})}$, simplified or un-simplified, with/without $+c$

Note: $\int\frac{\frac{4(u-1)}{3}}{3\sqrt{u}}\times\frac{-du}{(u-1)}$ is 1st M0A0 unless the $(u-1)$ terms have been cancelled out later

but $\int\frac{\frac{4\,\cancel{(u-1)}}{3}}{3\sqrt{u}}\times\frac{-du}{\cancel{(u-1)}}$ is 1st M1A1.

(c)(ii)

M1: Applies limits of $x = -3\ln 2$ or -2.07... and $x = 0$ to an expression in the form $\pm A\sqrt{(1+3e^{-x})}$ and
subtracts either way round.

Or attempts to apply limits of $u = 25$ and $u = 4$ to an expression in the form $\pm\beta u^{\frac{1}{2}}$ and subtracts either way round.

A1: $\frac{8}{3}$ or anything that rounds to 2.67.

Note: The final A1 mark in (c)(ii) is dependent on (c)(i) B1M1A1M1 and (c)(ii) M1.

Question Number	Scheme		Marks
5.	$\dfrac{dy}{dx} = \dfrac{3y^2}{2\sin^2 2x} \quad y = 2 \text{ at } x = \dfrac{\pi}{8}$		
	$\displaystyle\int \frac{1}{y^2}\,dy = \int \frac{3}{2\sin^2 2x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs.	B1
	$\displaystyle\int \frac{1}{y^2}\,dy = \int \frac{3}{2}\operatorname{cosec}^2 2x\,dx$		
		$\dfrac{1}{y^2} \to -\dfrac{1}{y}$. **(See notes).**	B1
	$-\dfrac{1}{y} = \dfrac{3}{2}\left(-\dfrac{\cot 2x}{2}\right) \{+c\}$	$\pm\lambda\cot 2x$	M1
		$-\dfrac{1}{y} = \dfrac{3}{2}\left(-\dfrac{\cot 2x}{2}\right)$	A1 oe
	$\left\{y = 2,\ x = \dfrac{\pi}{8} \Rightarrow\right\} \quad -\dfrac{1}{2} = -\dfrac{3}{4}\cot\left(2\left(\dfrac{\pi}{8}\right)\right) + c$	Use of $x = \dfrac{\pi}{8}$ and $y = 2$ in an integrated equation containing c	M1
	$-\dfrac{1}{2} = -\dfrac{3}{4} + c \Rightarrow c = \dfrac{1}{4}$		
	$-\dfrac{1}{y} = -\dfrac{3}{4}\cot 2x + \dfrac{1}{4} = \dfrac{1 - 3\cot 2x}{4}$		
	So, $y = \dfrac{-1}{-\frac{3}{4}\cot 2x + \frac{1}{4}}$ or $y = \dfrac{4}{3\cot 2x - 1}$ or $y = \dfrac{4\tan 2x}{3 - \tan 2x}$		A1 oe
			[6] 6

Notes

B1: Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The numbers "3" and "2" may appear on either side.

Eg: $\displaystyle\int \frac{2}{y^2}\,dy = \int \frac{3}{\sin^2 2x}\,dx$, $\displaystyle\int \frac{2}{3y^2}\,dy = \int \frac{1}{\sin^2 2x}\,dx$,

$\displaystyle\int \frac{1}{3y^2}\,dy = \int \frac{1}{2\sin^2 2x}\,dx$ are all fine for B1.

B1: $\dfrac{1}{y^2} \to -\dfrac{1}{y}$ or $\dfrac{2}{y^2} \to -\dfrac{2}{y}$ or $\dfrac{2}{3y^2} \to -\dfrac{2}{3y}$ or $\dfrac{1}{3y^2} \to -\dfrac{1}{3y}$

M1: $\dfrac{1}{\sin^2 2x}$ or $\operatorname{cosec}^2 2x \to \pm\lambda\cot 2x$, $\lambda \neq 0$

A1: $-\dfrac{1}{y} = \dfrac{3}{2}\left(-\dfrac{\cot 2x}{2}\right)$ with/without $+c$ or equivalent. Eg: $\dfrac{4}{3y} = \cot 2x$

M1: Some evidence of using both $x = \dfrac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing c.

Note that is mark can be implied by the correct value of c.

