Version 1.0: 0106

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)(i)	$f(1) = 0$	B1	1	
(ii)	$f(-2) = -24 + 8 + 14 + 2 = 0$	B1	1	
(iii)	$\dfrac{(x-1)(x+2)}{3x^3+2x^2-7x+2} = \dfrac{(x-1)(x+2)}{(x-1)(x+2)(ax+b)}$	B1		Recognising $(x-1), (x+2)$ as factors PI
	$ax^3 = 3x^3$ $\quad -2b = 2$	B1	3	a
	$a = 3$ $\quad b = -1$	B1		b
				Or By division M1 attempt started M1 complete division A1 Correct answers
(b)	Use $\frac{1}{3}$	B1		
	$3\left(\frac{1}{3}\right)^3 + 2\left(\frac{1}{3}\right)^2 - 7\times\frac{1}{3} + d = 2$	M1		Remainder Th$^{\underline{M}}$ with $\pm\frac{1}{3,}\pm 3$
	$d = 4$	A1F	3	Ft on $-\frac{1}{3}\left(\text{answer} - \frac{4}{9}\right)$ **Or** by division M1 M1 A1 as above
	Total		**8**	
2(a)	$\dfrac{dy}{dt} = \dfrac{-2}{t^2}$ $\quad \dfrac{dx}{dt} = -4$	M1A1		
	$\dfrac{dy}{dx} = \dfrac{dy}{dt}\cdot\dfrac{1}{\frac{dx}{dt}} = \dfrac{1}{2t^2}$	m1		Use chain rule
		A1F	4	Follow on use of chain rule (if f (t)) **Or** eliminate t : M1 $y = f(x)$ attempt to differentiate M1A1 chain rule A1F reintroduce t
(b)	$t = 2$ $\quad m_T = \frac{1}{8}$	B1F		follow on gradient (possibly used later)
	$x = -5$ $\quad y = 2$	B1		
	$y - 2 = \frac{1}{8}(x+5)$	M1		Their $(x, y), m$
	$x - 8y + 21 = 0$	A1F	4	Ft on (x, y) and m
(c)	$x - 3 = -4t$ $\quad y - 1 = \dfrac{2}{t}$	M1		PI
	$(x-3)(y-1) = -4t\times\dfrac{2}{t} = (-8)$	M1		Attempt to eliminate t
		A1	3	AG convincingly obtained
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$R=\sqrt{13}$ Or 3.6	B1	1	
(b)	$\frac{\sin\alpha}{\cos\alpha}=\tan\alpha=\frac{2}{3}$ $\alpha\approx 33.7$	M1A1	2	Allow M1 for tan $\alpha=\frac{-2}{3}$ or $\pm\frac{3}{2}$ AG convincingly obtained
(c)	maximum value $=\sqrt{13}$	B1F		
	$\cos(\theta+33.7)=1$ $(\theta=-33.7)$	M1		
	$\theta=326.3$	A1	3	AWRT 326
	Total		**6**	
4(a)	$A=80$	B1	1	
(b)	$5000=80\times k^{56}$	M1		SC1 Verification. Need 62.51 or better
	$k=\sqrt[56]{\frac{5000}{80}}\approx 1.07664$	M1A1	3	**Or** using logs: M1 ln $\left(\frac{5000}{80}\right)=56\ln k$ A1 $k=e^{\ln\left(\frac{62.5}{56}\right)}$ **Or** 3/3 for $k=1.076636$ **Or** 1.076637 seen
(c)(i)	$V=80\times k^{106}=200707$	M1A1	2	200648 using full register k
(ii)	$\ln 10000=\ln k^t$	M1		
	$t=\frac{\ln 10000}{\ln k}=124.7\Rightarrow 2024$	M1A1	3	M1 $t\ln k=\ln 10000$ A1 CAO **Or** trial and improvement M1expression M1 125, 124, A1 2024
	Total		**9**	
5(a)(i)	$(1-x)^{-1}=1+(-1)(-x)+\frac{(-1)(-2)}{2}(-x)^2$	M1		First two terms $+kx^2$
	$=1+x+x^2$	A1	2	
(ii)	$\frac{1}{(3-2x)}=\frac{1}{3}\left(1-\frac{2}{3}x\right)^{-1}$	B1		**Or** directly substitute into formula;
	$\approx *\left(1+\frac{2}{3}x+\left(\frac{2}{3}x\right)^2\right)$	M1		M1 power of 3 M1 other coefficients (allow one error) A1 CAO
	$\approx\frac{1}{3}+\frac{2}{9}x+\frac{4}{27}x^2$	A1	3	AG convincingly obtained
(b)	$(1-x)^{-2}=1+(-2)(-x)+\frac{(-2)(-3)(-x)^2}{2}$	M1		First two terms $+kx^2$
	$=1+2x+3x^2$	A1	2	

