

C3 TRIGONOMETRY

Worksheet H

- 1 a Solve the equation

$$2 \sec x - 3 \operatorname{cosec} x = 0,$$

for x in the interval $-180^\circ \leq x \leq 180^\circ$. (4)

- b Find all values of θ in the interval $0 \leq \theta \leq 2\pi$ for which

$$\cot^2 \theta - \cot \theta + \operatorname{cosec}^2 \theta = 4. \quad (6)$$

- 2 For values of θ in the interval $0 \leq \theta \leq 360^\circ$, solve the equation

$$2 \sin (\theta + 30^\circ) = \sin (\theta - 30^\circ). \quad (6)$$

- 3 a Given that $\sin A = 2 - \sqrt{3}$, find in the form $a + b\sqrt{3}$ the exact value of

i $\operatorname{cosec} A$,

ii $\cot^2 A$. (5)

- b Solve the equation

$$3 \cos 2x - 8 \sin x + 5 = 0,$$

for values of x in the interval $0 \leq x \leq 360^\circ$, giving your answers to 1 decimal place. (5)

- 4 $f: x \rightarrow \frac{\pi}{2} + 2 \arcsin x, x \in \mathbb{R}, -1 \leq x \leq 1.$

a Find the exact value of $f(\frac{1}{2})$. (2)

b State the range of f . (2)

c Sketch the curve $y = f(x)$. (2)

d Solve the equation $f(x) = 0$. (3)

- 5 a Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the values of R and α to 3 significant figures. (4)

b State the minimum value of $2 \sin x - 3 \cos x$ and the smallest positive value of x for which this minimum occurs. (3)

- c Solve the equation

$$2 \sin 2x - 3 \cos 2x + 1 = 0,$$

for x in the interval $0 \leq x \leq \pi$, giving your answers to 2 decimal places. (5)

- 6 a Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1. \quad (3)$$

- b Solve the equation

$$\frac{\sin x}{1 + \cos x} = 3 \cot \frac{x}{2},$$

for values of x in the interval $0 \leq x \leq 360^\circ$. (7)

C3 TRIGONOMETRY

Worksheet H continued

- 7 a Prove the identity

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta, \quad \theta \neq n\pi, \quad n \in \mathbb{Z}. \quad (3)$$

- b Find the values of
- x
- in the interval
- $0 \leq x \leq 2\pi$
- for which

$$2 \sec x + \tan x = 2 \cos x,$$

giving your answers in terms of π . (6)

- 8 a Sketch on the same diagram the curves
- $y = 3 \sin x^\circ$
- and
- $y = 1 + \operatorname{cosec} x^\circ$
- for
- x
- in the interval
- $-180 \leq x \leq 180$
- . (4)

- b Find the
- x
- coordinate of each point where the curves intersect in this interval, giving your answers correct to 1 decimal place. (6)

- 9 a Prove the identity

$$(1 - \sin x)(\sec x + \tan x) \equiv \cos x, \quad x \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}. \quad (4)$$

- b Find the values of
- y
- in the interval
- $0 \leq y \leq \pi$
- for which

$$2 \sec^2 2y + \tan^2 2y = 3,$$

giving your answers in terms of π . (6)

- 10 a Express
- $4 \sin x^\circ - \cos x^\circ$
- in the form
- $R \sin (x - \alpha)^\circ$
- , where
- $R > 0$
- and
- $0 < \alpha < 90$
- .

Give the values of R and α to 3 significant figures. (4)

- b Show that the equation

$$2 \operatorname{cosec} x^\circ - \cot x^\circ + 4 = 0 \quad (I)$$

can be written in the form

$$4 \sin x^\circ - \cos x^\circ + 2 = 0. \quad (2)$$

- c Using your answers to parts a and b, solve equation (I) for
- x
- in the interval
- $0 \leq x \leq 360$
- . (4)

- 11 a Use the identities

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

$$\text{and} \quad \cos (A - B) \equiv \cos A \cos B + \sin A \sin B$$

to prove that

$$\cos P + \cos Q \equiv 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}. \quad (4)$$

- b Find, in terms of
- π
- , the values of
- x
- in the interval
- $0 \leq x \leq 2\pi$
- for which

$$\cos x + \cos 2x + \cos 3x = 0. \quad (7)$$

- 12 a Express
- $3 \cos \theta + 4 \sin \theta$
- in the form
- $R \cos (\theta - \alpha)$
- , where
- $R > 0$
- and
- $0 < \alpha < \frac{\pi}{2}$
- . (4)

- b Given that the function
- f
- is defined by

$$f(\theta) \equiv 1 - 3 \cos 2\theta - 4 \sin 2\theta, \quad \theta \in \mathbb{R}, \quad 0 \leq \theta \leq \pi,$$

- i state the range of
- f
- ,

- ii solve the equation
- $f(\theta) = 0$
- . (6)

- c Find the coordinates of the turning points of the curve with equation
- $y = \frac{2}{3 \cos x + 4 \sin x}$

in the interval $[0, 2\pi]$. (3)