A1: $y = \dfrac{-1}{-\frac{3}{4}\cot 2x + \frac{1}{4}}$ or $y = \dfrac{4}{3\cot 2x - 1}$ or $y = \dfrac{4\tan 2x}{3 - \tan 2x}$ **or any equivalent correct answer.**

Note: You can ignore subsequent working which follows from a correct answer.

Question Number	Scheme		Marks
6.	From question, $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48$		
	$V = \pi r^2(0.3)$	$V = 0.3\pi r^2$ (Can be implied.)	B1 oe
	$\frac{\mathrm{d}V}{\mathrm{d}r} = 0.6\pi r$		B1 ft
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t}\right\} \Rightarrow (0.6\pi r)\frac{\mathrm{d}r}{\mathrm{d}t} = 0.48$ $\left\{\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}r}\right\} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = (0.48)\frac{1}{0.6\pi r}; \left\{= \frac{4}{5\pi r}\right\}$	$\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}\right) \times \frac{\mathrm{d}r}{\mathrm{d}t} = 0.48$ or $0.48 \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}$;	M1; oe
	When $r = 5\text{cm}$, $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{0.48}{0.6\pi(5)}$ $\left\{= \frac{4}{5\pi(5)}\right\}$	Substitutes $r = 5$ into an equation containing $\frac{\mathrm{d}r}{\mathrm{d}t}$.	dM1
	Hence, $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.05092958179...(\text{cm s}^{-1})$	anything that rounds to 0.0509	A1
			[5] **5**

Notes

B1: $V = \pi r^2(0.3)$ or equivalent.

B1ft: Correct follow through differentiation of their V or their A with respect to r.

M1: $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}\right) \times \frac{\mathrm{d}r}{\mathrm{d}t} = 0.48$ or $0.48 \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}r}$

dM1: (dependent on the previous method mark) Substitutes $r = 5$ into an equation containing $\frac{\mathrm{d}r}{\mathrm{d}t}$.

A1: anything that rounds to 0.0509

Example 1: Using thickness = 3 (cm) and not 0.3 (cm)

$V = 3\pi r^2 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 6\pi r$ leading to $\left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{t=5} = \frac{0.48}{6\pi(5)} = 0.005092958179...$ gets B0B1ftM1M1A0.

Example 2: Using thickness = 0.03 (cm) and not 0.3 (cm)

$V = 0.03\pi r^2 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 0.06\pi r$ leading to $\left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{t=5} = \frac{0.48}{0.06\pi(5)} = 0.5092958179...$ gets B0B1ftM1M1A0.

Alternative method 1 First 3 marks

$A = \pi r^2$ **and** $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{0.48}{0.3}\{= 1.6\}$	Can be implied.	B1 oe
$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$	ft $\frac{\mathrm{d}A}{\mathrm{d}r}$	B1 ft
$\left\{\frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}t}\right\} \Rightarrow (2\pi r)\frac{\mathrm{d}r}{\mathrm{d}t} = 1.6$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = (1.6)\frac{1}{2\pi r}; \left\{= \frac{4}{5\pi r}\right\}$		M1; oe

Alternative method 2 First 3 marks

$A = \pi r^2$ **and** $V = 0.3A$	Can be implied.	B1 oe
$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$, $\left\{\frac{\mathrm{d}V}{\mathrm{d}A} = 0.3\right\}$	ft $\frac{\mathrm{d}A}{\mathrm{d}r}$	B1 ft
$\left\{\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}\right\} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi r}\left(\frac{1}{0.3}\right)(0.48); \left\{= \frac{4}{5\pi r}\right\}$		M1; oe

