

C3 TRIGONOMETRY

Answers - Worksheet F

- 1 a** $\cos x + \sin x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 1$
 $\therefore R = \sqrt{1+1} = \sqrt{2} = 1.4$
 $\tan \alpha = 1, \alpha = 45$
 $\therefore \cos x^\circ + \sin x^\circ = 1.4 \cos (x - 45)^\circ$
- b** $3 \cos x + 4 \sin x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 4$
 $\therefore R = \sqrt{9+16} = 5$
 $\tan \alpha = \frac{4}{3}, \alpha = 53.1$
 $\therefore 3 \cos x^\circ + 4 \sin x^\circ = 5 \cos (x - 53.1)^\circ$
- c** $2 \sin x + \cos x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 2$
 $\therefore R = \sqrt{1+4} = \sqrt{5} = 2.2$
 $\tan \alpha = 2, \alpha = 63.4$
 $\therefore 2 \sin x^\circ + \cos x^\circ = 2.2 \cos (x - 63.4)^\circ$
- d** $\cos x + \sqrt{3} \sin x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = \sqrt{3}$
 $\therefore R = \sqrt{1+3} = 2$
 $\tan \alpha = \sqrt{3}, \alpha = 60$
 $\therefore \cos x^\circ + \sqrt{3} \sin x^\circ = 2 \cos (x - 60)^\circ$
- 2 a** $5 \cos x - 12 \sin x$
 $= R \cos x \cos \alpha - R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 5, R \sin \alpha = 12$
 $\therefore R = \sqrt{25+144} = 13$
 $\tan \alpha = \frac{12}{5}, \alpha = 67.4$
 $\therefore 5 \cos x^\circ - 12 \sin x^\circ = 13 \cos (x + 67.4)^\circ$
- b** $4 \sin x + 2 \cos x$
 $= R \sin x \cos \alpha + R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 4, R \sin \alpha = 2$
 $\therefore R = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$
 $\tan \alpha = \frac{1}{2}, \alpha = 26.6$
 $\therefore 4 \sin x^\circ + 2 \cos x^\circ = 2\sqrt{5} \sin (x + 26.6)^\circ$
- c** $\sin x - 7 \cos x$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 7$
 $\therefore R = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$
 $\tan \alpha = 7, \alpha = 81.9$
 $\therefore \sin x^\circ - 7 \cos x^\circ = 5\sqrt{2} \sin (x - 81.9)^\circ$
- d** $8 \cos 2x - 15 \sin 2x$
 $= R \cos 2x \cos \alpha - R \sin 2x \sin \alpha$
 $\Rightarrow R \cos \alpha = 8, R \sin \alpha = 15$
 $\therefore R = \sqrt{64+225} = 17$
 $\tan \alpha = \frac{15}{8}, \alpha = 61.9$
 $\therefore 8 \cos 2x^\circ - 15 \sin 2x^\circ = 17 \cos (2x + 61.9)^\circ$
- 3 a** $3 \sin x - 2 \cos x$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 2$
 $\therefore R = \sqrt{9+4} = \sqrt{13}$
 $\tan \alpha = \frac{2}{3}, \alpha = 0.59$
 $\therefore 3 \sin x - 2 \cos x = \sqrt{13} \sin (x - 0.59)$
- b** $3 \cos x + \sqrt{3} \sin x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = \sqrt{3}$
 $\therefore R = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$
 $\tan \alpha = \frac{1}{\sqrt{3}}, \alpha = \frac{\pi}{6}$
 $\therefore 3 \cos x + \sqrt{3} \sin x = 2\sqrt{3} \cos (x - \frac{\pi}{6})$
- c** $8 \sin 3x + 6 \cos 3x$
 $= R \sin 3x \cos \alpha + R \cos 3x \sin \alpha$
 $\Rightarrow R \cos \alpha = 8, R \sin \alpha = 6$
 $\therefore R = \sqrt{64+36} = 10$
 $\tan \alpha = \frac{3}{4}, \alpha = 0.64$
 $\therefore 8 \sin 3x + 6 \cos 3x = 10 \sin (3x + 0.64)$
- d** $\cos x + \frac{1}{2} \sin x$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = \frac{1}{2}$
 $\therefore R = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$
 $\tan \alpha = \frac{1}{2}, \alpha = 0.46$
 $\therefore \cos x + \frac{1}{2} \sin x = \frac{1}{2}\sqrt{5} \cos (x - 0.46)$

