

Question Number	Scheme	Marks
1.	$\int_1^3 x^2 dx = \left[\frac{x^3}{3} \ln x \right] - \int \frac{x^3}{3} \frac{1}{x} dx$ $= \left[\frac{x^3}{3} \ln x \right] - \int \frac{x^2}{3} dx$ $= \left[\frac{x^3}{3} \ln x \right] - \frac{1}{9} x^3$ $= 9 \ln 3 - 3 + \frac{1}{9} = 9 \ln 3 - 2\frac{8}{9}$	M1 A1 M1 A1 M1 A1 (6) (6 marks)
2.	<p>(a) $\frac{dV}{dt} = \pm c\sqrt{V}$ or $\frac{dV}{dt} \propto \sqrt{V}$</p> <p>As $V = Ah$, $\frac{dV}{dh} = A$ or $V \propto h$</p> <p>\therefore use chain rule to obtain $\frac{dh}{dt} = -\frac{c}{A}\sqrt{V} = \frac{-c}{\sqrt{A}}\sqrt{h} = -k\sqrt{h}$</p> <p>(b) $\int \frac{dh}{h} = -\int k dt$</p> $2h^{\frac{1}{2}} = A - kt$ $h^{\frac{1}{2}} = \frac{A}{2} - \frac{kt}{2}$ $h = (A - Bt)^2 \quad *$ <p>(c) $t = 0, h = 1: \quad A = 1$</p> $t = 5, h = 0.5: \quad 0.5 = (1 - 5B)^2$ $B = \frac{(1 - \sqrt{0.5})}{5} \quad (B = 0.0586)$ $h = 0, t = \frac{A}{B} = \frac{5}{1 - \sqrt{0.5}} = 17.1 \text{ min}$ <p>(d) $h = \frac{A^2}{4} = 0.25 \text{ m}$</p>	M1 M1 A1 (3) M1 M1 A1 A1 (4) B1 B1 B1 (3) M1 A1 (2) (12 marks)

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3.	<p>(a) $\cos(A + A) = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1$</p> <p>(b) $[x = 2, \theta = \frac{\pi}{4}; x = \sqrt{6}, \theta = \frac{\pi}{3}]$ $x = 2\sqrt{2} \sin \theta, \frac{dx}{d\theta} = 2\sqrt{2} \cos \theta$ $\int \sqrt{8 - x^2} dx = \int 2\sqrt{2} \cos \theta \cdot 2\sqrt{2} \cos \theta d\theta = \int 8 \cos^2 \theta d\theta$ Using $\cos 2\theta = 2 \cos^2 \theta - 1$ to give $\int 4(1 + \cos 2\theta) d\theta = 4\theta + 2 \sin 2\theta$ Substituting limits to give $\frac{1}{3}\pi + \sqrt{3} - 2$ or given result</p> <p>(c) $\frac{dy}{d\theta} = \frac{-2 \sin 2\theta}{1 + \cos 2\theta}$ Using the chain rule, with $\frac{dx}{d\theta} = \sec \theta \tan \theta$ to give $\frac{dy}{dx} (= -2 \cos \theta)$ Gradient at the point where $\theta = \frac{\pi}{3}$ is -1 Equation of tangent is $y + \ln 2 = -(x - 2)$ (or equivalent answer)</p>	<p>M1 A1 (2)</p> <p>B1 B1</p> <p>M1 A1 M1 A1 ft (7)</p> <p>B1 M1 A1 ft M1 A1 (5)</p> <p>(14 marks)</p>
4.	<p>(a) A: $y = 16$, B: $y = 2$</p> <p>(b) $y(x - 3) = 4$, $yx - 3y = 4$ $x = \frac{3y + 4}{y}$ (*)</p> <p>(c) $x^2 = \left(3 + \frac{4}{y}\right)^2 = 9 + \frac{24}{y} + \frac{16}{y^2}$ $\int x^2 dy = \int (9 + 24y^{-1} + 16y^{-2}) dy$ $= 9y - \frac{16}{y} + 24 \ln y$ $\left[9y - \frac{16}{y} + 24 \ln y\right]_2^{16} = (144 + 24 \ln 16 - 1) - (18 + 24 \ln 2 - 8)$ $V = \pi(133 + 24 \ln 8)$</p> <p>(d) $V \times 27 \approx 15\,500$ (*)</p>	<p>B1 (1)</p> <p>M1 (1)</p> <p>M1 A1 M1 A1ft, A1ft M1 A1 (7)</p> <p>M1 A1 (2)</p> <p>(11 marks)</p>

