

C4 JNE 09

$$1) f(x) = (4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{2} \left(1 + (-\frac{1}{2})\left(\frac{x}{4}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2} \left(\frac{x}{4}\right)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6} \left(\frac{x}{4}\right)^3 \right)$$

$$= \frac{1}{2} - \frac{1}{8}x + \frac{3}{128}x^2 - \frac{5}{1024}x^3$$

$$2) x = \frac{9\pi}{8} \quad y = 3 \cos\left(\frac{9\pi}{24}\right) = 1.14805$$

$$\text{Area} \hat{=} \frac{1}{2} \left(\frac{3\pi}{8} \right) \left(3 + 0 + 2(2.77164 + 2.12132 + 1.14805) \right) \\ = 8.884$$

$$\text{Area} = \int_0^{\frac{3\pi}{2}} 3 \cos\left(\frac{x}{3}\right) dx = \left[-9 \sin\left(\frac{x}{3}\right) \right]_0^{\frac{3\pi}{2}} = (-9) - (0) \\ = \underline{9}$$

$$3) f(x) = \frac{A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)}{(2x+1)(x+1)(x+3)} = 4-2x$$

$$x = -1 \Rightarrow -2B = 6 \Rightarrow B = -3$$

$$x = -3 \Rightarrow 10C = 10 \Rightarrow C = 1$$

$$x = 0 \Rightarrow 3A + 3B + C = 4 \Rightarrow 3A = 12 \Rightarrow A = 4$$

$$b) \int f(x) dx = \int \frac{4}{2x+1} dx + \int \frac{-3}{x+1} dx + \int \frac{1}{x+3} dx$$

$$= 4 \times \frac{1}{2} \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) + C$$

$$(ii) \int_0^2 f(x) dx = (2 \ln 5 - 3 \ln 3 + \ln 5) - (2 \ln 1 - 3 \ln 1 + \ln 3)$$

$$= 3 \ln 5 - 4 \ln 3 = \ln 125 - \ln 81 = \ln\left(\frac{125}{81}\right)$$

$$4) \frac{d}{dx} ye^{-2x} = \frac{d}{dx} 2x + \frac{d}{dx} y^2$$

$$u = y \quad v = e^{-2x}$$

$$u' = \frac{dy}{dx} \quad v' = -2e^{-2x} \Rightarrow e^{-2x} \frac{dy}{dx} - 2e^{-2x} y = 2 + 2y \frac{dy}{dx}$$

$$\Rightarrow e^{-2x} \frac{dy}{dx} - 2y \frac{dy}{dx} = 2 + 2e^{-2x} y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 + 2e^{-2x} y}{e^{-2x} - 2y}$$

$$b) (0,1) \quad M_t = \frac{2 + 2(1)}{1 - 2} = \frac{4}{-1} = -4 \Rightarrow M_n = \frac{1}{4}$$

$$y - 1 = \frac{1}{4}(x - 0) \Rightarrow 4y - 4 = x \Rightarrow \underline{x - 4y + 4 = 0}$$

$$5) x = 2 \cos 2t$$

$$y = 6 \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dx}{dt} = -4 \sin 2t$$

$$\frac{dy}{dt} = 6 \cos t$$

$$\frac{dy}{dx} = \frac{-6 \cos t}{4 \sin 2t}$$

$$\text{at } t = \frac{\pi}{3} \quad m = \frac{-3}{2\sqrt{3}} = \frac{-3\sqrt{3}}{6} = \underline{\underline{-\frac{1}{2}\sqrt{3}}}$$

$$5b) \quad \frac{y}{6} = \sin t \quad \frac{x}{2} = \cos 2t = 1 - 2\sin^2 t$$

$$\Rightarrow \frac{x}{2} = 1 - \frac{2y^2}{36} \Rightarrow x = 2 - \frac{y^2}{9} \Rightarrow \frac{y^2}{9} = 2 - x$$

$$\Rightarrow y^2 = 9(2-x) \Rightarrow y = \sqrt{9(2-x)} \Rightarrow y = 3\sqrt{2-x}$$

$$x = 2\cos 2t \Rightarrow -2 \leq x \leq 2 \Rightarrow \underline{h=2}$$

$$5c) \quad y \geq 0 \quad y \in \mathbb{R}.$$

$$6) \quad \int \sqrt{5-x} \, dx \quad u = (5-x)^{\frac{3}{2}} \quad x^{-\frac{2}{3}}$$

$$= \int (5-x)^{\frac{1}{2}} \, dx \quad \frac{du}{dx} = \frac{3}{2}(5-x)^{\frac{1}{2}} \cdot (-1) \quad x^{-\frac{2}{3}}$$

$$= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C$$

$$b) \quad \int (x-1)\sqrt{5-x} \, dx \quad u = (x-1) \quad v = -\frac{2}{3}(5-x)^{\frac{3}{2}}$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \sqrt{5-x}$$