MPC4 (Cont)

Q	Solution	Marks	Total	Comments
5(c)	$2x^2-3=$			
	$A(1-x)^2+B(3-2x)(1-x)+C(3-2x)$	M1		**Or** by equating coefficients
	$x=1 \quad -1=C\times1 \quad x=\frac{3}{2} \quad \frac{3}{2}=A\times\frac{1}{4}$	M1		M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct
	$C=-1 \qquad A=6$	A1		Follow on A and C
	$x=0 \quad (-3=6+3B-3)$			
	or other value $\Rightarrow$ equation in $A,\ B,\ C$	m1		
	$B=-2$	A1	5	
(d)	$\frac{6}{3-2x}-\frac{2}{1-x}-\frac{1}{(1-x)^2}$			
	$\approx\frac{6}{3}\left(1+\frac{2}{3}x+\frac{4}{9}x^2\right)-2\left(1+x+x^2\right)$	M1A1F		Follow on A B C and expansions
	$-\left(1+2x+3x^2\right) \quad \approx-1-\frac{8}{3}x-\frac{37}{9}x^2$	A1	3	CAO
	Total		**15**	
6(a)	$\cos 2x=2\cos^2 x-1$	B1B1	2	
(b)	$\cos^2 x=\frac{1}{2}(\cos 2x+1)$	M1		Attempt to express $\cos^2 x$ in terms of $\cos 2x$
	$\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos 2x+1\,\mathrm{d}x=\left[\frac{1}{4}\sin 2x+\frac{x}{2}\right]_0^{\frac{\pi}{2}}$	A1 A1		
	$=\frac{\pi}{4}$	M1A1F	5	Use limits. Ft on integer *a*.
	Total		**7**	
7(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}6\\5\\3\end{bmatrix}-\begin{bmatrix}2\\1\\3\end{bmatrix}=\begin{bmatrix}4\\4\\0\end{bmatrix}$	M1 A1	2	Penalise use of co-ordinates at first occurrence only
(ii)	$\begin{bmatrix}4\\4\\0\end{bmatrix}=4\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow$ parallel	E1	1	Needs comment "same direction" Or "same gradient" (**Or** by scalar product)
(iii)	$\begin{bmatrix}2\\-3\\-1\end{bmatrix}=\begin{bmatrix}6\\1\\-1\end{bmatrix}+\lambda\begin{bmatrix}1\\1\\0\end{bmatrix}$	M1		
	is satisfied by $\lambda=-4$	A1	2	$\lambda=-4$ satisfies 2 equations

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(b)(i)	l_2 has equation $\mathrm{r} = \begin{bmatrix}4\\1\\1\end{bmatrix} + \lambda\left[\begin{bmatrix}4\\1\\1\end{bmatrix} - \begin{bmatrix}2\\-3\\-1\end{bmatrix}\right] = \begin{bmatrix}4\\1\\1\end{bmatrix} + \lambda\begin{bmatrix}2\\4\\2\end{bmatrix}$	M1A1	2	**Or** $r = \begin{bmatrix}2\\-3\\-1\end{bmatrix} + t\begin{bmatrix}2\\4\\2\end{bmatrix}$ M1 calculate and use direction vector A1 all correct
(ii)	$\begin{bmatrix}1\\2\\1\end{bmatrix} \bullet \begin{bmatrix}4\\0\\-4\end{bmatrix} = 4 - 4 = 0$	M1A1		Clear attempt to use directions of AC and l_2 in scalar product
	$\Rightarrow 90°$ (or perpendicular)	A1F	3	Accept a correct ft value of $\cos\theta$
	Total		**10**	
8(a)	$\int \frac{\mathrm{d}x}{\sqrt{x-6}}\mathrm{d}x = \int -2\mathrm{d}t$	M1		Attempt to separate and integrate
	$2\sqrt{x-6} = -2t + c$	A1A1		c on either side
	$t = 0 \quad x{=}70 \quad \Rightarrow \quad c = 16$	m1A1F		Follow on c from sensible attempt at integrals ($\sqrt{}$ not ln)
	$t = 8 - \sqrt{x-6}$	A1	6	CAO (or AEF)
(b)(i)	The liquid level stops falling/flowing/ at minimum depth	B1	1	
	$x = 22 \qquad t = 8 - \sqrt{22-6}$	M1		Use $x = 22$ in their equation provided there is a c **Or** start again using limits M1 $2\sqrt{64} - 2\sqrt{16} = \pm 2t$, A1 $t = 4$
	$t = 4$	A1	2	CAO
	Total		**9**	
	Total		**75**	