

# C3 PROOF

## Worksheet B

- 1 Prove, by counter-example, that each of the following statements is false.
- a For all positive real values of  $x$ ,  $\sqrt[3]{x} \leq x$ . (2)
- b For all positive integer values of  $n$ ,  $(n^3 - n + 7)$  is prime. (2)
- 2 Use proof by contradiction to prove that  $\sqrt{\pi}$  is irrational.  
(You may assume that  $\pi$  is irrational). (4)
- 3 Find a counter-example to prove that the statement  
“ $15x^2 - 11x + 2 \geq 0$  for all real values of  $x$ ”  
is false. (4)
- 4 a Given that  $n = 2m + 1$ , find and simplify an expression in terms of  $m$  for  $n^2 + 2n$ . (1)
- b Hence, use proof by contradiction to prove that if  $(n^2 + 2n)$  is even, where  $n$  is an integer, then  $n$  is even. (5)
- 5 a Prove that if the equation  
 $k \cos x - \operatorname{cosec} x = 0$ ,  
where  $k$  is a constant, has real solutions, then  $|k| \geq 2$ . (5)
- b Find the values of  $x$  in the interval  $0 \leq x \leq 360$  for which  
 $3 \cos x^\circ - \operatorname{cosec} x^\circ = 0$ . (3)
- 6 Use proof by contradiction to prove that there are no positive integers,  $x$  and  $y$ , such that  
 $x^2 - y^2 = 1$ . (6)
- 7 For each statement, either prove that it is true or find a counter-example to prove that it is false.
- a If  $a$  and  $b$  are irrational and  $a \neq b$ , then  $(a + b)$  is irrational. (2)
- b If  $m$  and  $n$  are consecutive odd integers, then  $(m + n)$  is divisible by 4. (3)
- c For all real values of  $x$ ,  $\cos x \leq 1 + \sin x$ . (2)
- 8 a Show that if  $\log_2 3 = \frac{p}{q}$ , then  
 $2^p = 3^q$ . (2)
- b Use proof by contradiction to prove that  $\log_2 3$  is irrational. (4)
- c Prove, by counter-example, that the statement  
“if  $a$  is rational and  $b$  is irrational then  $\log_a b$  is irrational”  
is false. (2)
- 9 The function  $f$  is defined by  
 $f: x \rightarrow \frac{x-2}{4x}, x \in \mathbb{R}, x \neq 0$ .
- a Find an expression for the inverse function,  $f^{-1}(x)$ , and state its domain. (5)
- b Prove that there are no real values of  $x$  for which  
 $f(x) = f^{-1}(x)$ . (4)