Question Number	Scheme		Marks
7. (a)	$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$		
	$\dfrac{dy}{dx} = \dfrac{-2\sqrt{3}\sin 2t}{-2\sin t}\left\{= \dfrac{\sqrt{3}\sin 2t}{\sin t} = 2\sqrt{3}\cos t\right\}$	Candidate"s $\dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1
		Correct simplified or un-simplified result.	A1 oe **cso**
			[2]
(b)	$\left\{\text{When } t = \dfrac{2\pi}{3},\right\}$ $x = -1,\ y = -\dfrac{\sqrt{3}}{2}$ (need values)	The point $\left(-1, -\dfrac{\sqrt{3}}{2} \text{ or awrt } -0.87\right)$. **These coordinates can be implied.**	B1
	$m(\mathbf{T}) = \dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin\left(2\left(\frac{2\pi}{3}\right)\right)}{\sin\left(\frac{2\pi}{3}\right)} = \dfrac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\left\{= -\sqrt{3}\right\}$	Inserts $t = \dfrac{2\pi}{3}$ into their $\dfrac{dy}{dx}$. Can be implied.	M1
	So, $m(\mathbf{N}) = \dfrac{1}{\sqrt{3}}$	Applies $m(\mathbf{N}) = -\dfrac{1}{m(\mathbf{T})}$	M1
	N: $y - -\dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}(x - -1)$ or $-\dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}(-1) + c \Rightarrow c = -\dfrac{\sqrt{3}}{6}$	See notes.	M1
	N: $y + \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}(x + 1)$		
	N: $2\sqrt{3}y + 3 = 2x + 2$		
	N: $2x - 2\sqrt{3}y - 1 = 0$	Proves the result $2x - 2\sqrt{3}y - 1 = 0$ using exact values.	A1 * cso
			[5]
(c)	$2(2\cos t) - 2\sqrt{3}\left(\sqrt{3}\cos 2t\right) - 1 = 0$	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into **N** to form an equation in variable t.	M1
	$4\cos t - 6\cos 2t - 1 = 0$		
	$6\cos 2t - 4\cos t + 1 = 0$		
	$6(2\cos^2 t - 1) - 4\cos t + 1 = 0$	Applies $\cos 2t = 2\cos^2 t - 1$	M1
	$12\cos^2 t - 4\cos t - 5 = 0$	$12\cos^2 t - 4\cos t - 5\ \{= 0\}$. See notes.	A1 oe
	$(6\cos t - 5)(2\cos t + 1) = 0 \Rightarrow \cos t = ...$		ddM1
	$\cos t = \dfrac{5}{6},\ \left\{\cos t = -\dfrac{1}{2}\right\}$		
	So $(x, y) = \left(\dfrac{5}{3}, \dfrac{7}{18}\sqrt{3}\right)$	At least one of either x or y correct. (See notes)	A1 oe
		Both x and y correct.	A1 oe
			[6] **13**

Notes for Question 7	
7(a)	**Note:** Award M1A0 for a candidate who writes (explicitly) $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sqrt{3}\sin 2t$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\sin t$ followed by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$. **Note:** Award M1A1for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ with no explicit reference to $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$. **Note:** Also award M1A1 for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ with no explicit reference to $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$.
(b)	**B1:** $x = -1$, $y = -\frac{\sqrt{3}}{2}$ or $\left(-1, -\frac{\sqrt{3}}{2} \text{ or awrt } -0.87\right)$. You can imply these coordinates from later working. **M1:** Inserts $t = \frac{2\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. This mark can be implied by a correct ft value from their $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=\frac{2\pi}{3}}$. **M1:** Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\mathrm{m}(\mathbf{T})}$. Numerical value for $\mathrm{m}(\mathbf{N})$ is required here. **M1:** Use $y - (\text{their } y_1) = (\text{their } m_N)(x - (\text{their } x_1))$. or ***finds c*** by substituting $\left(\text{their } -1, \text{ their } -\frac{\sqrt{3}}{2}\right)$ into $y = (\text{their } m_N)x + c$ where $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -\text{their m}(\mathbf{T})$. **Note:** Numerical values for their x_1 , y_1 and $\mathrm{m}(\mathbf{N})$ are required here. **A1: (correct solution only from** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ **)** Convincing proof of $2x - 2\sqrt{3}y - 1 = 0$ **(answer given)** with no errors. Eg 1: $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1) \Rightarrow 2\sqrt{3}y + 3 = 2x + 2 \Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ Eg 2: $y = \frac{1}{\sqrt{3}}x - \frac{\sqrt{3}}{6} \Rightarrow \sqrt{3}y = x - \frac{1}{2} \Rightarrow 2\sqrt{3}y = 2x - 1 \Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ **Note:** Candidate need to work in exact values to prove $2x - 2\sqrt{3}y - 1 = 0$ for the final A1.
7. (c)	**M1:** Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to form an equation in only one variable. **M1:** Applies $\cos 2t = 2\cos^2 t - 1$ **A1:** For obtaining either $12\cos^2 t - 4\cos t - 5\ \{= 0\}$ or $-12\cos^2 t + 4\cos t + 5\ \{= 0\}$ This mark can also awarded for a correct three term equation eg. $12\cos^2 t - 4\cos t = 5$ or $12\cos^2 x = 4\cos x + 5$ etc.

