

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Bronze Level B1

Time: 1 hour 30 minutes**Materials required for examination papers**

Mathematical Formulae (Green)

Items included with question

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
73	69	60	54	49	45

1.

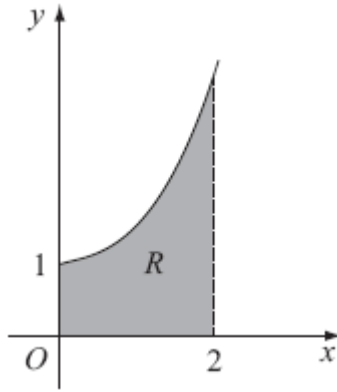


Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the y -axis and the line $x = 2$.

(a) Copy and complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R , giving your answer to 4 significant figures.

(3)

2.

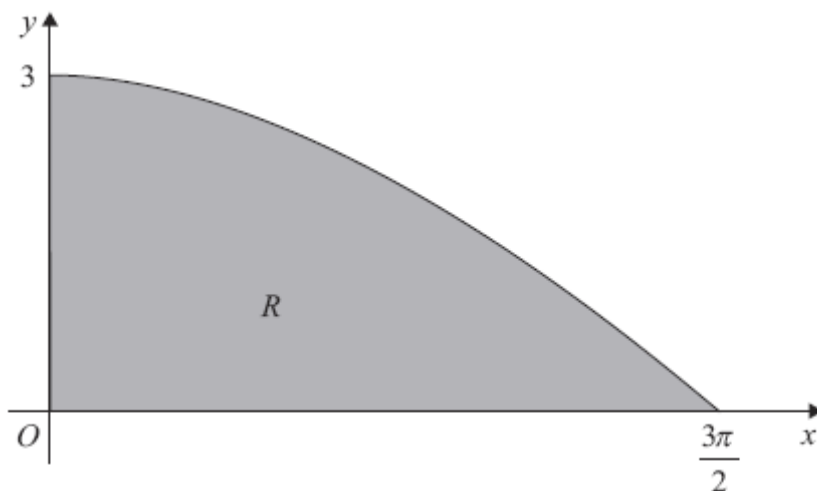


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Copy and complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. (4)
- (c) Use integration to find the exact area of R . (3)

3.

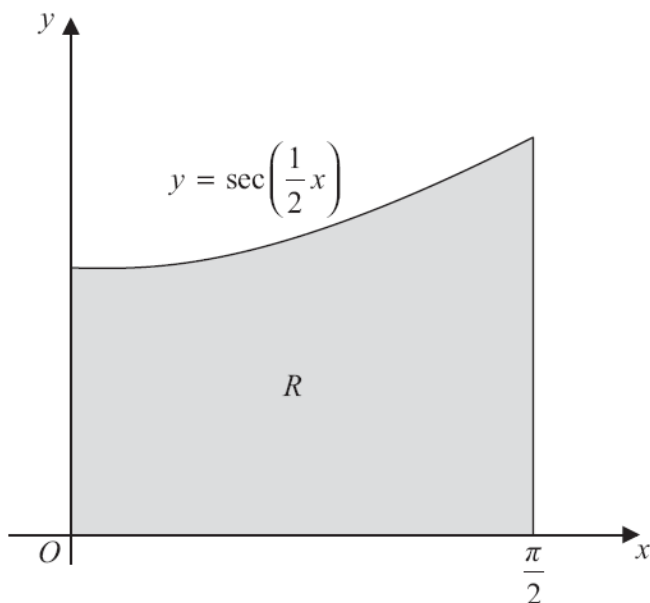


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places. (1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

(c) Use calculus to find the exact volume of the solid formed. (4)

4. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The point P on C has coordinates $(0, 1)$.

(b) Find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

6.

