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Mathematics MPC4

(Specification 6360)

Pure Core 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)(i)	$5-8x = A(1-3x)+B(2+x)$ $x=-2 \quad\quad x=\frac{1}{3}$ $A=3 \quad\quad B=1$	M1 m1 A1	3	Two values of x used to find values for A and B
(ii)	$\int_{-1}^{0} \frac{3}{2+x}+\frac{1}{1-3x}\,\mathrm{d}x$ $= 3\ln(2+x)-\frac{1}{3}\ln(1-3x)$ $=\left(3\ln 2-\frac{1}{3}\ln 1\right)-\left(3\ln 1-\frac{1}{3}\ln 4\right)$ $=3\ln 2+\frac{1}{3}\ln 4$ $=\frac{11}{3}\ln 2$	 M1 m1 A1ft A1ft	4	$a\ln(2+x)+b\ln(1-3x)$ where a and b are constants $\mathrm{f}(0)-\mathrm{f}(-1)$ used ft A and B ft $\left(A+\frac{2}{3}B\right)\ln 2$
(b)(i)	$(C=)\,2$	B1	1	
(ii)	$\int \frac{9-18x-6x^2}{2-5x-3x^2}\,\mathrm{d}x = \int C\mathrm{d}x + \int \frac{5-8x}{2-5x-3x^2}\,\mathrm{d}x$ $\int_{-1}^{0} \frac{9-18x-6x^2}{2-5x-3x^2}\,\mathrm{d}x = 2+\frac{11}{3}\ln 2$	M1 A1ft	2	Seen or implied. Allow $\pm C+\int \frac{5-8x}{2-5x-3x^2}\mathrm{d}x$ Accept $2+3\ln 2+\frac{1}{3}\ln 4$ ft $2+$ candidate's answer to part (a)(ii) if exact.
(a)(i)	**Alternative** $5-8x = A(1-3x)+B(2+x)$ $5=A+2B$ $-8=-3A+B$ $A=3 \quad B=1$	 (M1) (m1) (A1)	(3)	Set up simultaneous equations and solve.
	Total		**10**	

Q	Solution	Marks	Total	Comments
2(a)(i)	$h^2 = 2^2 + \sqrt{5}^2 = 9 \Rightarrow h = 3 \Rightarrow \sin\alpha = \frac{2}{3}$	B1		Pythagoras used or all of $2, \sqrt{5}, 3$ seen correctly on triangle **AG**
	$\cos\alpha = \frac{\sqrt{5}}{3}$	B1	2	$\frac{\sqrt{5}}{3}$ or $\sqrt{\frac{5}{9}}$ or $\frac{5}{3\sqrt{5}}$ seen
(ii)	$\sin 2\alpha = 2\sin\alpha\cos\alpha$	M1		Correct formula seen or implied
	$= \left(2 \times \frac{2}{3} \times \frac{\sqrt{5}}{3}\right) = \frac{4}{9}\sqrt{5}$	A1	2	Must see $\frac{\sqrt{5}}{3}$ here or in part (a)(i) Accept $\frac{4}{3}\sqrt{\frac{5}{9}}$
(b)	$\cos\beta = \frac{2}{\sqrt{5}}$ or $\sin\beta = \frac{1}{\sqrt{5}}$	B1		Either correct. Accept $\sqrt{\frac{4}{5}}$, $\frac{\sqrt{5}}{5}$
	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	M1		Correct formula seen or implied.
	$= \frac{\sqrt{5}}{3} \times \frac{2}{\sqrt{5}} + \frac{2}{3} \times \frac{1}{\sqrt{5}}$	A1		All correct
	$= \frac{2}{15}\left(5 + \sqrt{5}\right)$	A1	4	$k = 5$ with previous A mark awarded
(a)(i)	**Alternative** $\text{cosec}^2\alpha = 1 + \cot^2\alpha = 1 + \frac{5}{4} = \frac{9}{4}$			
	$\text{cosec}\,\alpha = \frac{3}{2}$ $\sin\alpha = \frac{2}{3}$	(B1)		Must be positive
	$\sec^2\alpha = 1 + \tan^2\alpha = 1 + \frac{4}{5} = \frac{9}{5}$			
	$\sec\alpha = \frac{3}{\sqrt{5}}$ $\cos\alpha = \frac{\sqrt{5}}{3}$	(B1)		Must be posiitve
	Total		**8**	

