General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MPC4

Unit Pure Core 4

Tuesday 15 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 8x^3 + 6x^2 - 14x - 1$.

Find the remainder when $f(x)$ is divided by $(4x - 1)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 8x^3 + 6x^2 - 14x + d$.

(i) Given that $(4x - 1)$ is a factor of $g(x)$, find the value of the constant d. *(2 marks)*

(ii) Given that $g(x) = (4x - 1)(ax^2 + bx + c)$, find the values of the integers a, b and c. *(3 marks)*

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2 A curve is defined by the parametric equations

$$x = 1 - 3t, \qquad y = 1 + 2t^3$$

(a) Find $\dfrac{dy}{dx}$ in terms of t. *(3 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(2 marks)*

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3 (a) (i) Express $\dfrac{7x - 3}{(x + 1)(3x - 2)}$ in the form $\dfrac{A}{x + 1} + \dfrac{B}{3x - 2}$. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{7x - 3}{(x + 1)(3x - 2)}\, dx$. *(2 marks)*

(b) Express $\dfrac{6x^2 + x + 2}{2x^2 - x + 1}$ in the form $P + \dfrac{Qx + R}{2x^2 - x + 1}$. *(3 marks)*

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4 (a) (i) Find the binomial expansion of $(1 + x)^{\frac{3}{2}}$ up to and including the term in x^2. *(2 marks)*

(ii) Find the binomial expansion of $(16 + 9x)^{\frac{3}{2}}$ up to and including the term in x^2. *(3 marks)*

(b) Use your answer to part **(a)(ii)** to show that $13^{\frac{3}{2}} \approx 46 + \dfrac{a}{b}$, stating the values of the integers a and b. *(2 marks)*

5 (a) (i) Show that the equation $3\cos 2x + 2\sin x + 1 = 0$ can be written in the form

$$3\sin^2 x - \sin x - 2 = 0$$ *(3 marks)*

(ii) Hence, given that $3\cos 2x + 2\sin x + 1 = 0$, find the possible values of $\sin x$. *(2 marks)*

(b) (i) Express $3\cos 2x + 2\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving α to the nearest 0.1°. *(3 marks)*

(ii) Hence solve the equation

$$3\cos 2x + 2\sin 2x + 1 = 0$$

for all solutions in the interval $0° < x < 180°$, giving x to the nearest 0.1°. *(3 marks)*

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6 A curve has equation $x^3y + \cos(\pi y) = 7$.

(a) Find the exact value of the x-coordinate at the point on the curve where $y = 1$. *(2 marks)*

(b) Find the gradient of the curve at the point where $y = 1$. *(5 marks)*

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7 The point A has coordinates $(4, -3, 2)$.

The line l_1 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$.

The point B lies on l_2 where $\mu = 2$.

(a) Find the vector $\overrightarrow{AB}$. *(3 marks)*

(b) (i) Show that the lines l_1 and l_2 intersect. *(4 marks)*

(ii) The lines l_1 and l_2 intersect at the point P. Find the coordinates of P. *(1 mark)*

(c) The point C lies on a line which is parallel to l_1 and which passes through the point B. The points A, B, C and P are the vertices of a parallelogram.

Find the coordinates of the two possible positions of the point C. *(4 marks)*

Turn over ▶

8 (a) Solve the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

given that $x = 80$ when $t = 0$. Give your answer in the form $x = \mathrm{f}(t)$. *(6 marks)*

(b) A fungus is spreading on the surface of a wall. The proportion of the wall that is unaffected after time t hours is x%. The rate of change of x is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

At $t = 0$, the proportion of the wall that is unaffected is 80%. Find the proportion of the wall that will still be unaffected after 60 hours. *(2 marks)*

(c) A biologist proposes an alternative model for the rate at which the fungus is spreading on the wall. The total surface area of the wall is $9\,\mathrm{m}^2$. The surface area that is **affected** at time t hours is $A\,\mathrm{m}^2$. The biologist proposes that the rate of change of A is proportional to the product of the surface area that is affected and the surface area that is unaffected.

(i) Write down a differential equation for this model.

(You are not required to solve your differential equation.) *(2 marks)*

(ii) A solution of the differential equation for this model is given by

$$A = \frac{9}{1 + 4\mathrm{e}^{-0.09t}}$$

Find the time taken for 50% of the area of the wall to be affected. Give your answer in hours to three significant figures. *(4 marks)*

END OF QUESTIONS