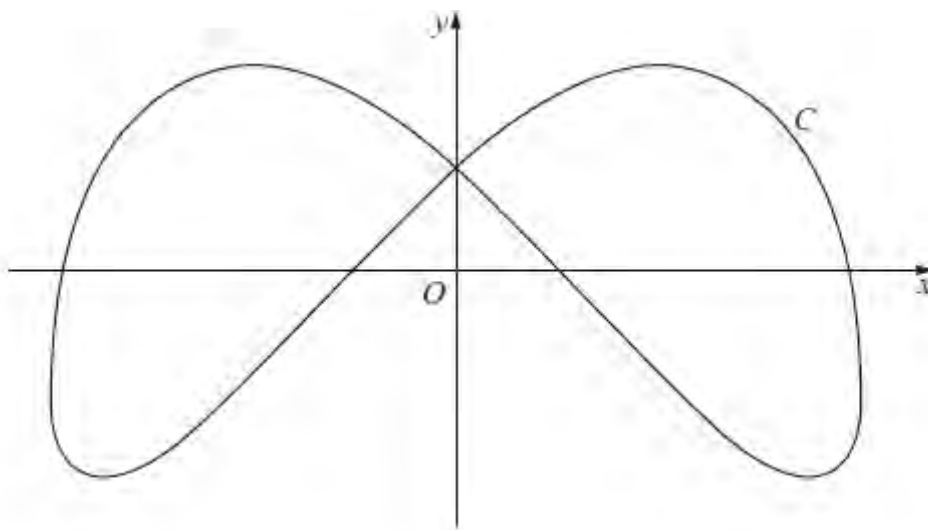
**Figure 2**

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \sin \left(t + \frac{\pi}{6} \right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$.

(5)

7. The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0.$$

Find

(a) an equation of the normal to C at the point where $t = 3$, (6)

(b) a cartesian equation of C . (3)

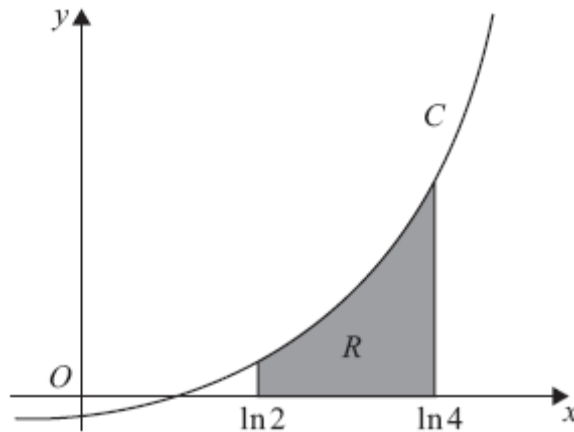


Figure 1

The finite area R , shown in Figure 1, is bounded by C , the x -axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x -axis.

(c) Use calculus to find the exact volume of the solid generated. (6)

8.

$$I = \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx.$$

- (a) Given that $y = \frac{1}{4 + \sqrt{x-1}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

- (b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I , giving your answer to 3 decimal places.

(4)

- (c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I .

(8)

TOTAL FOR PAPER: 75 MARKS
END

Question Number	Scheme						Marks		
1.	(a)	x	0	0.4	0.8	1.2	1.6	2	B1 (1)
		y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2	
		or v	1	1.08329	1.37713	2.05443	3.59664	7.38906	
	(b)	$\text{Area} \approx \frac{1}{2} \times 0.4 \times [e^0 + 2(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}) + e^2]$ $= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922} \text{ (4sf)}$						B1; M1	
								A1 (3)	
(4 marks)									

2.	(a)	1.14805	awrt 1.14805	B1 (1)
	(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8} (\dots)$ $= \dots (3 + 2(2.77164 + 2.12132 + 1.14805) + 0)$ $= \frac{3\pi}{16} (3 + 2(2.77164 + 2.12132 + 1.14805))$ $= \frac{3\pi}{16} \times 15.08202 \dots = 8.884$	0 can be implied ft their (a) cao	B1 M1 A1 ft A1 (4)
	(c)	$\int 3 \cos\left(\frac{x}{3}\right) dx = \frac{3 \sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ $= 9 \sin\left(\frac{x}{3}\right)$ $A = \left[9 \sin\left(\frac{x}{3}\right) \right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$	cao	M1 A1 A1 (3)
(8 marks)				