Q	Solution	Marks	Total	Comments
3(a)	$(1+6x)^{-\frac{1}{3}} = 1 + \left(-\tfrac{1}{3}\right)6x + kx^2$	M1		
	$= 1 - 2x + 8x^2$	A1	2	
(b)(i)	$(27+6x)^{-\frac{1}{3}} = 27^{-\frac{1}{3}}\left(1+\tfrac{6}{27}x\right)^{-\frac{1}{3}}$	B1		
	$\left(1+\tfrac{6}{27}x\right)^{-\frac{1}{3}} = 1 + \left(-\tfrac{1}{3}\times\tfrac{6}{27}x\right) + \left(-\tfrac{1}{3}\times-\tfrac{4}{3}\right)\tfrac{1}{2}\left(\tfrac{6}{27}x\right)^2$	M1		Condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}} = \dfrac{1}{3} - \dfrac{2}{81}x + \dfrac{8}{2187}x^2$	A1	3	
(ii)	$\left(\sqrt[3]{\dfrac{2}{7}} = \dfrac{2}{\sqrt[3]{28}} \Rightarrow 27+6x = 28 \Rightarrow x = \dfrac{1}{6}\right)$			
	$\sqrt[3]{\dfrac{1}{28}} = \dfrac{1}{3} - \dfrac{2}{81}\times\dfrac{1}{6} + \dfrac{8}{2187}\times\left(\dfrac{1}{6}\right)^2 \ (\approx 0.3293..)$	M1		Substitute $x = \frac{1}{6}$ into expansion from (b)(i)
	$\left(\sqrt[3]{\dfrac{2}{7}} \approx 2\times 0.3293197.. = 0.6586394...\right)$			
	$= 0.658639 \quad (6\text{dp})$	A1	2	CSO
	Alternatives			
(b)(i)	$(27+6x)^{-\frac{1}{3}} = 27^{-\frac{1}{3}}\left(1+\tfrac{6}{27}x\right)^{-\frac{1}{3}}$	(B1)		
	$\left(1+\tfrac{6}{27}x\right)^{-\frac{1}{3}} = 1 - 2\times\tfrac{1}{27}x + 8\times\left(\tfrac{1}{27}\right)^2 x^2$	(M1)		Replace x with $\frac{1}{27}x$,**not** $\frac{6}{27}x$, in expansion from (a); condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}} = \tfrac{1}{3} - \tfrac{2}{81}x + \tfrac{8}{2187}x^2$	(A1)	(3)	
(b)(i)	$(27+6x)^{-\frac{1}{3}} = 27^{-\frac{1}{3}} + \left(-\tfrac{1}{3}\right)27^{-\frac{4}{3}}\times 6x$ $+\left(-\tfrac{1}{3}\right)\times\left(-\tfrac{4}{3}\right)\tfrac{1}{2}27^{-\frac{7}{3}}\times(6x)^2$	(M1)		Use result from formula book; Condone missing brackets and one error
	$(27+6x)^{-\frac{1}{3}} = \tfrac{1}{3} - \tfrac{2}{81}x + \tfrac{8}{2187}x^2$	(A2)	(3)	A1 not available
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)	$\left(\frac{dx}{dt}=\right)-16e^{-2t}$ $\left(\frac{dy}{dt}=\right)4e^{2t}$	B1		Both derivatives correct
	$\frac{dy}{dx}=\frac{\text{candidate's }\frac{dy}{dt}}{\text{candidate's }\frac{dx}{dt}}$	M1		chain rule used correctly
	$\frac{dy}{dx}=\frac{4e^{2t}}{-16e^{-2t}}$ $\left(=-\frac{1}{4}e^{4t}\right)$	A1	3	Simplification not required $4e^{2t}$ and $-16e^{-2t}$ must be seen. ISW.
(b)(i)	$t=\ln 2$ gradient at $P=-4$	B1ft	1	B0 if ISW result is used.
(ii)	coordinates of P $x=-2$	B1		
	$y=12$	B1	2	
(iii)	gradient of normal $=\frac{1}{4}$	B1ft		ft gradient at P
	equation of normal $\frac{y-12}{x--2}=\frac{1}{4}$	M1		Set up equation of normal
	at $y=0$ $x=-50$	A1	3	$(-50,0)$ CSO
(c)	$xy+4y-4x=\left(8e^{-2t}-4\right)\left(2e^{2t}+4\right)$ $+4\left(2e^{2t}+4\right)-4\left(8e^{-2t}-4\right)$	M1		Write $xy+4y-4x$ in terms of t.
	$=16+32e^{-2t}-8e^{2t}-16$ $+8e^{2t}+16-32e^{-2t}+16$	m1		Multiply out and simplify using $e^{-2t}e^{2t}=1$ PI
	$(xy+4y-4x)=32$	A1	3	Correct working to $k=32$ $k=32$ NMS; SC1
(c)	**Alternative** $e^{-2t}=\frac{x+4}{8}$ or $e^{2t}=\frac{y-4}{2}$	(M1)		Write e^{-2t} in terms of x or e^{2t} in terms of y. Condone sign errors
	$e^{-2t}e^{2t}=\left(\frac{x+4}{8}\right)\left(\frac{y-4}{2}\right)$ $=\frac{xy+4y-4x-16}{16}=1$	(m1)		Multiply out and use $e^{-2t}e^{2t}=1$
	$xy+4y-4x=32$	(A1)	(3)	All correct with $k=32$
	Other alternatives are possible			
	Total		**12**	