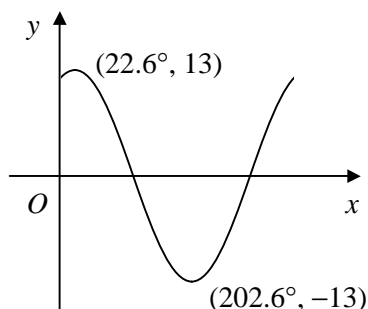
4 a $24 \sin x - 7 \cos x = R \sin(x - \alpha)$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 24, R \sin \alpha = 7$
 $\therefore R = \sqrt{576 + 49} = 25$
 $\tan \alpha = \frac{7}{24}, \alpha = 16.3^\circ$
 $\therefore 24 \sin x - 7 \cos x = 25 \sin(x - 16.3^\circ)$
 $\therefore \text{max.} = 25$ when $x - 16.3 = 90$
 $x = 106.3^\circ$ (1dp)

c $3 \cos x - 5 \sin x = R \cos(x + \alpha)$
 $= R \cos x \cos \alpha - R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 5$
 $\therefore R = \sqrt{9 + 25} = \sqrt{34}$
 $\tan \alpha = \frac{5}{3}, \alpha = 59.0^\circ$
 $\therefore 3 \cos x - 5 \sin x = \sqrt{34} \cos(x + 59.0^\circ)$
 $\therefore \text{max.} = \sqrt{34}$ when $x + 59.0 = 360$
 $x = 301.0^\circ$ (1dp)

5 a $3 \sin x - 3 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 3$
 $\therefore R = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
 $\tan \alpha = 1, \alpha = 45^\circ$
 $\therefore 3 \sin x - 3 \cos x = 3\sqrt{2} \sin(x - 45^\circ)$

b translation by 45 units in positive x -direction and stretch by a factor of $3\sqrt{2}$ in y -direction

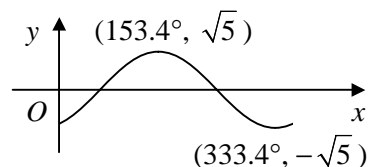
6 a $12 \cos x + 5 \sin x = R \cos(x - \alpha)$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 12, R \sin \alpha = 5$
 $\therefore R = \sqrt{144 + 25} = 13$
 $\tan \alpha = \frac{5}{12}, \alpha = 22.6^\circ$ (1dp)
 $\therefore y = 13 \cos(x - 22.6^\circ)$



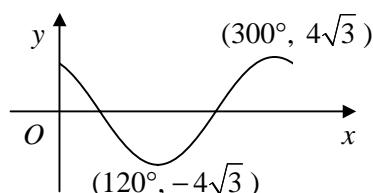
b $4 \cos 2x + 4 \sin 2x = R \cos(2x - \alpha)$
 $= R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$
 $\Rightarrow R \cos \alpha = 4, R \sin \alpha = 4$
 $\therefore R = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$
 $\tan \alpha = 1, \alpha = 45^\circ$
 $\therefore 4 \cos 2x + 4 \sin 2x = 4\sqrt{2} \cos(2x - 45^\circ)$
 $\therefore \text{max.} = 4\sqrt{2}$ when $2x - 45 = 0$
 $x = 22.5^\circ$

d $5 \sin 3x + \cos 3x = R \sin(3x + \alpha)$
 $= R \sin 3x \cos \alpha + R \cos 3x \sin \alpha$
 $\Rightarrow R \cos \alpha = 5, R \sin \alpha = 1$
 $\therefore R = \sqrt{25 + 1} = \sqrt{26}$
 $\tan \alpha = \frac{1}{5}, \alpha = 11.3^\circ$
 $\therefore 5 \sin 3x + \cos 3x = \sqrt{26} \sin(3x + 11.3^\circ)$
 $\therefore \text{max.} = \sqrt{26}$ when $3x + 11.3 = 90$
 $x = 26.2^\circ$ (1dp)