Question Number	Scheme	Marks
3. (a)	1.154701	B1 cao (1)
(b)	$\text{Area} \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 2(1.035276 + \text{their } 1.154701) + 1.414214]$ $= \frac{\pi}{12} \times 6.794168 = 1.778709023... = 1.7787 \text{ (4 dp)}$	B1; <u>M1</u> A1 (3)
(c)	$V = \pi \int_0^{\frac{\pi}{2}} \left(\sec\left(\frac{x}{2}\right) \right)^2 dx$ $= \{\pi\} \left[2 \tan\left(\frac{x}{2}\right) \right]_0^{\frac{\pi}{2}}$ $= 2\pi$	For $\pi \int \left(\sec\left(\frac{x}{2}\right) \right)^2$ Ignore limits and dx. Can be implied. $\pm \lambda \tan\left(\frac{x}{2}\right)$ $2 \tan\left(\frac{x}{2}\right)$ or equivalent 2π B1 M1 A1 A1 cao cs0 (4) [8]

4. (a)	$e^{-2x} \frac{dy}{dx} - 2ye^{-2x} = 2 + 2y \frac{dy}{dx}$ $\frac{d}{dx}(ye^{-2x}) = e^{-2x} \frac{dy}{dx} - 2ye^{-2x}$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2 + 2ye^{-2x}$ $\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	A1 correct RHS	M1 A1
(b)	At P, $\frac{dy}{dx} = \frac{2 + 2e^0}{e^0 - 2} = -4$ Using $mm' = -1$ $m' = \frac{1}{4}$ $y - 1 = \frac{1}{4}(x - 0)$ $x - 4y + 4 = 0$	or any integer multiple	B1 M1 M1 A1 (5) M1 A1 (4) (9 marks)

Question Number	Scheme	Marks
<p>5. (a)</p>	$x^3 - 4y^2 = 12xy \quad (\text{eqn } *)$ $x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$ $4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$ $(y - 16)(y - 8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$ $y = 16 \text{ or } y = 8.$	<p>M1</p> <p>Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.</p> <p>dM1</p> <p>An attempt to solve the quadratic in y by either factorising or by the formula or by completing the square.</p> <p>A1</p> <p>Both $y = 16$ and $y = 8$. or $(-8, 8)$ and $(-8, 16)$.</p> <p>[3]</p>
<p>(b)</p>	$\left\{ \begin{array}{l} \cancel{\frac{dy}{dx}} \\ \cancel{\frac{dy}{dx}} \end{array} \right\} \times 3x^2 - 8y \frac{dy}{dx} = \left(12y + 12x \frac{dy}{dx} \right)$ $\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$ $\text{@ } (-8, 8), \frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = -3,$ $\text{@ } (-8, 16), \frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = 0.$	<p>M1</p> <p>Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $12x \frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \dots$</p> <p>A1;</p> <p><u>Correct LHS equation;</u> <u>Correct application of product rule</u></p> <p>(B1)</p> <p>not necessarily required.</p> <p>dM1</p> <p>Substitutes $x = -8$ and <i>at least one</i> of their y-values to attempt to find any one of $\frac{dy}{dx}$.</p> <p>A1</p> <p>One gradient found.</p> <p>A1 cso</p> <p>Both gradients of <u>-3</u> and <u>0</u> correctly found.</p> <p>[6]</p> <p>9 marks</p>

Question Number	Scheme	Marks
<p>6.</p> <p>(a)</p> <p>(b)</p>	$x = 4 \sin\left(t + \frac{\pi}{6}\right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$ $\frac{dx}{dt} = 4 \cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6 \sin 2t$ <p>So, $\frac{dy}{dx} = \frac{-6 \sin 2t}{4 \cos\left(t + \frac{\pi}{6}\right)}$</p> $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} -6 \sin 2t = 0$ <p>@ $t = 0$, $x = 4 \sin\left(\frac{\pi}{6}\right) = 2$, $y = 3 \cos 0 = 3 \rightarrow (2, 3)$</p> <p>@ $t = \frac{\pi}{2}$, $x = 4 \sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}$, $y = 3 \cos \pi = -3 \rightarrow (2\sqrt{3}, -3)$</p> <p>@ $t = \pi$, $x = 4 \sin\left(\frac{7\pi}{6}\right) = -2$, $y = 3 \cos 2\pi = 3 \rightarrow (-2, 3)$</p> <p>@ $t = \frac{3\pi}{2}$, $x = 4 \sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}$, $y = 3 \cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$</p>	<p>B1 B1</p> <p>B1 $\sqrt{\quad}$ oe (3)</p> <p>M1 oe</p> <p>M1</p> <p>A1 A1 A1 (5)</p> <p>(8 marks)</p>

