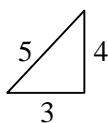


# C3 TRIGONOMETRY

## Answers - Worksheet A

- 1**    **a** 1.09                      **b** -11.47                      **c** 0.33                      **d** 1.89
- 2**    **a**  $= 1 \div \sin 30^\circ$                       **b**  $= 1 \div \tan 45^\circ$                       **c**  $= 1 \div \cos 150^\circ$                       **d**  $= 1 \div \sin 300^\circ$   
 $= 1 \div \frac{1}{2}$                        $= 1 \div 1$                        $= 1 \div (-\cos 30^\circ)$                        $= 1 \div (-\sin 60^\circ)$   
 $= 2$                        $= 1$                        $= 1 \div \left(-\frac{\sqrt{3}}{2}\right)$                        $= 1 \div \left(-\frac{\sqrt{3}}{2}\right)$   
 $= -\frac{2}{\sqrt{3}}$                        $= -\frac{2}{\sqrt{3}}$
- e**  $= \cos 90^\circ \div \sin 90^\circ$                       **f**  $= 1 \div \cos 225^\circ$                       **g**  $= 1 \div \sin 270^\circ$                       **h**  $= 1 \div \tan 330^\circ$   
 $= 0 \div 1$                        $= 1 \div (-\cos 45^\circ)$                        $= 1 \div (-\sin 90^\circ)$                        $= 1 \div (-\tan 30^\circ)$   
 $= 0$                        $= 1 \div \left(-\frac{1}{\sqrt{2}}\right)$                        $= 1 \div (-1)$                        $= 1 \div \left(-\frac{1}{\sqrt{3}}\right)$   
 $= -\sqrt{2}$                        $= -1$                        $= -\sqrt{3}$
- i**  $= 1 \div \cos 660^\circ$                       **j**  $= 1 \div \sin (-45^\circ)$                       **k**  $= 1 \div \tan (-240^\circ)$                       **l**  $= 1 \div \cos (-315^\circ)$   
 $= 1 \div \cos 60^\circ$                        $= 1 \div (-\sin 45^\circ)$                        $= 1 \div (-\tan 60^\circ)$                        $= 1 \div \cos 45^\circ$   
 $= 1 \div \frac{1}{2}$                        $= 1 \div \left(-\frac{1}{\sqrt{2}}\right)$                        $= 1 \div (-\sqrt{3})$                        $= 1 \div \frac{1}{\sqrt{2}}$   
 $= 2$                        $= -\sqrt{2}$                        $= -\frac{1}{\sqrt{3}}$                        $= \sqrt{2}$
- 3**    **a** 1.60                      **b** 1.01                      **c** -2.09                      **d** 2.54
- 4**    **a**  $= 1 \div \cos 0$                       **b**  $= 1 \div \sin \frac{\pi}{4}$                       **c**  $= 1 \div \tan \frac{3\pi}{4}$                       **d**  $= 1 \div \cos \frac{4\pi}{3}$   
 $= 1 \div 1$                        $= 1 \div \frac{1}{\sqrt{2}}$                        $= 1 \div (-\tan \frac{\pi}{4})$                        $= 1 \div (-\cos \frac{\pi}{3})$   
 $= 1$                        $= \sqrt{2}$                        $= 1 \div (-1)$                        $= 1 \div \left(-\frac{1}{2}\right)$   
 $= -1$                        $= -2$
- e**  $= 1 \div \sin \frac{2\pi}{3}$                       **f**  $= \cos \frac{7\pi}{2} \div \sin \frac{7\pi}{2}$                       **g**  $= 1 \div \cos \frac{5\pi}{4}$                       **h**  $= 1 \div \sin \left(-\frac{5\pi}{6}\right)$   
 $= 1 \div \sin \frac{\pi}{3}$                        $= \cos \frac{\pi}{2} \div (-\sin \frac{\pi}{2})$                        $= 1 \div (-\cos \frac{\pi}{4})$                        $= 1 \div (-\sin \frac{\pi}{6})$   
 $= 1 \div \frac{\sqrt{3}}{2}$                        $= 0 \div (-1)$                        $= 1 \div \left(-\frac{1}{\sqrt{2}}\right)$                        $= 1 \div \left(-\frac{1}{2}\right)$   
 $= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$                        $= 0$                        $= -\sqrt{2}$                        $= -2$   
 $= \frac{2}{3}\sqrt{3}$
- i**  $= 1 \div \tan \frac{11\pi}{6}$                       **j**  $= 1 \div \cos (-4\pi)$                       **k**  $= 1 \div \sin \frac{13\pi}{4}$                       **l**  $= 1 \div \tan \left(-\frac{7\pi}{3}\right)$   
 $= 1 \div \left(-\tan \frac{\pi}{6}\right)$                        $= 1 \div \cos 0$                        $= 1 \div (-\sin \frac{\pi}{4})$                        $= 1 \div \left(-\tan \frac{\pi}{3}\right)$   
 $= 1 \div \left(-\frac{1}{\sqrt{3}}\right)$                        $= 1 \div 1$                        $= 1 \div \left(-\frac{1}{\sqrt{2}}\right)$                        $= 1 \div (-\sqrt{3})$   
 $= -\sqrt{3}$                        $= 1$                        $= -\sqrt{2}$                        $= -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$   
 $= -\frac{1}{3}\sqrt{3}$