Notes for Question 7 continued

ddM1 (dependent on the previous 2 M marks)

See page 4: Method mark for solving a 3 term quadratic.

- $(6\cos t - 5)(2\cos t + 1) = 0 \Rightarrow \cos t = ...$
- $\cos t = \dfrac{4 \pm \sqrt{16 - 4(12)(-5)}}{2(12)}$
- $\cos^2 t - \frac{1}{3}\cos t - \frac{5}{12} = 0 \Rightarrow \left(\cos t - \frac{1}{6}\right)^2 - \frac{1}{36} - \frac{5}{12} = 0 \Rightarrow \cos t = ...$
- Or writes down at least one correct root from their quadratic equation.

A1: Any one of either $x = \frac{5}{3}$ or 1.66 or awrt 1.67 or $y = \frac{7}{18}\sqrt{3}$ or $\frac{21}{18\sqrt{3}}$ or awrt 0.67

A1: Both $x = \frac{5}{3}$ and $y = \frac{7}{18}\sqrt{3}$ or $\frac{21}{18\sqrt{3}}$ (both exact values required here for the final A1.)

Note: A candidate cannot obtain any of the final two accuracy marks unless the first three marks (M1M1A1) have already been awarded.

(c): Alternative Method 1 Forming a Cartesian equation from $x = 2\cos t,\ y = \sqrt{3}\cos 2t$

$y = \sqrt{3}\cos 2t = \sqrt{3}\left(2\cos^2 t - 1\right)$ — **2nd M1:** For applying $\cos 2t = 2\cos^2 t - 1$

So $y = \sqrt{3}\left(\frac{2x^2}{4} - 1\right) = \frac{\sqrt{3}}{2}x^2 - \sqrt{3}$

$2x - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}x^2 - \sqrt{3}\right) - 1 = 0$ — **1st M1:** For substituting their $y = \frac{\sqrt{3}}{2}x^2 - \sqrt{3}$ into **N**.

$3x^2 - 2x - 5 = 0$ — **A1:** For $3x^2 - 2x - 5\ \{= 0\}$ or $3x^2 - 2x = 5$, etc.

$(x+1)(3x-5) = 0 \Rightarrow x = ...$ — **ddM1:** For attempting to solve a quadratic equation.

$x = \frac{5}{3},\ y = \frac{7}{18}\sqrt{3}$ — **A1A1:** As above.

(c): Alternative Method 2 Forming a Cartesian equation from $x = 2\cos t,\ y = \sqrt{3}\cos 2t$

$y = \sqrt{3}\cos 2t = \sqrt{3}\left(2\cos^2 t - 1\right)$ — **2nd M1:** For applying $\cos 2t = 2\cos^2 t - 1$

So $y = \frac{\sqrt{3}}{2}x^2 - \sqrt{3} \Rightarrow x^2 = \frac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)$

$2x = 2\sqrt{3}\,y + 1 \Rightarrow 4x^2 = 12y^2 + 4\sqrt{3}y + 1$

$4\left(\frac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)\right) = 12y^2 + 4\sqrt{3}y + 1$ — **1st M1:** For substituting their $x^2 = \frac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)$ or $x = \sqrt{\frac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)}$ into **N**.

$36y^2 + 4\sqrt{3}\,y - 21 = 0$ — **A1:** For $36y^2 + 4\sqrt{3}\,y - 21\ \{= 0\}$ or $12y^2 + \frac{4\sqrt{3}}{3}y - 7\ \{= 0\}$ etc.

$\left(18y - 7\sqrt{3}\right)\left(2y + \sqrt{3}\right) = 0 \Rightarrow y = ...$ — **ddM1:** For attempting to solve a quadratic equation.

$y = \frac{7}{18}\sqrt{3},\ x = \frac{5}{3}$ — **A1A1:** As above.