Question Number	Scheme	Marks
<p>7.</p> <p>(a)</p>	$\frac{dx}{dt} = \frac{1}{t}, \quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = 2t^2$ <p>Using $mm' = -1$, at $t = 3$</p> $m' = -\frac{1}{18}$ $y - 7 = -\frac{1}{18}(x - \ln 3)$	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (6)</p>
<p>(b)</p>	$x = \ln t \Rightarrow t = e^x$ $y = e^{2x} - 2$	<p>B1</p> <p>M1 A1 (3)</p>
<p>(c)</p>	$V = \pi \int (e^{2x} - 2)^2 dx$ $\int (e^{2x} - 2)^2 dx = \int (e^{4x} - 4e^{2x} + 4) dx$ $= \frac{e^{4x}}{4} - \frac{4e^{2x}}{2} + 4x$ $\pi \left[\frac{e^{4x}}{4} - \frac{4e^{2x}}{2} + 4x \right]_{\ln 2}^{\ln 4} = \pi [(64 - 32 + 4 \ln 4) - (4 - 8 + 4 \ln 2)]$ $= \pi(36 + 4 \ln 2)$	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(6)</p> <p>[15]</p>

Question Number	Scheme	Marks
8. (a)	$x = 3 \Rightarrow y = 0.1847$ $x = 5 \Rightarrow y = 0.1667$	awrt B1 awrt or $\frac{1}{6}$ B1 (2)
(b)	$I \approx \frac{1}{2} [0.2 + 0.1667 + 2(0.1847 + 0.1745)]$ ≈ 0.543	B1 M1 A1ft 0.542 or 0.543 A1 (4)
(c)	$\frac{dx}{du} = 2(u-4)$ $\int \frac{1}{4 + \sqrt{(x-1)}} dx = \int \frac{1}{u} \times 2(u-4) du$ $= \int \left(2 - \frac{8}{u} \right) du$ $= 2u - 8 \ln u$ $x = 2 \Rightarrow u = 5, \quad x = 5 \Rightarrow u = 6$ $[2u - 8 \ln u]_5^6 = (12 - 8 \ln 6) - (10 - 8 \ln 5)$ $= 2 + 8 \ln \left(\frac{5}{6} \right)$	B1 M1 A1 M1 A1 B1 M1 A1 (8) [14]

Question 1

A significant majority of candidates were able to score full marks on this question. In part (a), very few candidates failed to find either one or both of the y -coordinates required. In part (b), some candidates incorrectly used the formula $h = \frac{b-a}{n}$, with $n = 6$ instead of $n = 5$ to give the width of each trapezium as $\frac{1}{3}$. Many candidates, however, were able to look at the given table and deduce the value of h . A few candidates wrote down e^0 as 0 instead of 1. Nearly all answers were given to 4 significant figures as requested in the question.

Question 2

Most candidates could gain the mark in part (a) although 2.99937, which arises from the incorrect angle mode, was seen occasionally. The main error seen in part (b) was finding the width of the trapezium incorrectly, $\frac{3\pi}{10}$ being commonly seen instead of $\frac{3\pi}{8}$. This resulted from confusing the number of values of the ordinate, 5, with the number of strips, 4. Nearly all candidates gave the answer to the specified accuracy. In part (c), the great majority of candidates recognised that they needed to find $\int 3\cos\left(\frac{x}{3}\right)dx$ and most could integrate correctly. However $\sin x$, $9\sin x$, $3\sin\left(\frac{x}{3}\right)$, $-9\sin\left(\frac{x}{3}\right)$, $-\sin\left(\frac{x}{3}\right)$ and $-3\sin\left(\frac{x}{3}\right)$ were all seen from time to time. Candidates did not seem concerned if their answers to part (b) and part (c) were quite different, possibly not connecting the parts of the question. Despite these difficulties, full marks were common and, generally, the work on these topics was sound.