$$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{2}{3} \int (5-x)^{\frac{3}{2}} \, dx$$

$$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3} \left(-\frac{2}{5}(5-x)^{\frac{5}{2}} \right) + C$$

$$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} + C$$

$$c) \quad \int_1^5 (x-1)(\sqrt{5-x}) \, dx = (0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}} \right) = \frac{4}{15} \times 32 = \frac{128}{15}$$

$$7) \vec{AB} = b - a = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

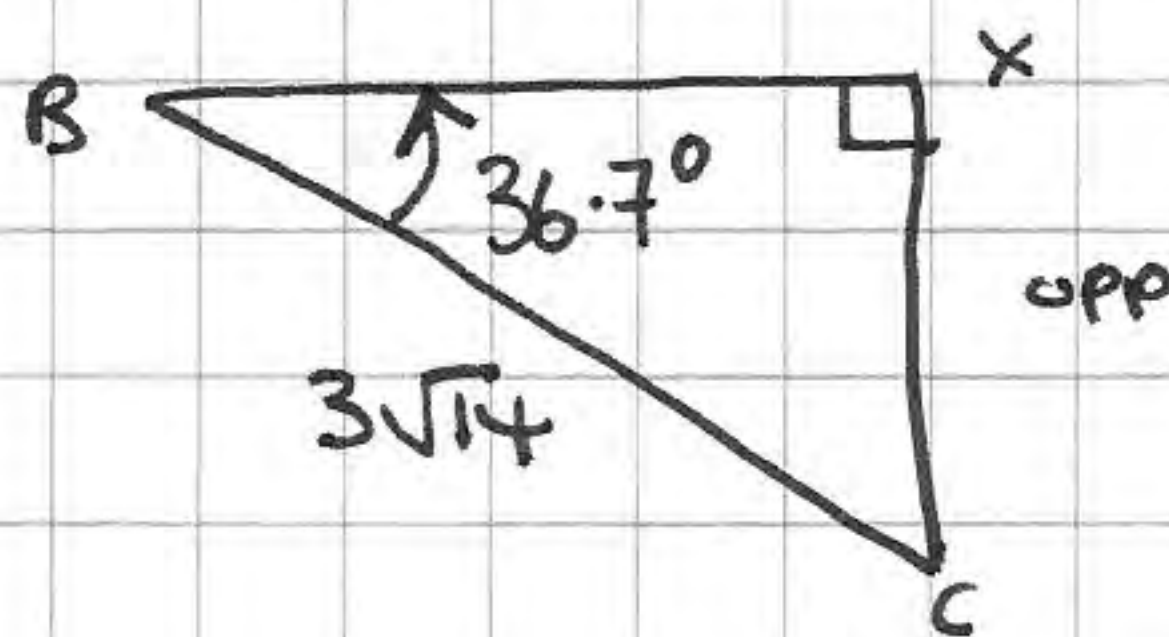
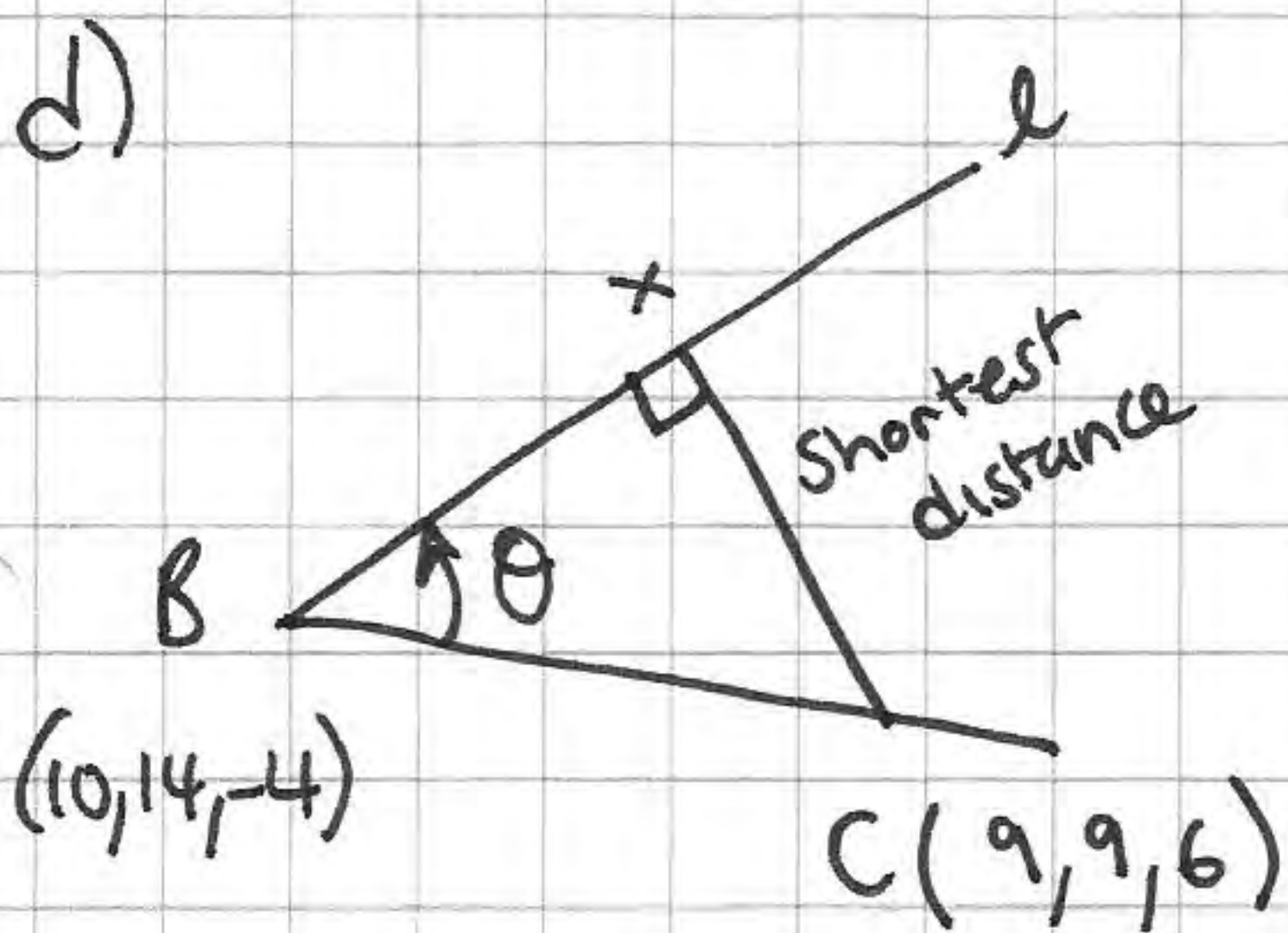
$$l = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$b) \vec{CB} = b - c = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$$