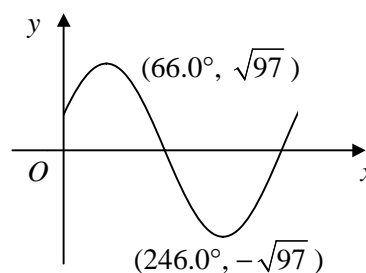
b $\sin x - 2 \cos x = R \sin(x - \alpha)$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 2$
 $\therefore R = \sqrt{1 + 4} = \sqrt{5}$
 $\tan \alpha = 2, \alpha = 63.4^\circ$ (1dp)
 $\therefore y = \sqrt{5} \sin(x - 63.4^\circ)$



$$\begin{aligned} \text{c } 2\sqrt{3} \cos x - 6 \sin x &= R \cos(x + \alpha) \\ &= R \cos x \cos \alpha - R \sin x \sin \alpha \\ \Rightarrow R \cos \alpha &= 2\sqrt{3}, R \sin \alpha = 6 \\ \therefore R &= \sqrt{12+36} = \sqrt{48} = 4\sqrt{3} \\ \tan \alpha &= \sqrt{3}, \alpha = 60^\circ \\ \therefore y &= 4\sqrt{3} \cos(x + 60^\circ) \end{aligned}$$



$$\begin{aligned} \text{d } 9 \sin x + 4 \cos x &= R \sin(x + \alpha) \\ &= R \sin x \cos \alpha + R \cos x \sin \alpha \\ \Rightarrow R \cos \alpha &= 9, R \sin \alpha = 4 \\ \therefore R &= \sqrt{81+16} = \sqrt{97} \\ \tan \alpha &= \frac{4}{9}, \alpha = 24.0^\circ \text{ (1dp)} \\ \therefore y &= \sqrt{97} \sin(x + 24.0^\circ) \end{aligned}$$



$$\begin{aligned} 7 \text{ a } \sqrt{3} \cos x - \sin x &= R \cos x \cos \alpha - R \sin x \sin \alpha \\ \Rightarrow R \cos \alpha &= \sqrt{3}, R \sin \alpha = 1 \\ \therefore R &= \sqrt{3+1} = 2 \\ \tan \alpha &= \frac{1}{\sqrt{3}}, \alpha = \frac{\pi}{6} \\ \therefore \sqrt{3} \cos x - \sin x &= 2 \cos(x + \frac{\pi}{6}) \end{aligned}$$

$$\begin{aligned} \text{b } 2 \cos(x + \frac{\pi}{6}) &= 1 \\ \cos(x + \frac{\pi}{6}) &= \frac{1}{2} \\ x + \frac{\pi}{6} &= \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3} \\ x &= \frac{\pi}{6}, \frac{3\pi}{2} \end{aligned}$$

$$\begin{aligned} 8 \text{ a } 6 \sin x + 8 \cos x &= R \sin(x + \alpha) \\ &= R \sin x \cos \alpha + R \cos x \sin \alpha \\ \Rightarrow R \cos \alpha &= 6, R \sin \alpha = 8 \\ \therefore R &= \sqrt{36+64} = 10 \\ \tan \alpha &= \frac{4}{3}, \alpha = 0.9273 \\ \therefore 10 \sin(x + 0.9273) &= 5 \\ \sin(x + 0.9273) &= \frac{1}{2} \\ x + 0.9273 &= \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6} \\ &= \frac{5\pi}{6}, \frac{13\pi}{6} \\ x &= 1.69, 5.88 \end{aligned}$$

$$\begin{aligned} \text{b } 2 \cos x - 2 \sin x &= R \cos(x + \alpha) \\ &= R \cos x \cos \alpha + R \sin x \sin \alpha \\ \Rightarrow R \cos \alpha &= 2, R \sin \alpha = 2 \\ \therefore R &= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \\ \tan \alpha &= 1, \alpha = \frac{\pi}{4} \\ \therefore 2\sqrt{2} \cos(x + \frac{\pi}{4}) &= 1 \\ \cos(x + \frac{\pi}{4}) &= \frac{1}{2\sqrt{2}} \\ x + \frac{\pi}{4} &= 1.2094, 2\pi - 1.2094 \\ &= 1.2094, 5.0738 \\ x &= 0.42, 4.29 \end{aligned}$$