Question Number	Scheme	Marks
8.	$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $l_2: \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$.	
	$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$.	
(a)	So $\mathbf{d}_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$ — Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm\left(\dfrac{-5-4+5}{\sqrt{(-1)^2+(2)^2+(1)^2}.\sqrt{(5)^2+(-2)^2+(5)^2}}\right)$ — Correct **equation**.	A1
	$\cos\theta = \dfrac{-4}{18} \Rightarrow \theta = 102.8395884...$	
	So acute angle $= 77.16041159...$ — awrt 77.2	A1
		[3]
(b)	$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$. — Substitutes candidate's $\lambda = 2$ into l_1 and finds $\mathbf{j} + 6\mathbf{k}$.	B1
	$(= \overrightarrow{OA}$. Hence the point A lies on $l_1)$. — The conclusion on this occasion is not needed.	
		[1]
(c)	$\{l_1 = l_2 \Rightarrow\}$ So $X(2, -3, 4)$. — $X(2, -3, 4)$	B1
		[1]
(d)	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$	
	$AX = \sqrt{(2)^2 + (-4)^2 + (-2)^2} = \sqrt{24}\ \{= 2\sqrt{6}\}$ — Full method for finding AX.	M1
	$\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90	A1
		[2]
(e)	Area $AB_1B_2 = \left(\frac{1}{2}(\sqrt{24})^2 \sin 77.1604...^\circ\right); \times 2 = 23.3990503...$ — awrt 23.4 or $\dfrac{8}{3}\sqrt{77}$	M1; dM1 A1
		[3]

Question Number	Scheme		Marks
8. (f)	$\overrightarrow{XB} = \begin{pmatrix} 5\mu \\ -2\mu \\ 5\mu \end{pmatrix}$ and $XB = \sqrt{24}$		
	$\{AX^2 =\}\ (5\mu)^2 + (-2\mu)^2 + (5\mu)^2 = 24$		M1
	$\left\{\Rightarrow 54\mu^2 = 24 \Rightarrow \mu^2 = \frac{4}{9} \Rightarrow\right\}\ \mu = \pm\frac{2}{3}$	Either $\mu = \frac{2}{3}$ or $\mu = -\frac{2}{3}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \pm \frac{2}{3}\begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$	Substitutes at least one of their values of μ into l_2.	dM1
	$\{\overrightarrow{OB_1}\} = \begin{pmatrix} \frac{16}{3} \\ -\frac{13}{3} \\ \frac{22}{3} \end{pmatrix}$ or $\begin{pmatrix} 5\frac{1}{3} \\ -4\frac{1}{3} \\ 7\frac{1}{3} \end{pmatrix}$, $\{\overrightarrow{OB_2}\} = \begin{pmatrix} -\frac{4}{3} \\ -\frac{5}{3} \\ \frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} -1\frac{1}{3} \\ -1\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5] **15**

Notes

8. (a)

M1: Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d_2}$. Allow one copying slip.

A1: Correct application of the dot product formula $\mathbf{d_1} \bullet \mathbf{d_2} = \pm|\mathbf{d_1}||\mathbf{d_2}|\cos\theta$ or $\cos\theta = \pm\left(\frac{\mathbf{d_1} \bullet \mathbf{d_2}}{|\mathbf{d_1}||\mathbf{d_2}|}\right)$

The dot product must be correctly applied, and the square roots although they can be un-simplified must be correctly applied.

A1: awrt 77.2 $\theta = 1.3467...^c$ or $\theta = 1.7948...^c$ is A0.

<u>Alternative Method: Vector Cross Product</u>

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$$\left\{\mathbf{d_1} \times \mathbf{d_2} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 1 \\ 5 & -2 & 5 \end{vmatrix} = 12\mathbf{i} + 10\mathbf{j} - 8\mathbf{k}\right\}$$

<u>M1</u>: Realisation that the vector cross product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d_2}$. Allow one copying slip.

$$\sin\theta = \frac{\sqrt{(12)^2 + (10)^2 + (-8)^2}}{\sqrt{(-1)^2 + (2)^2 + (1)^2}\,.\sqrt{(5)^2 + (-2)^2 + (5)^2}}$$

A1: Correct applied **equation**.

$$\sin\theta = \frac{\sqrt{308}}{\sqrt{6}.\sqrt{54}} \Rightarrow \theta = 77.16041159... = 77.2 \text{ (1 dp)}$$

A1: awrt 77.2

(b) **B1:** Substitutes candidate's $\lambda = 2$ into l_1 and finds $\mathbf{j} + 6\mathbf{k}$. The conclusion on this occasion is not needed.

