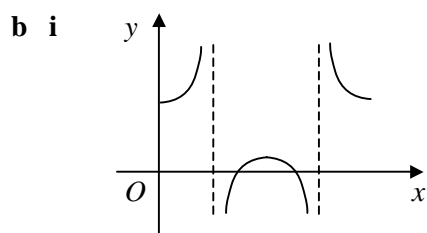


### C3 TRIGONOMETRY

### Answers - Worksheet G

- 1**  $\sec^2 x - 1 - \sec x = 1$   
 $\sec^2 x - \sec x - 2 = 0$   
 $(\sec x + 1)(\sec x - 2) = 0$   
 $\sec x = -1$  or  $2$   
 $\cos x = -1$  or  $\frac{1}{2}$   
 $x = 180$  or  $60, 360 - 60$   
 $x = 60^\circ, 180^\circ, 300^\circ$
- 2** **a**  $2 \cos x + 5 \sin x$   
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$   
 $\Rightarrow R \cos \alpha = 2, R \sin \alpha = 5$   
 $\therefore R = \sqrt{4 + 25} = \sqrt{29} = 5.39$   
 $\tan \alpha = \frac{5}{2}, \alpha = 68.2$   
 $\therefore 2 \cos x + 5 \sin x = 5.39 \cos(x - 68.2)^\circ$
- b**  $\sqrt{29} \cos(x - 68.199) = 3$   
 $\cos(x - 68.199) = \frac{3}{\sqrt{29}} = 0.5571$   
 $x - 68.199 = 56.145, -56.145$   
 $x = 12.1, 124.3$
- 3** **a**  $\arctan 2x = \frac{\pi}{6}$   
 $2x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$   
 $x = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$
- b**  $4 \sin x \cos x = 3 \cos x$   
 $\cos x(4 \sin x - 3) = 0$   
 $\cos x = 0$  or  $\sin x = \frac{3}{4}$   
 $x = 90, 360 - 90$  or  $48.6, 180 - 48.6$   
 $x = 48.6^\circ$  (1dp),  $90^\circ, 131.4^\circ$  (1dp),  $270^\circ$
- 4** **a**  $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$   
 $\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$   
 subtracting  
 $\sin(A + B) - \sin(A - B) \equiv 2 \cos A \sin B$   
 let  $P = A + B$  (1) and  $Q = A - B$  (2)  
 $(1) + (2) \Rightarrow 2A = P + Q \Rightarrow A = \frac{P + Q}{2}$   
 $(1) - (2) \Rightarrow 2B = P - Q \Rightarrow B = \frac{P - Q}{2}$   
 $\therefore \sin P - \sin Q \equiv 2 \cos \frac{P + Q}{2} \sin \frac{P - Q}{2}$
- b**  $\sin 4x - \sin 2x = 0$   
 $2 \cos \frac{4x + 2x}{2} \sin \frac{4x - 2x}{2} = 0$   
 $\cos 3x \sin x = 0$   
 $\cos 3x = 0$  or  $\sin x = 0$   
 $3x = 90, 360 - 90, 360 + 90$  or  $x = 0, 180$   
 $3x = 90, 270, 450$  or  $x = 0, 180$   
 $x = 0, 30^\circ, 90^\circ, 150^\circ, 180^\circ$

$$\begin{aligned}
 5 \quad \mathbf{a} \quad \text{LHS} &= 4 \sin^2 \theta - 4 + \operatorname{cosec}^2 \theta \\
 &= 4(1 - \cos^2 \theta) - 4 + \operatorname{cosec}^2 \theta \\
 &= \operatorname{cosec}^2 \theta - 4 \cos^2 \theta = \text{RHS}
 \end{aligned}$$



**ii**  $(0, 5)$

**iii**  $3 + 2 \sec x = 0$

$$\sec x = -\frac{3}{2}, \quad \cos x = -\frac{2}{3}$$

$$x = \pi - 0.841, \pi + 0.841 = 2.30, 3.98$$

$$\therefore (2.30, 0) \text{ and } (3.98, 0) \text{ [} x \text{ to 2dp]}$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad \text{LHS} &= \frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x} \\
 &= \frac{\cos 2x + 1}{\sin 2x} \\
 &= \frac{2 \cos^2 x - 1 + 1}{2 \sin x \cos x} \\
 &= \frac{2 \cos^2 x}{2 \sin x \cos x} \\
 &= \frac{\cos x}{\sin x} \\
 &= \cot x = \text{RHS}
 \end{aligned}$$

**b**  $\cot x = 6 - \cot^2 x$   
 $\cot^2 x + \cot x - 6 = 0$

$$(\cot x + 3)(\cot x - 2) = 0$$

$$\cot x = -3 \text{ or } 2$$

$$\tan x = -\frac{1}{3} \text{ or } \frac{1}{2}$$

$$x = \pi - 0.3218, 2\pi - 0.3218$$

$$\text{or } 0.4636, \pi + 0.4636$$

$$x = 0.46, 2.82, 3.61, 5.96$$

$$\begin{aligned}
 6 \quad \mathbf{a} \quad \cos x - \sin x &= R \cos x \cos \alpha - R \sin x \sin \alpha \\
 \Rightarrow R \cos \alpha &= 1, \quad R \sin \alpha = 1 \\
 \therefore R &= \sqrt{1+1} = \sqrt{2} \\
 \tan \alpha &= 1, \quad \alpha = \frac{\pi}{4}
 \end{aligned}$$