Q	Solution	Marks	Total	Comments
5(a)	$\text{f}\left(-\frac{3}{2}\right)=4\left(-\frac{3}{2}\right)^3-11\left(-\frac{3}{2}\right)-3$	M1		$x=-\frac{3}{2}$ substituted
	$=-4\times\dfrac{27}{8}+\dfrac{33}{2}-3=0\Rightarrow$ factor	A1	2	Processing, $=0$ and conclusion
(b)	$2x^2-3x-1$	M1A1	2	M1 for any two of a,b,c correct
(c)(i)	$2\cos 2\theta\sin\theta+9\sin\theta+3$ $=2\left(1-2\sin^2\theta\right)\sin\theta+9\sin\theta+3$	M1		$\cos 2\theta$ expanded ; ACF and substituted
	$=2\sin\theta-4\sin^3\theta+9\sin\theta+3$	m1		All in terms of $\sin\theta$ or x and simplified to a cubic expression.
	$\sin\theta=x\Rightarrow 4x^3-11x-3=0$	A1	3	Reverse signs and express in x correctly **AG**
(c)(ii)	$2x^2-3x-1=0\Rightarrow x=\dfrac{3\pm\sqrt{17}}{4}$	M1		Use formula correctly to solve $ax^2+bx+c=0$ from part (b)
	$x=\dfrac{3-\sqrt{17}}{4}$ or $-0.28...$	A1		
	$\theta=196°$ and $344°$	A1		Both required and no others in range; condone greater accuracy Ignore solutions out of range.
	$x=\dfrac{3+\sqrt{17}}{4}$ no solutions for $\sin\theta$ $x=-\dfrac{3}{2}$ no solutions for $\sin\theta$	E1	4	Must have three correct roots and reject both other roots from cubic equation.
	Total		**11**	

Q	Solution	Marks	Total	Comments
6(a)	$\lambda = -1$	B1		$\lambda = -1$ seen or implied
	$\lambda = -1$ verified in all three components	B1	2	Shown
(b)	$\pm\begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$	B1		$\overrightarrow{AB}$ or $\overrightarrow{BA}$ correct
	$\mathbf{r} = \overrightarrow{OA} + \mu\overrightarrow{AB} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \mu\begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$	M1		$\mathbf{a} + \mu\mathbf{d}$
		A1ft	3	OE; ft on $\overrightarrow{AB}$ or $\overrightarrow{BA}$
(c)	$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ $= \begin{bmatrix} 3-2\mu \\ -2-3\mu \\ 4+2\mu \end{bmatrix} - \begin{bmatrix} -4 \\ 5 \\ -1 \end{bmatrix} \quad \left(= \begin{bmatrix} 7-2\mu \\ -7-3\mu \\ 5+2\mu \end{bmatrix} \right)$	B1		$\pm\overrightarrow{CD}$ in terms of μ OE
	$\overrightarrow{CD}.\overrightarrow{AB} = 0$ or $\overrightarrow{CD}.\overrightarrow{AD} = 0$ $= \left(\begin{bmatrix} 3-2\mu \\ -2-3\mu \\ 4+2\mu \end{bmatrix} - \begin{bmatrix} -4 \\ 5 \\ -1 \end{bmatrix} \right) . \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} = 0$	M1		Candidate's $\overrightarrow{CD}$ sp with candidate's $\overrightarrow{AB}$ or $\overrightarrow{AD}$ $= 0$ PI by a solution for μ
	$-14 + 4\mu + 21 + 9\mu + 10 + 4\mu = 0$ $17 + 17\mu = 0$ $\mu = -1$	m1A1		Expand sp to an equation in μ and solve for μ
	D is at $(5,1,2)$	A1	5	Accept as a column vector
(d)	$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = \overrightarrow{OA} + 3\overrightarrow{AD}$	M1		Accept $AE = 3AD$
	$\overrightarrow{OE} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ $\quad E$ is at $(9,7,-2)$	A1		Accept as a column vector
	Or $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = \overrightarrow{OA} + 3\overrightarrow{DA}$	M1		Accept $AE = 3DA$
	$\overrightarrow{OE} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + 3\begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$ $\quad E$ is at $(-3,-11,10)$	A1	4	Accept as a column vector.