Question 3

This was a straightforward question with about 55% of candidates gaining all 8 marks with an overwhelmingly majority of candidates realising that they needed to use radians in this question.

Although most candidates correctly computed 1.154701 in part (a), a significant number wrote 1.154700, suggesting that truncation rather than rounding was applied by some at this stage. Also an answer of 1.000004 was occasionally seen (a consequence of having the calculator in degrees mode).

In applying the trapezium rule in part (b), a small minority of candidates multiplied $\frac{1}{2}$ by $\frac{\pi}{8}$ instead of $\frac{1}{2}$ by $\frac{\pi}{6}$. Whilst the table of values clearly shows an interval width of $\frac{\pi}{6}$, the application of a formula $h = \frac{b-a}{n}$ with $n = 4$ instead of $n = 3$ sometimes caused this error. Other errors included the occasional bracketing mistake, use of the x -value of 0 rather than the ordinate of 1, and the occasional calculation error following a correctly written expression.

In part (c), the majority of candidates were able to apply volume formula $\pi \int y^2 dx$, although a number of candidates used incorrect formulae such as $2\pi \int y^2 dx$ or $\int y^2 dx$ or even $\int y dx$.

Few candidates incorrectly wrote $\left(\sec\left(\frac{x}{2}\right)\right)^2$ as either $\sec\left(\frac{x^2}{4}\right)$ or $\sec^2\left(\frac{x^2}{4}\right)$. A minority of candidates integrated $\sec^2\left(\frac{x}{2}\right)$ incorrectly to give expressions such as $\frac{1}{2}\tan\left(\frac{x}{2}\right)$ or $\tan\left(\frac{x}{2}\right)$. The majority of candidates, however, were able to apply the limits correctly and recognised the need to give an exact final answer.

Question 4

Work on this topic has shown a marked improvement and the median mark scored by candidates on this question was 8 out of 9. The only errors frequently seen were in differentiating ye^{-2x} implicitly with respect to x . A few candidates failed to read the question correctly and found the equation of the tangent instead of the normal or failed to give their answer to part (b) in the form requested.

Question 5

This question was generally well done with many candidates scoring at least seven or eight of the nine marks available.

In part (a), the majority of candidates were able to use algebra to gain all three marks available with ease. It was disappointing, however, to see a significant minority of candidates at A2 level who were unable to correctly substitute $y = -8$ into the given equation or solve the resulting quadratic to find the correct values for y .

In part (b), implicit differentiation was well handled, with most candidates appreciating the need to apply the product rule to the $12xy$ term although errors in sign occurred particularly with those candidates who had initially rearranged the given equation so that all terms were on the LHS. A few candidates made errors in rearranging their correctly differentiated equation to make $\frac{dy}{dx}$ the subject.

Also some candidates lost either one or two marks when manipulating their correctly substituted $\frac{dy}{dx}$ expressions to find the gradients.

Question 6

This question was generally well answered with about 40% of the candidature gaining all 8 marks.

Whilst a large number of fully correct solutions were seen in part (a), there were a significant number of candidates who struggled to differentiate sine and cosine functions, with expressions such as $\frac{dx}{dt} = -4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = \frac{3}{2}\sin 2t$ being encountered frequently.

Most candidates were able to apply the chain rule to find an expression for $\frac{dy}{dx}$, although the application of $\frac{dy}{dt} \times \frac{dx}{dt}$ was occasionally seen. A small proportion of candidates used the

compound angle formula to rewrite x as $2\sqrt{3}\sin t + 2\cos t$, or likewise the double angle formulae to rewrite y as either $3(2\cos^2 t - 1)$ or $3(\cos^2 t - \sin^2 t)$ or $3(1 - 2\sin^2 t)$ before going on to find their $\frac{dy}{dx}$.