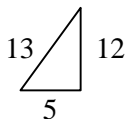
5



$\therefore \cos x = \pm \frac{3}{5}, \tan x = \pm \frac{4}{3}$   
 $0 < x < 90^\circ \Rightarrow \cos x = \frac{3}{5}, \tan x = \frac{4}{3}$

**a**  $= \frac{3}{5}$                       **b**  $= \frac{4}{3}$                       **c**  $= 1 \div \frac{4}{5} = \frac{5}{4}$                       **d**  $= 1 \div \frac{3}{5} = \frac{5}{3}$

6



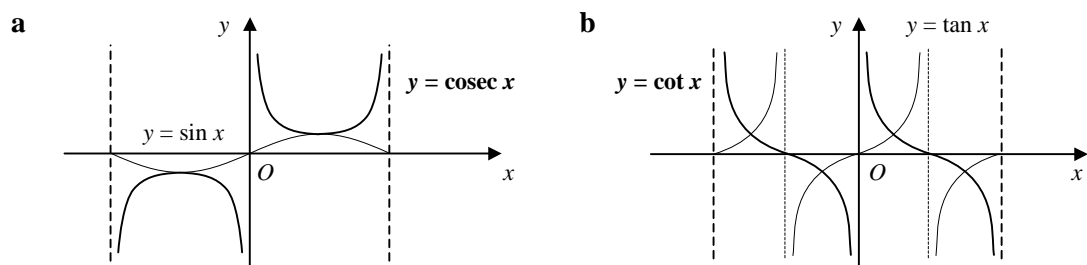
$\therefore \sin x = \pm \frac{12}{13}, \tan x = \pm \frac{12}{5}$   
 $90^\circ < x < 180^\circ \Rightarrow \sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

**a**  $= \frac{12}{13}$                       **b**  $= 1 \div (-\frac{5}{13}) = -\frac{13}{5}$                       **c**  $= 1 \div \frac{12}{13} = \frac{13}{12}$                       **d**  $= 1 \div -\frac{12}{5} = -\frac{5}{12}$