Q	Solution	Marks	Total	Comments
6(c)	**Alternative using Pythagoras** $\overrightarrow{CD} = \overrightarrow{OD} - \mu\overrightarrow{OC}$ $= \begin{bmatrix} 3-2\mu \\ -2-3\mu \\ 4+2\mu \end{bmatrix} - \begin{bmatrix} -4 \\ 5 \\ -1 \end{bmatrix} \quad \left(= \begin{bmatrix} 7-2\mu \\ -7-3\mu \\ 5+2\mu \end{bmatrix} \right)$	(B1)		$\pm\overrightarrow{CD}$ in terms of μ
	$AC^2 = AD^2 + CD^2$ $\left(7^2+7^2+5^2\right) = \mu^2\left(2^2+3^2+2^2\right)$ $+\left(\left(7-2\mu\right)^2+\left(7+3\mu\right)^2+\left(5+2\mu\right)^2\right)$	(M1)		$\overrightarrow{AC} = \begin{bmatrix} -7 \\ 7 \\ -5 \end{bmatrix} \quad \overrightarrow{AD} = \begin{bmatrix} -2\mu \\ -3\mu \\ 2\mu \end{bmatrix}$ Correct Pythagoras expression in terms of μ;
	$123 = 17\mu^2 + 123 + 34\mu + 17\mu^2$ $0 = 34\mu^2 + 34\mu$	(m1)		Multiply out and solve to find a value for μ
	$\mu = -1 \quad (\mu = 0$ is point $A)$	(A1)		$\mu = -1$
	D is at $(5,1,2)$	(A1)	(5)	
6(d)	**Alternative** $\lvert\overrightarrow{DE}\rvert = 2\lvert\overrightarrow{AD}\rvert \Rightarrow \overrightarrow{OE} = \overrightarrow{OD} + 2\overrightarrow{AD}$	(M1)		
	$\overrightarrow{OE} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \quad E$ is at $(9,7,-2)$	(A1)		
	$\lvert DE\rvert = 4\lvert DA\rvert \Rightarrow \overrightarrow{OE} = \overrightarrow{OD} + 4\overrightarrow{DA}$	(M1)		
	$\overrightarrow{OE} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix} \quad E$ is at $(-3,-11,10)$	(A1)	(4)	
	Total		**14**	

Q	Solution	Marks	Total	Comments
7	$\frac{dh}{dt}$ $a = 1.3$ or $a = -1.3$ $k = \frac{\pi}{6}$ or $k = \frac{2\pi}{12}$	B1 B1 B1	1 1 1	$\frac{dh}{dt}$ seen
	Total		**3**	
8 (a)	$\int t\cos\left(\frac{\pi}{4}t\right)\,dt$	M1		Clear attempt to use parts $u = t$ $\frac{dv}{dt} = \cos\left(\frac{\pi}{4}t\right)$ $\frac{du}{dt} = 1$ $v = k\sin\left(\frac{\pi}{4}t\right)$
	$= t \times \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right) - \frac{4}{\pi}\int \sin\left(\frac{\pi}{4}t\right)\,(dt)$	A1		Must be in terms of π
	$= pt\sin\left(\frac{\pi}{4}t\right) + q\cos\left(\frac{\pi}{4}t\right)$	m1		Correct form, any non-zero values for p, q
	$= t \times \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right) + \frac{4}{\pi} \times \frac{4}{\pi}\cos\left(\frac{\pi}{4}t\right)$	A1	4	Any correct unsimplified form. Constant not required
(b)	$\int 32x\,dx = \int t\cos\left(\frac{\pi}{4}t\right)\,dt$	B1		Correct separation and notation.
	$16x^2 =$	B1		$\frac{x^2}{2}$ if 32 not brought over; allow $32 \times \frac{x^2}{2}$
	$t \times \frac{4}{\pi}\sin\left(\frac{\pi}{4}t\right) + \frac{16}{\pi^2}\cos\left(\frac{\pi}{4}t\right) + C$	M1		Equate to result from part (a) with constant and use $(0,4)$ to find a value for the constant
	$C = 256 - \frac{16}{\pi^2}$	A1		Accept $C = 254$ or better $(254.37886...)$
	$t = 45$ $16x^2 = -40.514... - 1.146... + 254.378...$ $= 212.718...$ $x^2 = 13.294...$ $x = 3.646... = 3.65$ m or $(\text{height} =)$ 365 cm	m1A1	6	Substitute $t = 45$ into $kx^2 = pt\sin\left(\frac{\pi}{4}t\right) + q\cos\left(\frac{\pi}{4}t\right) + C$ $p \neq 0$, $q \neq 0$ and calculate x. CSO
	Total		**10**	
	TOTAL		**75**	