

1. The function f is given by

$$f(x) = \frac{3(x+1)}{(x+2)(x-1)}, x \in \mathbb{R}, x \neq -2, x \neq 1.$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence, or otherwise, prove that $f'(x) < 0$ for all values of x in the domain.

(3)

2. The curve C is described by the parametric equations

$$x = 3 \cos t, \quad y = \cos 2t, \quad 0 \leq t \leq \pi$$

(a) Find a cartesian equation of the curve C .

(2)

(b) Draw a sketch of the curve C .

(2)

3. Use the substitution $x = \sin \theta$ to show that, for $|x| \leq 1$,

$$\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx = \frac{x}{(1-x^2)^{\frac{1}{2}}} + c, \text{ where } c \text{ is an arbitrary constant.}$$

(6)

4. A measure of the effective voltage, M volts, in an electrical circuit is given by

$$M^2 = \int_0^1 V^2 dt$$

where V volts is the voltage at time t seconds. Pairs of values of V and t are given in the following table.

t	0	0.25	0.5	0.75	1
V	-48	207	37	-161	-29
V^2					

Use the trapezium rule with five values of V^2 to estimate the value of M .

(6)

5.

Figure 1

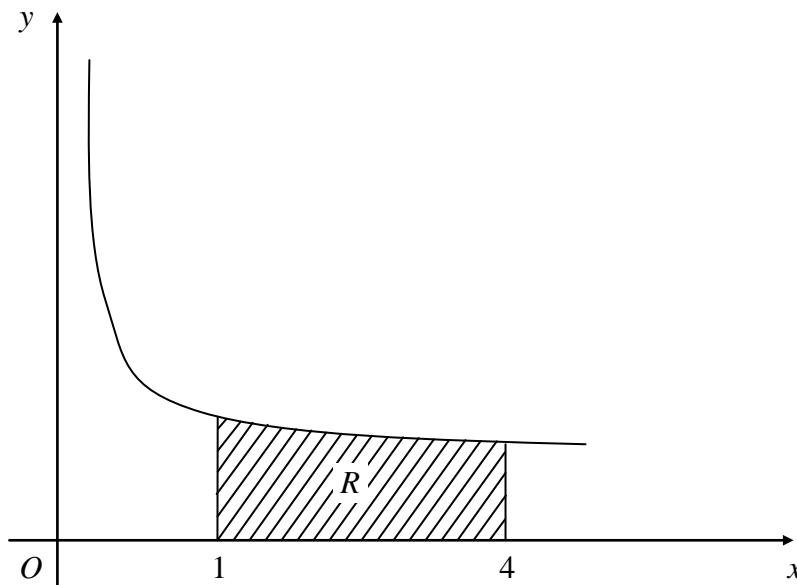


Figure 1 shows part of the curve with equation $y = 1 + \frac{1}{2\sqrt{x}}$. The shaded region R , bounded by the curve, that x -axis and the lines $x = 1$ and $x = 4$, is rotated through 360° about the x -axis. Using integration, show that the volume of the solid generated is $\pi(5 + \frac{1}{2} \ln 2)$.

(8)

6. Liquid is poured into a container at a constant rate of $30 \text{ cm}^3 \text{ s}^{-1}$. At time t seconds liquid is leaking from the container at a rate of $\frac{2}{15} V \text{ cm}^3 \text{ s}^{-1}$, where $V \text{ cm}^3$ is the volume of liquid in the container at that time.

(a) Show that

$$-15 \frac{dV}{dt} = 2V - 450. \tag{3}$$

Given that $V = 1000$ when $t = 0$,

(b) find the solution of the differential equation, in the form $V = f(t)$. (7)

(c) Find the limiting value of V as $t \rightarrow \infty$. (1)

7. The curve C has equation $y = \frac{x}{4 + x^2}$.

(a) Use calculus to find the coordinates of the turning points of C . (5)

Using the result $\frac{d^2y}{dx^2} = \frac{2x(x^2 - 12)}{(4 + x^2)^3}$, or otherwise,

(b) determine the nature of each of the turning points. (3)

(c) Sketch the curve C . (3)

8. (i) Given that $\cos(x + 30)^\circ = 3 \cos(x - 30)^\circ$, prove that $\tan x^\circ = -\frac{\sqrt{3}}{2}$. (5)

(ii) (a) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$. (3)

(b) Verify that $\theta = 180^\circ$ is a solution of the equation $\sin 2\theta = 2 - 2 \cos 2\theta$. (1)

(c) Using the result in part (a), or otherwise, find the other two solutions, $0 < \theta < 360^\circ$, of the equation using $\sin 2\theta = 2 - 2 \cos 2\theta$. (4)

9. The equations of the lines l_1 and l_2 are given by

$$l_1: \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

$$l_2: \mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$

where λ and μ are parameters.

- (a) Show that l_1 and l_2 intersect and find the coordinates of Q , their point of intersection.

(6)

- (b) Show that l_1 is perpendicular to l_2 .

(2)

The point P with x -coordinate 3 lies on the line l_1 and the point R with x -coordinate 4 lies on the line l_2 .

- (c) Find, in its simplest form, the exact area of the triangle PQR .

(6)

END