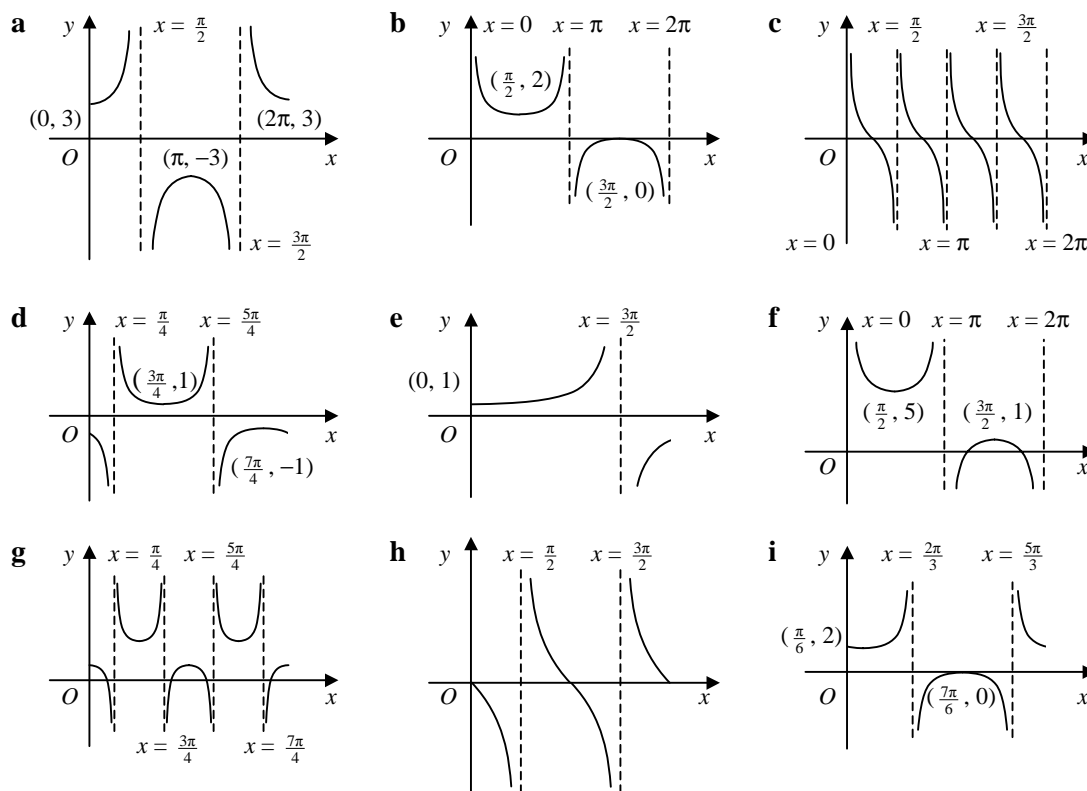
7

- a** (0, 1), (180, -1), (360, 1), (540, -1), (720, 1)  
**b**  $x = 90, x = 270, x = 450, x = 630$

8



9



TP: (0, 0),  $(\frac{\pi}{2}, 2)$ ,  $(\pi, 0)$ ,  
 $(\frac{3\pi}{2}, 2)$ ,  $(2\pi, 0)$

- 10**
- a**  $\tan x = 1$   
 $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$   
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$
- b**  $\cos x = \frac{1}{2}$   
 $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$   
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$
- c**  $\sin x = \frac{1}{\sqrt{2}}$   
 $x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$   
 $x = \frac{\pi}{4}, \frac{3\pi}{4}$
- d**  $\cos x = 0$   
 $x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$   
 $x = \frac{\pi}{2}, \frac{3\pi}{2}$
- e**  $\cos x = -1$   
 $x = \pi$
- f**  $\sin x = -\frac{1}{2}$   
 $x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$   
 $x = \frac{7\pi}{6}, \frac{11\pi}{6}$
- g**  $\tan x = -\frac{1}{\sqrt{3}}$   
 $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$   
 $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
- h**  $\cos x = -\frac{1}{\sqrt{2}}$   
 $x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$   
 $x = \frac{3\pi}{4}, \frac{5\pi}{4}$
- 11**
- a**  $\cos \theta = 0.5556$   
 $\theta = 56.3, 360 - 56.3$   
 $\theta = 56.3^\circ, 303.7^\circ$
- b**  $\sin \theta = 0.3891$   
 $\theta = 22.9, 180 - 22.9$   
 $\theta = 22.9^\circ, 157.1^\circ$
- c**  $\tan \theta = 0.9434$   
 $\theta = 43.3, 180 + 43.3$   
 $\theta = 43.3^\circ, 223.3^\circ$
- d**  $\cos \theta = -0.3802$   
 $\theta = 180 - 67.7,$   
 $180 + 67.7$   
 $\theta = 112.3^\circ, 247.7^\circ$
- e**  $\sin \theta = 0.3333$   
 $\theta = 19.5, 180 - 19.5$   
 $\theta = 19.5^\circ, 160.5^\circ$
- f**  $\tan \theta = -1.0638$   
 $\theta = 180 - 46.8,$   
 $360 - 46.8$   
 $\theta = 133.2^\circ, 313.2^\circ$
- g**  $\cos \theta = 0.5297$   
 $\theta = 58.0, 360 - 58.0$   
 $\theta = 58.0^\circ, 302.0^\circ$
- h**  $\sin \theta = -0.8333$   
 $\theta = 180 + 56.4,$   
 $360 - 56.4$   
 $\theta = 236.4^\circ, 303.6^\circ$
- 12**
- a**  $\sin(x + 30) = 0.5$   
 $x + 30 = 30, 180 - 30$   
 $= 30, 150$   
 $x = 0, 120$
- b**  $\tan(x - 57) = 0.625$   
 $x - 57 = 32.0, 32.0 - 180$   
 $= -148.0, 32.0$   
 $x = -91.0, 89.0$
- c**  $\cos 2x = 0.4255$   
 $2x = 64.816, 360 - 64.816,$   
 $-64.816, 64.816 - 360$   
 $= -295.184, -64.816,$   
 $64.816, 295.184$   
 $x = -147.6, -32.4,$   
 $32.4, 147.6$
- d**  $\cot x = 2.5$   
 $\tan x = 0.4$   
 $x = 21.8, 21.8 - 180$   
 $x = -158.2, 21.8$
- e**  $\sec(x - 60) = \frac{2}{\sqrt{3}}$   
 $\cos(x - 60) = \frac{\sqrt{3}}{2}$   
 $x - 60 = 30, -30$   
 $x = 30, 90$
- f**  $\operatorname{cosec} \frac{1}{2}x = 3.5$   
 $\sin \frac{1}{2}x = 0.2857$   
 $\frac{1}{2}x = 16.602$   
 $x = 33.2$
- g**  $\cos(2x - 18) = -0.7692$   
 $2x - 18 = 180 - 39.715,$   
 $180 + 39.715,$   
 $39.715 - 180,$   
 $-39.715 - 180$   
 $= -219.715, -140.285,$   
 $140.285, 219.715$   
 $2x = -201.715, -122.285,$   
 $158.285, 237.715$   
 $x = -100.9, -61.1$   
 $79.1, 118.9$
- h**  $\sin 3x = -0.2941$   
 $3x = 180 + 17.105,$   
 $360 - 17.105,$   
 $-17.105,$   
 $17.105 - 180,$   
 $-360 - 17.105,$   
 $17.105 - 540$   
 $= -522.895, -377.105,$   
 $-162.895, -17.105,$   
 $197.105, 342.895$   
 $x = -174.3, -125.7, -54.3,$   
 $-5.7, 65.7, 114.3$
- i**  $\tan(2x + 135) = 1$   
 $2x + 135 = 45, 180 + 45,$   
 $360 + 45,$   
 $45 - 180,$   
 $= -135, 45,$   
 $225, 405$   
 $2x = -270, -90, 90, 270$   
 $x = -135, -45, 45, 135$