**b**  $\cos x - \sin x + \sqrt{2} \cos(3x - \frac{\pi}{4}) = 0$   
 $\sqrt{2} \cos(x + \frac{\pi}{4}) + \sqrt{2} \cos(3x - \frac{\pi}{4}) = 0$

$$2\sqrt{2} \cos \frac{4x}{2} \cos \frac{-2x + \frac{\pi}{2}}{2} = 0$$

$$\cos 2x \cos(\frac{\pi}{4} - x) = 0$$

$$\cos 2x \cos(x - \frac{\pi}{4}) = 0$$

$$\cos 2x = 0 \text{ or } \cos(x - \frac{\pi}{4}) = 0$$

$$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$$

$$\text{or } x - \frac{\pi}{4} = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ or } x - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\begin{aligned}
 8 \quad \mathbf{a} \quad \text{LHS} &= \cos x \cos 30 - \sin x \sin 30 + \sin x \\
 &= \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x + \sin x \\
 &= \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \\
 &= \cos x \cos 30 + \sin x \sin 30 \\
 &= \cos(x - 30)^\circ = \text{RHS}
 \end{aligned}$$

**b** let  $x = 45$

$$\cos 75^\circ + \sin 45^\circ = \cos 15^\circ$$

$$\therefore \cos 75^\circ - \cos 15^\circ = -\sin 45^\circ$$

$$= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$$

**c**  $3 \cos(x + 30) + 3 \sin x - 2 \sin x$   
 $= 3 \cos(x - 30) + 1$

$$-2 \sin x = 1$$

$$\sin x = -\frac{1}{2}$$

$$x = -30, 30 - 180$$

$$x = -150, -30$$

9 a  $a = 3$

$b \sin x^\circ + c \cos x^\circ$  can be expressed in the form  $k \sin(x + \alpha)^\circ$  which will vary between  $-k$  and  $+k$

$\therefore a + k = 5$  and  $a - k = 1$ , hence  $a = 3$

b  $3 + k = 5 \therefore k = 2$

$60 + \alpha = 90 \therefore \alpha = 30$

c  $f(x) = 3 + 2 \sin(x + 30)$

$$= 3 + 2 \sin x \cos 30 + 2 \cos x \sin 30$$

$$= 3 + \sqrt{3} \sin x + \cos x$$

$\therefore b = \sqrt{3}, c = 1$

11 a  $6 \cot^2 x - \operatorname{cosec} x + 5 = 0$

$$\Rightarrow 6(\operatorname{cosec}^2 x - 1) - \operatorname{cosec} x + 5 = 0$$

$$6 \operatorname{cosec}^2 x - \operatorname{cosec} x - 1 = 0$$

$$(3 \operatorname{cosec} x + 1)(2 \operatorname{cosec} x - 1) = 0$$

$$\operatorname{cosec} x = -\frac{1}{3}, \frac{1}{2}$$

for real  $x$ ,  $|\operatorname{cosec} x| \geq 1$

$\therefore$  no real solutions

b  $\cos 5y - \cos y = 0$

$$-2 \sin \frac{5y+y}{2} \sin \frac{5y-y}{2} = 0$$

$$\sin 3y \sin 2y = 0$$

$$\sin 3y = 0 \text{ or } \sin 2y = 0$$

$$3y = 0, 180, 360, 540 \text{ or } 2y = 0, 180, 360$$

$$y = 0, 60^\circ, 90^\circ, 120^\circ, 180^\circ$$

10 a  $\text{LHS} = \frac{1 - (1 - 2 \sin^2 \frac{x}{2})}{1 + (2 \cos^2 \frac{x}{2} - 1)}$

$$= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= \tan^2 \frac{x}{2} = \text{RHS}$$

b i let  $x = \frac{\pi}{6}$ ,  $\frac{1 - \cos \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}} = \tan^2 \frac{\pi}{12}$

$$\tan^2 \frac{\pi}{12} = \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{4 - 4\sqrt{3} + 3}{4 - 3} = 7 - 4\sqrt{3}$$

ii  $\tan^2 \frac{x}{2} = 1 - \sec \frac{x}{2}$

$$\sec^2 \frac{x}{2} - 1 = 1 - \sec \frac{x}{2}$$

$$\sec^2 \frac{x}{2} + \sec \frac{x}{2} - 2 = 0$$

$$(\sec \frac{x}{2} + 2)(\sec \frac{x}{2} - 1) = 0$$

$$\sec \frac{x}{2} = -2 \text{ or } 1$$

$$\cos \frac{x}{2} = -\frac{1}{2} \text{ or } 1$$

$$\frac{x}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ or } 0$$

$$x = 0, \frac{4\pi}{3}$$

12 a  $\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

subtracting

$$\cos(A - B) - \cos(A + B) \equiv 2 \sin A \sin B$$

$$\therefore \sin A \sin B \equiv \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

b  $4 \sin(x + \frac{\pi}{3}) = \frac{1}{\sin(x - \frac{\pi}{6})}$

$$4 \sin(x + \frac{\pi}{3}) \sin(x - \frac{\pi}{6}) = 1$$

$$2[\cos \frac{\pi}{2} - \cos(2x + \frac{\pi}{6})] = 1$$

$$2[0 - \cos(2x + \frac{\pi}{6})] = 1$$

$$\cos(2x + \frac{\pi}{6}) = -\frac{1}{2}$$

$$2x + \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$2x = \frac{\pi}{2}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12}$$