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5.	(a) $\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}), \overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$ (or $\overrightarrow{BA}, \overrightarrow{BC},$ or $\overrightarrow{AB}, \overrightarrow{BC},$ stated in above form or column vector form)	M1 A1
	$\cos \hat{ABC} = \frac{\overrightarrow{CB} \cdot \overrightarrow{AB}}{ \overrightarrow{CB} \overrightarrow{AB} } = -\frac{4}{9}$	M1 A1 (4)
	(b) Area of $\triangle ABC = \frac{1}{2} \times 3 \times 3 \times \sin B$	M1
	$\sin B = \sqrt{1 - \frac{16}{81}} = \frac{\sqrt{65}}{9}$	M1
	$\therefore \text{Area} = \frac{1}{2} \sqrt{65}$	A1 (3)
	(c) $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} \quad \overrightarrow{DC} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ or given in alternative form with attempt at scalar product	M1
	$\overrightarrow{AC} \cdot \overrightarrow{DC} = 0,$ therefore the lines are perpendicular.	A1 (2)
	(d) $\overrightarrow{AD} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \quad \overrightarrow{DB} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ and $AD:DB = 2:-1$ (allow 2:1)	M1 A1 (2)
		(11 marks)

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<p>6. (a)</p> <table border="1" data-bbox="284 253 1246 331"> <tr> <td>Distance from one side (m)</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td>Height (m)</td> <td>0</td> <td>6.13</td> <td>7.80</td> <td>7.80</td> <td>6.13</td> <td>0</td> </tr> </table> <p style="text-align: right;">“y” = 7.80 when “x” = 4 or 6</p> <p style="text-align: right;">Symmetry</p> <p>(b) Estimate area = $\frac{2}{2} [0 + 2(6.13 + 7.80 + 7.80 + 6.13)]$ $= 55.7 \text{ m}^2$</p> <p>(c) $140 - (b) = 84.3 \text{ m}^2$</p> <p>(d) Over-estimate; reason, e.g. area under curve is under-estimate (due to curvature)</p>	Distance from one side (m)	0	2	4	6	8	10	Height (m)	0	6.13	7.80	7.80	6.13	0	<p>B1 B1 ft (2)</p> <p>B1 M1 A1ft A1 (4)</p> <p>A1 ft (1)</p> <p>B1 B1 (2)</p> <p style="text-align: right;">(9 marks)</p>
Distance from one side (m)	0	2	4	6	8	10									
Height (m)	0	6.13	7.80	7.80	6.13	0									
<p>7. (a) Method using either</p> $\frac{A}{(1-x)} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2} \text{ or } \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$ <p>$A = 1, C = 10, B = 2$ or $D = 4$ and $E = 16$</p> <p>(b) $\int \left[\frac{1}{1-x} + \frac{2}{2x+3} + 10(2x+3)^{-2} \right] dx$ or $\int \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2} dx$</p> <p>$-\ln 1-x + \ln 2x+3 - 5(2x+3)^{-1} (+c)$ or</p> <p>$-\ln 1-x + \ln 2x+3 - (2x+8)(2x+3)^{-1} (+c)$</p> <p>(c) $(1-x)^{-1} + 2(3+2x)^{-1} + 10(3+2x)^{-2} =$ $1 + x + x^2 + \dots$ $+ \frac{2}{3} \left(1 - \frac{2x}{3} + \frac{4x^2}{9} \dots \right)$ $+ \frac{10}{9} \left(1 + (-2) \left(\frac{2x}{3} \right) + \frac{(-2)(-3)}{2} \left(\frac{2x}{3} \right)^2 + \dots \right)$ $= \frac{25}{9} - \frac{25}{27}x + \frac{25}{9}x^2 \dots$</p>	<p>M1 B1 A1 A1 (4)</p> <p>M1 M1 A1ft A1ft A1ft (5)</p> <p>M1 A1 M1 A1 A1 M1 A1 (7)</p> <p style="text-align: right;">(16 marks)</p>														