Note: $\lambda = 2 \Rightarrow r = \mathbf{j} + 6\mathbf{k}$ is not sufficient working for B1.

Note: Writing $2 - \lambda = 0$, $2\lambda - 3 = 1$, $\lambda + 4 = 6$ followed by $\lambda = 2$ is ok for B1.

(c) **B1:** $(2, -3, 4)$ or $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ etc.

	Notes for Question 8 continued
8. (d)	**Working must occur in part (d) only.** **M1:** Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find either AX or AX^2. **A1:** $\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90 . **M1A1** can be awarded for seeing either $\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90 as their answer.
(e)	**NOTE: Parts (e) and (f) can be marked together.** **M1:** Either $\frac{1}{2}\left(\text{their "}\sqrt{24}\text{"}\right)^2 \sin\left(\text{their } (77.2^\circ) \text{ from (a)}\right)$ or $\frac{1}{2}\left(\text{their "}\sqrt{24}\text{"}\right)^2 \sin\left(\text{their } (180-77.2^\circ) \text{ from (a)}\right)$. awrt 11.7 will usually imply this mark. **dM1:** Multiplies one of their areas by 2 for triangle AB_1B_2 or writes down **both areas** for ΔAXB_1 and ΔAXB_2. **A1:** awrt 23.4 **Note: Award M1dM1 for** $\left(\text{their "}\sqrt{24}\text{"}\right)^2 \sin\left(\text{their } (77.2^\circ) \text{ from (a)}\right)$ **Note: Award M1dM1 for** $(24)\left(\frac{\sqrt{77}}{9}\right)$ **Alternative Method 1:** Some candidates may apply $\frac{1}{2}$(base)(height) "perpendicular" height = $\left(\text{their "}\sqrt{24}\text{"}\right)\sin\left(\text{their } (77.2^\circ) \text{ from (a)}\right)$ **Award M1dM1** for $\frac{1}{2}\left(2(\text{their "}\sqrt{24}\text{"})\right)\left(\text{their "}\sqrt{24}\text{"}\right)\sin\left(\text{their } (77.2^\circ) \text{ from (a)}\right)$, where their "$\sqrt{24}$" 's are consistent, i.e. the same. **Alternative Method 2:** **M1:** $\frac{1}{2}\left(\text{their "}\sqrt{24}\text{"}\right)\left(\text{their } AB = \text{"6.11"}\right)\sin A$, where $A = 51.42°$, or $A = \frac{1}{2}\left(180 - \text{their } (77.2^\circ) \text{ from (a)}\right)$. **Note:** there must be a full method for finding the length AB. (i.e. from either the sine rule or the cosine rule.) **dM1:** Multiplies one of their areas by 2 for triangle AB_1B_2 or writes down **both areas** for ΔAXB_1 and ΔAXB_2. **A1:** awrt 23.4
(f)	**M1:** Writes down an equation relating the $\lvert\overrightarrow{AX}\rvert$ to $\lvert\overrightarrow{XB}\rvert$ or $\lvert\overrightarrow{AX}\rvert^2$ to $\lvert\overrightarrow{XB}\rvert^2$. M1 can also be awarded for either $\frac{\text{their } \lvert\overrightarrow{AX}\rvert}{\text{their } \lvert\mathbf{d}_2\rvert}$ or $\frac{\text{their } \lvert\mathbf{d}_2\rvert}{\text{their } \lvert\overrightarrow{AX}\rvert}$. **A1:** Either $\mu = \frac{2}{3}$ or $\mu = -\frac{2}{3}$ **dM1: (Dependent on the previous method mark)** Substitutes at least one of their values of μ into l_2. If no working shown then two out of three of the components must be correctly followed through. **A1:** At least one set of coordinates are correct. Ignore labelling of B_1, B_2 **A1:** Both sets of coordinates are correct. Ignore labelling of B_1, B_2

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