- 13 a**  $\operatorname{cosec}^2 \theta = 4$   
 $\operatorname{cosec} \theta = \pm 2$   
 $\sin \theta = \pm \frac{1}{2}$   
 $\theta = 30, 180 - 30$  or  $180 + 30, 360 - 30$   
 $\theta = 30, 150, 210, 330$
- b**  $(\sec \theta + 1)(\sec \theta - 3) = 0$   
 $\sec \theta = -1$  or  $3$   
 $\cos \theta = -1$  or  $\frac{1}{3}$   
 $\theta = 180$  or  $70.5, 360 - 70.5$   
 $\theta = 70.5, 180, 289.5$
- c**  $\cot \theta (\operatorname{cosec} \theta - 6) = 0$   
 $\cot \theta = 0$  or  $\operatorname{cosec} \theta = 6$   
 $\cos \theta = 0$  or  $\sin \theta = \frac{1}{6}$   
 $\theta = 90, 360 - 90$  or  $9.6, 180 - 9.6$   
 $\theta = 9.6, 90, 170.4, 270$
- d**  $\frac{1}{\sin \theta} = \frac{4}{\cos \theta}$   
 $\frac{\sin \theta}{\cos \theta} = \frac{1}{4}$   
 $\tan \theta = \frac{1}{4}$   
 $\theta = 14.0, 180 + 14.0$   
 $\theta = 14.0, 194.0$
- e**  $2 \cos \theta = \frac{\cos \theta}{\sin \theta}$   
 $2 \cos \theta \sin \theta = \cos \theta$   
 $\cos \theta (2 \sin \theta - 1) = 0$   
 $\cos \theta = 0$  or  $\sin \theta = \frac{1}{2}$   
 $\theta = 90, 360 - 90$  or  $30, 180 - 30$   
 $\theta = 30, 90, 150, 270$
- f**  $5 \sin \theta - \frac{2}{\sin \theta} - 3 = 0$   
 $5 \sin^2 \theta - 3 \sin \theta - 2 = 0$   
 $(5 \sin \theta + 2)(\sin \theta - 1) = 0$   
 $\sin \theta = -\frac{2}{5}$  or  $1$   
 $\theta = 180 + 23.6, 360 - 23.6$  or  $90$   
 $\theta = 90, 203.6, 336.4$
- 14 a**  $(2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 4) = 0$   
 $\operatorname{cosec} x = -4$  or  $\frac{3}{2}$   
 $\sin x = -\frac{1}{4}$  or  $\frac{2}{3}$   
 $x = -0.2527, -\pi + 0.2527$  or  
 $0.7297, \pi - 0.7297$   
 $x = -2.89, -0.25, 0.73, 2.41$
- b**  $\frac{1}{\cos x} = \frac{3 \sin x}{\cos x}$   
 $\sin x = \frac{1}{3}$   
 $x = 0.3398, \pi - 0.3398$   
 $x = 0.34, 2.80$
- c**  $\frac{3}{\cos x} = \frac{2 \cos x}{\sin x}$   
 $3 \sin x = 2 \cos^2 x$   
 $3 \sin x = 2(1 - \sin^2 x)$   
 $2 \sin^2 x + 3 \sin x - 2 = 0$   
 $(2 \sin x - 1)(\sin x + 2) = 0$   
 $\sin x = \frac{1}{2}$  or  $-2$  [no solutions]  
 $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$   
 $x = 0.52, 2.62$
- d**  $4 + \tan x - \frac{5}{\tan x} = 0$   
 $\tan^2 x + 4 \tan x - 5 = 0$   
 $(\tan x + 5)(\tan x - 1) = 0$   
 $\tan x = -5$  or  $1$   
 $x = \pi - 1.3734, -1.3734$  or  $\frac{\pi}{4}, -\pi + \frac{\pi}{4}$   
 $x = -2.36, -1.37, 0.79, 1.77$
- e**  $\frac{1}{\sin x} = -\frac{5 \cos x}{\sin x}$   
 $\cos x = -\frac{1}{5}$   
 $x = \pi - 1.3694, -\pi + 1.3694$   
 $x = -1.77, 1.77$
- f**  $\frac{6 \sin x}{\cos x} = \frac{5}{\sin x}$   
 $6 \sin^2 x = 5 \cos x$   
 $6(1 - \cos^2 x) = 5 \cos x$   
 $6 \cos^2 x + 5 \cos x - 6 = 0$   
 $(3 \cos x - 2)(2 \cos x + 3) = 0$   
 $\cos x = \frac{2}{3}$  or  $-\frac{3}{2}$  [no solutions]  
 $x = -0.84, 0.84$

