

C3

NUMERICAL METHODS

Worksheet C

- 1 a Show that the equation $x^3 - 7x - 11 = 0$ has a real root in the interval (3, 4).
 b Using the iterative formula $x_{n+1} = \sqrt{7 + \frac{11}{x_n}}$, with $x_0 = 3.2$, find x_1, x_2 and x_3 , giving the value of x_3 correct to 2 decimal places.

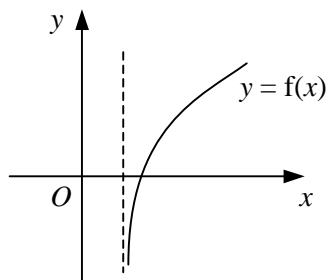
2
$$f(x) \equiv 4 \operatorname{cosec} x - 5 + 2x.$$

- a Find the values of $f(4)$ and $f(5)$.
 b Hence show that the equation $f(x) = 0$ has a root in the interval (4, 5).

The iterative formula $x_{n+1} = a + \frac{b}{\sin x_n}$, where a and b are constants, is used to find this root.

- c Find the values of a and b .
 d Starting with $x_0 = 4.5$, use the iterative formula with your values of a and b to find 3 further approximations of the root, giving your final answer correct to 3 decimal places.

3



The diagram shows the curve with equation $y = f(x)$ where

$$f: x \rightarrow 2x + \ln(3x - 1), \quad x \in \mathbb{R}, \quad x > \frac{1}{3}.$$

Given that $f(\alpha) = 0$,

- a show that $0.4 < \alpha < 0.5$,
 b use the iterative formula $x_{n+1} = \frac{1}{3}(1 + e^{-2x_n})$, with $x_0 = 0.45$, to find the value of α correct to 3 decimal places.
- 4 a On the same set of axes, sketch the curves $y = \cos x$ and $y = x^2$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
 b Show that the equation $\cos x - x^2 = 0$ has exactly one positive and one negative real root.
 c Show that the positive real root lies in the interval $[0.8, 0.9]$.
 d Use the iteration formula $x_{n+1} = \sqrt{\cos x_n}$ and the starting value $x_0 = 0.8$ to find the positive root correct to 2 decimal places.

5
$$f(x) \equiv e^{5-2x} - x^5.$$

Show that the equation $f(x) = 0$

- a has a root in the interval (1.4, 1.5),
 b can be written as $x = e^{1-kx}$, stating the value of k .
 c Using the iteration formula $x_{n+1} = e^{1-kx_n}$, with $x_0 = 1.5$ and the value of k found in part b, find x_1, x_2 and x_3 . Give the value of x_3 correct to 3 decimal places.

C3 NUMERICAL METHODS

Worksheet C continued

6 $f: x \rightarrow 2^x + x^3 - 5, x \in \mathbb{R}.$

- a Show that there is a solution of the equation $f(x) = 0$ in the interval $1.3 < x < 1.4$
- b Using the iterative formula $x_{n+1} = \sqrt[3]{5 - 2^{x_n}}$, with $x_0 = 1.4$, find x_1, x_2, x_3 and x_4 .
- c Hence write down an approximation for this solution of the equation $f(x) = 0$ to an appropriate degree of accuracy.

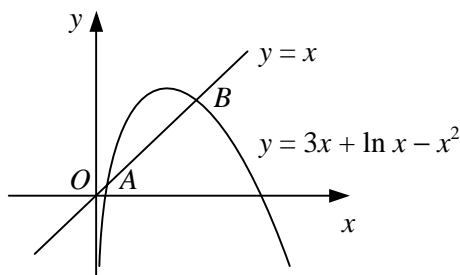
Another attempt is made to find the solution using the iterative formula $x_{n+1} = \frac{\ln(5 - x_n^3)}{\ln 2}$.

- d Describe the outcome of this attempt.

7 $f(x) = 2x^3 + 4x - 9.$

- a Find $f'(x)$.
- b Hence show that the equation $f(x) = 0$ has exactly one real root.
- c Show that this root lies in the interval $(1.2, 1.3)$.
- d Use the iterative formula $x_{n+1} = \sqrt[3]{4.5 - 2x_n}$, with $x_0 = 1.2$, to find the root of $f(x) = 0$ correct to 2 decimal places.
- e Justify the accuracy of your answer.

8



The diagram shows part of the curve with equation $y = 3x + \ln x - x^2$ and the line $y = x$. Given that the curve and line intersect at the points A and B , show that

- a the x -coordinates of A and B are the solutions of the equation $x = e^{x^2 - 2x}$,
- b the x -coordinate of A lies in the interval $(0.4, 0.5)$,
- c the x -coordinate of B lies in the interval $(2.3, 2.4)$.
- d Use the iteration formula $x_{n+1} = e^{x_n^2 - 2x_n}$, with $x_0 = 0.5$, to find the x -coordinate of A correct to 2 decimal places.
- e Justify the accuracy of your answer to part d.
- 9 a On the same set of axes, sketch the graphs of $y = x^4$ and $y = 5x + 2$.
- b Show that the equation $x^4 - 5x - 2 = 0$ has exactly one positive and one negative real root.
- c Use the iteration formula $x_{n+1} = \sqrt[4]{5x_n + 2}$, with $x_0 = 1.8$, to find x_1, x_2, x_3 and x_4 , giving the value of x_4 correct to 3 decimal places.
- d Show that the equation $x^4 - 5x - 2 = 0$ can be written in the form $x = \frac{a}{x^3 + b}$, stating the values of a and b .
- e Use the iteration formula $x_{n+1} = \frac{a}{x_n^3 + b}$, with $x_0 = -0.4$ and your values of a and b , to find the negative real root of the equation correct to 4 decimal places.