- c** $7 \sin x - 24 \cos x = R \sin(x - \alpha)$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 7, R \sin \alpha = 24$
 $\therefore R = \sqrt{49 + 576} = 25$
 $\tan \alpha = \frac{24}{7}, \alpha = 1.2870$
 $\therefore 25 \sin(x - 1.2870) - 10 = 0$
 $\sin(x - 1.2870) = \frac{2}{5}$
 $x - 1.2870 = 0.4115, \pi - 0.4115$
 $= 0.4115, 2.7301$
 $x = 1.70, 4.02$
- d** $3 \cos x + \sin x = R \cos(x - \alpha)$
 $= R \cos x \cos \alpha + R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 3, R \sin \alpha = 1$
 $\therefore R = \sqrt{9 + 1} = \sqrt{10}$
 $\tan \alpha = \frac{1}{3}, \alpha = 0.3218$
 $\therefore \sqrt{10} \cos(x - 0.3218) + 1 = 0$
 $\cos(x - 0.3218) = -\frac{1}{\sqrt{10}}$
 $x - 0.3218 = \pi - 1.2490, \pi + 1.2490$
 $= 1.8925, 4.3906$
 $x = 2.21, 4.71$
- e** $\cos 2x + 4 \sin 2x = R \cos(2x - \alpha)$
 $= R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 4$
 $\therefore R = \sqrt{1 + 16} = \sqrt{17}$
 $\tan \alpha = 4, \alpha = 1.3258$
 $\therefore \sqrt{17} \cos(2x - 1.3258) = 3$
 $\cos(2x - 1.3258) = \frac{3}{\sqrt{17}}$
 $2x - 1.3258 = 0.7560, 2\pi - 0.7560,$
 $2\pi + 0.7560, -0.7560$
 $= -0.7560, 0.7560, 5.5272, 7.0392$
 $2x = 0.5698, 2.0818, 6.8530, 8.3650$
 $x = 0.28, 1.04, 3.43, 4.18$
- f** $5 \sin x - 8 \cos x = R \sin(x - \alpha)$
 $= R \sin x \cos \alpha + R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 5, R \sin \alpha = 8$
 $\therefore R = \sqrt{25 + 64} = \sqrt{89}$
 $\tan \alpha = \frac{8}{5}, \alpha = 1.0122$
 $\therefore \sqrt{89} \sin(x - 1.0122) + 7 = 0$
 $\sin(x - 1.0122) = -\frac{7}{\sqrt{89}}$
 $x - 1.0122 = -0.8360, \pi + 0.8360$
 $= -0.8360, 3.9776$
 $x = 0.18, 4.99$
- 9 a** $\sin x + \cos x = R \sin(x + \alpha)$
 $= R \sin x \cos \alpha + R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 1$
 $\therefore R = \sqrt{1 + 1} = \sqrt{2}$
 $\tan \alpha = 1, \alpha = 45^\circ$
 $\therefore \sqrt{2} \sin(x + 45^\circ) = 1$
 $\sin(x + 45^\circ) = \frac{1}{\sqrt{2}}$
 $x + 45 = 45, 180 - 45$
 $= 45, 135$
 $x = 0, 90^\circ$
- b** $4 \cos x - \sin x = R \cos(x + \alpha)$
 $= R \cos x \cos \alpha - R \sin x \sin \alpha$
 $\Rightarrow R \cos \alpha = 4, R \sin \alpha = 1$
 $\therefore R = \sqrt{16 + 1} = \sqrt{17}$
 $\tan \alpha = \frac{1}{4}, \alpha = 14.04^\circ$
 $\therefore \sqrt{17} \cos(x + 14.04^\circ) + 2 = 0$
 $\cos(x + 14.04^\circ) = -\frac{2}{\sqrt{17}}$
 $x + 14.04 = 180 - 60.98, 60.98 - 180$
 $= -119.02, 119.02$
 $x = -133.1^\circ, 105.0^\circ$
- c** $\cos \frac{x}{2} + 5 \sin \frac{x}{2} = R \cos(\frac{x}{2} - \alpha)$
 $= R \cos \frac{x}{2} \cos \alpha + R \sin \frac{x}{2} \sin \alpha$
 $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 5$
 $\therefore R = \sqrt{1 + 25} = \sqrt{26}$
 $\tan \alpha = 5, \alpha = 78.69^\circ$
 $\therefore \sqrt{26} \cos(\frac{x}{2} - 78.69^\circ) - 4 = 0$
 $\cos(\frac{x}{2} - 78.69^\circ) = \frac{4}{\sqrt{26}}$
 $\frac{x}{2} - 78.69 = -38.33$
 $\frac{x}{2} = 40.36$
 $x = 80.7^\circ$
- d** $6 \sin x + 3 \cos x = R \sin(x + \alpha)$
 $= R \sin x \cos \alpha + R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 6, R \sin \alpha = 3$
 $\therefore R = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$
 $\tan \alpha = \frac{1}{2}, \alpha = 26.57^\circ$
 $\therefore 3\sqrt{5} \sin(x + 26.57^\circ) = 5$
 $\sin(x + 26.57^\circ) = \frac{\sqrt{5}}{3}$
 $x + 26.57 = 48.19, 180 - 48.19$
 $= 48.19, 131.81$
 $x = 21.6^\circ, 105.2^\circ$