C4 Mock (MA)

Q1) $\int_0^2 \left[\frac{2x}{(4+3x^2)^2}\right] dx$

$$\left[\begin{array}{l} u = 4+3x^2 \\ \frac{du}{dx} = 6x \\ dx = \frac{1}{6x}\,du \\ \begin{array}{c|c} x & u \\ \hline 0 & 4 \\ 2 & 16 \end{array} \end{array}\right]$$

$= \int_4^{16} \left[\frac{2\cancel{x}}{u^2} \times \frac{1}{6\cancel{x}}\right] du$

$= \frac{1}{3}\int_4^{16} \left[u^{-2}\right] du$

$= \frac{1}{3}\left[\frac{u^{-1}}{-1}\right]_4^{16} = \frac{1}{3}\left[-\frac{1}{u}\right]_4^{16}$

$= \frac{1}{3}\left[-\frac{1}{16}\right] - \frac{1}{3}\left[-\frac{1}{4}\right] = \frac{1}{3}\left[\frac{3}{16}\right] = \boxed{\frac{1}{16}}$

Q2) $\frac{d}{dx}\left(x^3 - 2xy - 4x + y^3 - 51 = 0\right)$

$\Rightarrow 3x^2 - 2x\frac{dy}{dx} - 2y - 4 + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}\left(3y^2 - 2x\right) = 4 + 2y - 3x^2$

$\frac{dy}{dx} = \frac{4+2y-3x^2}{3y^2-2x}$ //

at $(4,3)$, $\frac{dy}{dx} = \frac{4+2(3)-3(16)}{3(9)-2(4)}$

$= \frac{-38}{19} = -2$

$\therefore$ at normal, $m = \frac{1}{2}$ $\left(\frac{1}{2}x - 2 = -1\right)$

$\Rightarrow y - 3 = \frac{1}{2}(x-4)$

$\Rightarrow y = \frac{1}{2}x - 2 + 3$

$\Rightarrow y = \frac{1}{2}x + 1$

$\overset{\times 2}{\Rightarrow} 2y = x + 2$

$\Rightarrow \boxed{x - 2y + 2 = 0}$

Q3a) $f(x) = \frac{1+14x}{(1-x)(1+2x)} = \frac{A}{1-x} + \frac{B}{1+2x}$

$\Rightarrow 1 + 14x = A(1+2x) + B(1-x)$.

$\underline{x=1}$: $15 = 3A$ $\therefore A = 5$

$\underline{x=0}$: $1 = A + B$ $\therefore B = -4$

$\therefore f(x) = \boxed{\frac{5}{1-x} - \frac{4}{1+2x}}$

$$f(x) = 5(1-x)^{-1} - 4(1+2x)^{-1}$$

$$\therefore f(x) \simeq \begin{array}{cccc} 5 & +5x & +5x^2 & +5x^3 \\ -4 & +8x & -16x^2 & +32x^3 \end{array}$$

$$f(x) \simeq 1 + 13x - 11x^2 + 37x^3$$

(Q4a) <u>assume intersection</u>

$$\begin{pmatrix} 11+4\lambda \\ 5+2\lambda \\ 6+4\lambda \end{pmatrix} = \begin{pmatrix} 24+7\mu \\ 4+\mu \\ 13+5\mu \end{pmatrix} \begin{array}{l} —— ① \\ —— ② \\ —— ③ \end{array}$$

① : $4\lambda = 13 + 7\mu$

$\lambda = \frac{13}{4} + \frac{7}{4}\mu.$

$\hookrightarrow$ ② : $5 + \frac{13}{2} + \frac{7}{2}\mu = 4 + \mu$

$\frac{5}{2}\mu = -\frac{15}{2} \quad \therefore \mu = -3$

substitute $\mu = -3$ into ① : $\lambda = \frac{13}{4} + \frac{7}{4}(-3) = -2$

and into ③ : $4\lambda = 13 + 5(-3) - 6 = -8$

$\lambda = \frac{-8}{4} = -2$

$\therefore$ values of μ and λ are consistent so l_1 and l_2 do indeed intersect

b) $\underline{\lambda = -2}$: $\begin{pmatrix} 11-8 \\ 5-4 \\ 6-8 \end{pmatrix} = \boxed{\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}}$

c) $\underline{\text{using direction vectors}}$: $\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} = 28+2+20 = 50 //$

$$\left|\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}\right| = \sqrt{4^2+2^2+4^2} = 6 //$$

$$\left|\begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix}\right| = \sqrt{7^2+1^2+5^2} = 5\sqrt{3} //$$

$$\therefore \cos\theta = \frac{50}{6 \times 5\sqrt{3}} = \frac{5\sqrt{3}}{9} //$$

$$= \boxed{\frac{5}{9}\sqrt{3}}$$

Q5a) $x = \cos t \longrightarrow \frac{dx}{dt} = -\sin t$

$y = \sin 2t \longrightarrow \frac{dy}{dt} = 2\cos 2t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \boxed{\frac{2\cos 2t}{-\sin t}} //$$

b) $\underline{\frac{dy}{dx} = 0}$: $-2\cos 2t = 0$.