$$\begin{aligned}
 15 \quad \mathbf{a} \quad \text{LHS} &= \frac{1}{\cos x} - \cos x \\
 &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \sin x \times \frac{\sin x}{\cos x} \\
 &= \sin x \tan x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \text{LHS} &= \frac{\sin x(\cot x - \cos x)}{\sin x(1 - \sin x)} \\
 &= \frac{\cos x - \sin x \cos x}{\sin x(1 - \sin x)} \\
 &= \frac{\cos x(1 - \sin x)}{\sin x(1 - \sin x)} \\
 &= \frac{\cos x}{\sin x} \\
 &= \cot x \\
 &= \text{RHS}
 \end{aligned}$$

$$16 \quad \mathbf{a} \quad x = 0 \Rightarrow y = 2 - 3 - 5 = -6 \quad \therefore (0, -6)$$

$$\begin{aligned}
 \mathbf{b} \quad y = 0 &\Rightarrow 2 \cos x - \frac{3}{\cos x} - 5 = 0 \\
 2 \cos^2 x - 5 \cos x - 3 &= 0 \\
 (2 \cos x + 1)(\cos x - 3) &= 0 \\
 \cos x = -\frac{1}{2} \quad \text{or} \quad 3 & \text{ [no solutions]} \\
 x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} \\
 x = \frac{2\pi}{3}, \frac{4\pi}{3} \\
 \therefore \left(\frac{2\pi}{3}, 0\right) \text{ and } \left(\frac{4\pi}{3}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \text{LHS} &= \operatorname{cosec} x - \cot x + \cot x - \cos x \cot x \\
 &= \frac{1}{\sin x} - \cos x \times \frac{\cos x}{\sin x} \\
 &= \frac{1 - \cos^2 x}{\sin x} \\
 &= \frac{\sin^2 x}{\sin x} \\
 &= \sin x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad \text{LHS} &= \sin x \cos x + \sin x \cot x + \tan x \cos x + 1 \\
 &= \sin x \cos x + \cos x + \sin x + 1 \\
 &= \sin x (\cos x + 1) + \cos x + 1 \\
 &= (\cos x + 1)(\sin x + 1) \\
 &= \text{RHS}
 \end{aligned}$$