$$|\vec{CB}| = \sqrt{1^2 + 5^2 + 10^2} = \sqrt{126} = 3\sqrt{14}$$

$$c) \theta = \cos^{-1} \left| \frac{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \right| \cdot 3\sqrt{14}} \right| = \cos^{-1} \left| \frac{2 \times 1 + 1 \times 5 + -2 \times -10}{\sqrt{2^2 + 1^2 + 2^2} \times 3\sqrt{14}} \right|$$

$$\theta = \cos^{-1} \left(\frac{27}{9\sqrt{14}} \right) \Rightarrow \theta = \cos^{-1} \left(\frac{3\sqrt{14}}{14} \right) \Rightarrow \theta = 36.7^\circ$$



$$\text{opp} = 3\sqrt{14} \sin 36.7^\circ \dots = \underline{6.71}$$

$$e) \text{Area} = \frac{1}{2} (BX)(6.71 \dots)$$

$$BX = \sqrt{(3\sqrt{14})^2 - (6.71 \dots)^2} = 9$$

$$\text{Area} = \frac{1}{2} (9)(6.71 \dots) = \underline{30.2}$$

$$8) \int \sin^2 \theta d\theta = \int \frac{1}{2} - \frac{1}{2} \cos 2\theta d\theta = \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta + C$$

$$b) \quad x = \tan \theta \quad y = 2 \sin 2\theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta \quad \frac{dy}{d\theta} = 4 \cos 2\theta$$

$$\text{Volume} = \pi \int_0^{\frac{1}{\sqrt{3}}} y^2 dx = \pi \int_0^{\frac{1}{\sqrt{3}}} (2 \sin 2\theta)^2 \frac{dx}{d\theta} d\theta$$

$$x = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \tan \theta \Rightarrow \theta = \frac{\pi}{6}$$

$$x = 0 \Rightarrow 0 = \tan \theta \Rightarrow \theta = 0$$

$$\text{Volume} = \pi \int_0^{\frac{\pi}{6}} (4 \sin \theta \cos \theta)^2 \sec^2 \theta d\theta$$

$$\text{Volume} = 16\pi \int_0^{\frac{\pi}{6}} \sin^2 \theta \cos^2 \theta \times \frac{1}{\cos^2 \theta} d\theta = 16\pi \int_0^{\frac{\pi}{6}} \sin^2 \theta d\theta \quad \#$$

$$c) \quad 16\pi \left[\frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \right]_0^{\frac{\pi}{6}}$$

$$= 16\pi \left[\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) - (0 - 0) \right] = \frac{4\pi^2}{3} - 2\pi\sqrt{3}$$