$\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$$\boxed{t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

c) $x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$y = \sin\frac{\pi}{2} = 1$

(symmetry)

$$\boxed{\begin{array}{c} \left(\frac{\sqrt{2}}{2}, 1\right) \\ \left(\frac{\sqrt{2}}{2}, -1\right) \\ \left(-\frac{\sqrt{2}}{2}, 1\right) \\ \left(-\frac{\sqrt{2}}{2}, -1\right) \end{array}}$$

d) $0 \leq t < \pi$ is where $x \geq 0$.

$y = 2\sin t\cos t$

$x = \cos t$

$x^2 = \cos^2 t$

$1 - x^2 = 1 - \cos^2 t$

$1 - x^2 = \sin^2 t$

$\therefore \sqrt{1-x^2} = \sin t$

$y = 2(\sqrt{1-x^2})(x)$ $\square$

e) same curve but reflected in y-axis...

$\Rightarrow y = -2x\sqrt{1-x^2}$

Q6a) $R = \int_{\pi}^{2\pi} x^2 \sin\left(\frac{x}{2}\right) dx$

By parts

$\frac{dv}{dx} = \sin\left(\frac{x}{2}\right) \quad v = -2\cos\left(\frac{x}{2}\right)$

$u = x^2 \quad u' = 2x$

$= \left[-2x^2\cos\left(\frac{x}{2}\right)\right]_{\pi}^{2\pi} + 4\int_{\pi}^{2\pi}\left[x\cos\left(\frac{x}{2}\right)\right]dx$

By parts

$\frac{dv}{dx} = \cos\left(\frac{x}{2}\right) \rightarrow v = 2\sin\left(\frac{x}{2}\right)$

$u = x \quad u' = 1$

$= \left[2x\sin\frac{x}{2}\right]_{\pi}^{2\pi} - 2\int_{\pi}^{2\pi}\sin\left(\frac{x}{2}\right)dx$

$= -2\pi - 2\left[-2\cos\left(\frac{x}{2}\right)\right]_{\pi}^{2\pi} = -2\pi - 2[2] + 2[0]$

$= -2\pi - 4$ //

$\therefore R = 2(4\pi^2) + 4(-2\pi - 4)$

$\boxed{R = 8\pi^2 - 8\pi - 16}$

b) $G = \left(\frac{7\pi}{4}\right)^2 \cdot \sin\left(\frac{7\pi}{8}\right) = \boxed{11.567}$

cii) $h = \frac{2\pi - \pi}{4} = \frac{\pi}{4}$ //

$\text{Area} \simeq \frac{1}{2} \times \frac{\pi}{4}\left[9.8696 + 2(14.247 + 15.702 + 11.567)\right]$

$\simeq \boxed{36.48}$

ci) $h = \frac{2\pi - \pi}{2} = \frac{\pi}{2}$ //

$\text{Area} \simeq \frac{1}{2} \times \frac{\pi}{2}\left[9.8696 + 2(15.702)\right]$

$\simeq \boxed{32.42}$

Q7a) $\boxed{\frac{dM}{dt} = -kM}$

b) $\frac{dM}{dt} = 10(0.98^t)\ln 0.98 = (10\ln 0.98) \cdot 0.98^t$ //

$= -0.202 \cdot 0.98^t$

$= -0.02 \cdot 10(0.98^t) = -0.02 \cdot M$

$(M = 10(0.98^t))$

(fits our model).

c) $10 \frac{dM}{dt} = -k(10M - 1)$

$\left(\frac{10}{10M-1}\right) \frac{dM}{dt} = -k$

$10 \int \left(\frac{1}{10M-1}\right) dM = -k \int (1) dt$

$\frac{10}{10} \ln |10M - 1| = -kt + c$

$\ln |10M - 1| = -kt + c$

$\underline{t=0, M=10}$: $c = \ln 99$

$\therefore \ln|10M - 1| = \ln 99 - kt$

$\underline{t=10, M=8.5}$: $\ln(84) - \ln(99) = -kt$

$\therefore \frac{1}{10} \ln \frac{99}{84} = k$

$\Rightarrow \ln |10M - 1| = \ln 99 - \frac{t}{10} \ln \frac{99}{84}$

$\underline{t=15}$: $\ln |10M - 1| = \ln 99 - \frac{3}{2} \ln \frac{99}{84} = 4.3487.$

$10M - 1 = e^{4.3487..}$

$M = \frac{e^{4.3487.} + 1}{10} = \boxed{7.8}\,g$