General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

Thursday 25 January 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. *(2 marks)*

(ii) Hence find $\frac{dy}{dx}$ in terms of t. *(2 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. *(2 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. *(2 marks)*

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. *(2 marks)*

3 (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3 \sin x - \cos 2x = 2 \sin^2 x + 3 \sin x - 1$ for all values of x. *(2 marks)*

(ii) Solve the equation $3 \sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

4 (a) (i) Express $\dfrac{3x-5}{x-3}$ in the form $A+\dfrac{B}{x-3}$, where A and B are integers. *(2 marks)*

(ii) Hence find $\displaystyle\int \frac{3x-5}{x-3}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Express $\dfrac{6x-5}{4x^2-25}$ in the form $\dfrac{P}{2x+5}+\dfrac{Q}{2x-5}$, where P and Q are integers. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{6x-5}{4x^2-25}\,\mathrm{d}x$. *(3 marks)*

5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2+\frac{1}{4}x-\frac{1}{32}x^2$ for small values of x. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \dfrac{599}{288}$. *(2 marks)*

6 The points A, B and C have coordinates $(3,-2,4)$, $(5,4,0)$ and $(11,6,-4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle ABC is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line l has equation $\mathbf{r}=\begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix}+\lambda\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.

(i) Verify that C lies on l. *(2 marks)*

(ii) Show that AB is parallel to l. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D. *(3 marks)*

Turn over for the next question

7 (a) Use the identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x, $\tan 2x \neq 0$. *(4 marks)*

8 (a) (i) Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}t} = y\sin t$ to obtain y in terms of t. *(4 marks)*

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50\mathrm{e}^{-(1+\cos t)}$. *(3 marks)*

(b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\sin t$$

(i) Given that this height is 50 cm after π seconds, find, to the nearest centimetre, the height of the water after 6 seconds. *(2 marks)*

(ii) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$ and hence verify that the water reaches a maximum height after π seconds. *(4 marks)*

END OF QUESTIONS