

C3 PROOF

Worksheet A

- 1 Give a counter-example to prove that each of the following statements is false.
 - a If $a^2 - b^2 > 0$, where a and b are real, then $a - b > 0$.
 - b There are no prime numbers divisible by 7.
 - c If x and y are irrational and $x \neq y$, then xy is irrational.
 - d For all real values of x , $\cos(90 - |x|)^\circ = \sin x^\circ$.

- 2 For each statement, either prove that it is true or find a counter-example to prove that it is false.
 - a There are no prime numbers divisible by 6.
 - b $(3^n + 2)$ is prime for all positive integer values of n .
 - c \sqrt{n} is irrational for all positive integers n .
 - d If a , b and c are integers such that a is divisible by b and b is divisible by c , then a is divisible by c .

- 3 Use proof by contradiction to prove each of the following statements.
 - a If n^3 is odd, where n is a positive integer, then n is odd.
 - b If x is irrational, then \sqrt{x} is irrational.
 - c If a , b and c are integers and bc is not divisible by a , then b is not divisible by a .
 - d If $(n^2 - 4n)$ is odd, where n is a positive integer, then n is odd.
 - e There are no positive integers, m and n , such that $m^2 - n^2 = 6$.

- 4 Given that x and y are integers and that $(x^2 + y^2)$ is divisible by 4, use proof by contradiction to prove that
 - a x and y are not both odd,
 - b x and y are both even.

- 5 For each statement, either prove that it is true or find a counter-example to prove that it is false.
 - a If a and b are positive integers and $a \neq b$, then $\log_a b$ is irrational.
 - b The difference between the squares of any two consecutive odd integers is divisible by 8.
 - c $(n^2 + 3n + 13)$ is prime for all positive integer values of n .
 - d For all real values of x and y , $x^2 - 2y(x - y) \geq 0$.

- 6
 - a Prove that if

$$\sqrt{2} = \frac{p}{q},$$
 where p and q are integers, then p must be even.
 - b Use proof by contradiction to prove that $\sqrt{2}$ is irrational.