Candidates found part (b) more challenging and a variable range of marks was awarded in this part. Although a few candidates could not proceed further from setting their $\frac{dy}{dx}$ to zero, most

candidates appreciated that the numerator from their $\frac{dy}{dx}$ expression needed to be equated to zero, so resulting in the first method mark. A number of candidates who solved $\sin 2t = 0$, found only one value of t (usually $t = 0$) and then one point (usually $(2, 3)$). A surprisingly large number of candidates found all four correct values of t , but did not realise that they needed to use these values in order to find four sets of coordinates for (x, y) . Candidates who found more than one value for t often relied on the symmetry of the diagram to find all four points rather than a full solution, and this was permitted. A surprising number of stronger candidates did not relate the diagram to the question in part (b). These candidates stopped at finding only two or three sets of coordinates when it was clear from the diagram that there was a total of four points where $\frac{dy}{dx} = 0$. Few candidates also set the denominator equal to zero and used the resulting values of t to find erroneous coordinates.

Question 7

Although there were many correct solutions to part (a), a surprising number of candidates made mistakes in establishing $\frac{dy}{dx}$ from their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$. Both 2 and $2t^3$ were seen and, in many cases, the method used was not clearly shown and this resulted in the loss of both the method and the accuracy marks. If $\frac{dy}{dx}$ was correctly found, the majority were able to complete this part correctly. A significant number, however, failed to read the question and gave the equation of the tangent rather than that of the normal. Part (b) was well done and nearly all could eliminate the parameter. Quite a number of candidates thought that $(e^x)^2$ was e^{x^2} and this often caused a major loss of marks in part (c).

In part (c), the majority of candidates knew the volume formula but the attempts at integration were of a very variable quality. Many used the lead given in part (b) but the resulting squaring out of the brackets was often incorrect. Examples of errors seen are $(e^{2x})^2 = e^{4x^2}$ and $(e^{2x} - 2)^2 = e^{4x} + 4$ or $e^{4x} - 4$. There were also attempts at direct integration, for example

$\int (e^{2x} - 2)^2 dx = \frac{(e^{2x} - 2)^3}{3}$. Those who used parameters often made similar mistakes and

sometimes the $\frac{dx}{dt}$ was omitted. The choice of limits also gave some difficulty; those who integrated using the variable x using the t limits and vice versa. Despite these frequent mistakes, there were many completely correct solutions.

Question 8

Parts (a) and (b) were usually fully correct and few lost marks through failing to work to the accuracy specified in the question. The trapezium rule is well known and the only error commonly seen was obtaining an incorrect width of an individual trapezium.

Part (c) proved more demanding. Many got off to a bad start. The substitution was deliberately given in the form $x = (u - 4)^2 + 1$ so that the essential $\frac{dx}{du} = 2(u - 4)$ could be found easily.

Many, however, rearranged the substitution and obtained $\frac{dx}{du} = \frac{1}{2}(x - 1)^{-\frac{1}{2}}$ and the resulting algebraic manipulations often proved beyond candidates. Those who did complete the substitution often failed to complete the definite integral. An unexpected difficulty was that a number of candidates failed, in the context, to simplify $4 + (u - 4)$ to u .

Many did not see that $\frac{2u - 8}{u}$ reduced to $2 - \frac{8}{u}$ and embarked upon complicated solutions using integration by parts which, although theoretically possible, were rarely completed. The choice of limits also gave some difficulties. The convention is used that a surd is taken as the positive square root. If this were not the case the expression given at the head of the question would be ambiguous. Some however produce limits resulting from negative square roots, 3 and 2, as well as the correct 5 and 6, and did not know which to choose. As in question 6, there was also some confusion in choosing the limits, some choosing the x limits when the u limits were appropriate and vice versa. Despite these difficulties, nearly 34% of the candidates gained full marks for this question.

Statistics for C4 Practice Paper Bronze Level B1

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	4		86	3.43		3.77	3.53	3.27	2.96	2.62	1.98
2	8		78	6.24		7.53	6.69	5.57	4.33	3.17	1.95
3	8	8	81	6.44	7.83	7.51	6.98	6.13	5.01	3.68	1.94
4	9		74	6.68		8.20	7.21	6.01	4.46	2.87	1.25
5	9		83	7.50		8.55	7.84	6.81	5.98	4.85	2.86
6	8		69	5.53	7.62	6.51	5.21	4.09	3.28	2.54	1.20
7	15		75	11.27	14.75	12.91	10.99	8.77	6.67	4.26	2.17
8	14		75	10.54	13.82	11.96	9.77	7.64	6.04	5.14	3.45
	75		77	57.63		66.94	58.22	48.29	38.73	29.13	16.80