

C3 NUMERICAL METHODS

Answers - Worksheet B

- 1 a** $9 + 4x - 2x^3 = 0$
 $x^3 = 2x + 4.5$
 $x = \sqrt[3]{2x + 4.5}$
 $\therefore x_{n+1} = \sqrt[3]{2x_n + 4.5}$
 $x_1 = 2.040828$
 $x_2 = 2.047342$
 $x_3 = 2.048377 = 2.0484$ (4dp)
- b** $e^x - 8x + 5 = 0$
 $e^x = 8x - 5$
 $x = \ln(8x - 5)$
 $\therefore x_{n+1} = \ln(8x_n - 5)$
 $x_1 = 2.944439$
 $x_2 = 2.920767$
 $x_3 = 2.910508 = 2.9105$ (4dp)
- c** $\tan x - 5x + 13 = 0$
 $\tan x = 5x - 13$
 $x = \arctan(5x - 13)$
 $\therefore x_{n+1} = \arctan(5x_n - 13)$
 $x_1 = -1.518213$
 $x_2 = -1.522270$
 $x_3 = -1.522317 = -1.5223$ (4dp)
- d** $\ln x + \sqrt{x} + 1.4 = 0$
 $\ln x = -(\sqrt{x} + 1.4)$
 $x = e^{-(\sqrt{x} + 1.4)}$
 $\therefore x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$
 $x_1 = 0.165299$
 $x_2 = 0.164216$
 $x_3 = 0.164436 = 0.1644$ (4dp)
- 2 a** $e^{2x-1} - 6x = 0$
 $e^{2x-1} = 6x$
 $2x - 1 = \ln 6x$
 $x = \frac{1}{2}(\ln 6x + 1)$
 $\therefore x_{n+1} = \frac{1}{2}(\ln 6x_n + 1), a = \frac{1}{2}, b = 6$
 $x_1 = 1.661194$
 $x_2 = 1.649648$
 $x_3 = 1.646161 = 1.646$ (3dp)
- b** $\frac{2}{x} + \cos x - 3 = 0$
 $\frac{2}{x} = 3 - \cos x$
 $2 = x(3 - \cos x)$
 $x = \frac{2}{3 - \cos x}$
 $\therefore x_{n+1} = \frac{2}{3 - \cos x_n}, a = 2, b = 3$
 $x_1 = 0.868322$
 $x_2 = 0.849657$
 $x_3 = 0.854789 = 0.855$ (3dp)
- c** $2x^3 - 6x - 11 = 0$
 $2x^3 = 6x + 11$
 $x^2 = 3 + \frac{11}{2x}$
 $x = \pm\sqrt{3 + \frac{11}{2x}}$
 $\therefore x_{n+1} = \sqrt{3 + \frac{11}{x_n}}, a = 3, b = 5.5$
 $x_1 = 2.397916$
 $x_2 = 2.300795$
 $x_3 = 2.321740 = 2.322$ (3dp)
- d** $15 \ln(x + 3) - 4x = 0$
 $\ln(x + 3) = \frac{4}{15}x$
 $x + 3 = e^{\frac{4}{15}x}$
 $x = e^{\frac{4}{15}x} - 3$
 $\therefore x_{n+1} = e^{\frac{4}{15}x_n} - 3, a = \frac{4}{15}, b = -3$
 $x_1 = -2.486583$
 $x_2 = -2.484743$
 $x_3 = -2.484490 = -2.484$ (3dp)

- 3 a** $x_1 = 0.428135$
 $x_2 = 0.433865$
 $x_3 = 0.431107$
 $x_4 = 0.432437$
 $x_5 = 0.431796$
 $\therefore \text{root} = 0.432$ (3dp)
 $f(0.4315) = -0.00465$
 $f(0.4325) = 0.00457$
 sign change, $f(x)$ continuous \therefore root
- b** $x_1 = 0.474342$
 $x_2 = 0.470474$
 $x_3 = 0.469923$
 $\therefore \text{root} = 0.47$ (2sf)
 $f(0.465) = -0.00428$
 $f(0.475) = 0.00463$
 sign change, $f(x)$ continuous \therefore root
- c** $x_1 = 5.892685$
 $x_2 = 5.859202$
 $x_3 = 5.850013$
 $x_4 = 5.847607$
 $x_5 = 5.846985$
 $x_6 = 5.846825$
 $\therefore \text{root} = 5.85$ (3sf)
 $f(5.845) = 0.00658$
 $f(5.855) = -0.0305$
 sign change, $f(x)$ continuous \therefore root
- d** $x_1 = 3.731246$
 $x_2 = 3.724839$
 $x_3 = 3.726145$
 $x_4 = 3.725879$
 $\therefore \text{root} = 3.726$ (3dp)
 $f(3.7255) = 0.000672$
 $f(3.7265) = -0.000912$
 sign change, $f(x)$ continuous \therefore root
- 4 a** $x_1 = -3.192595$
 $x_2 = -3.188214$
 $x_3 = -3.185620$
 $x_4 = -3.184084$
 $x_5 = -3.183174$
 $x_6 = -3.182635$
 $\therefore \text{root} = -3.18$ (2dp)
- b** $x^5 - 10x^3 + 4 = 0$
 $4 = 10x^3 - x^5 = x^3(10 - x^2)$
 $x^3 = \frac{4}{10 - x^2}$
 $x = \sqrt[3]{\frac{4}{10 - x^2}} \therefore a = 4, b = 10$
- c** $x_1 = 0.763143$
 $x_2 = 0.751692$
 $x_3 = 0.751231$
 $x_4 = 0.751212$
 $\therefore \text{root} = 0.751$ (3dp)
- 5 a** $\arcsin 2x - 0.5x - 0.7 = 0$
 $\arcsin 2x = 0.5x + 0.7$
 $2x = \sin(0.5x + 0.7)$
 $x = 0.5 \sin(0.5x + 0.7) \therefore a = 0.5, b = 0.5, c = 0.7$
- b** $x_1 = 0.391663$
 $x_2 = 0.390365$
 $x_3 = 0.390162$
 $x_4 = 0.390130$
 $\therefore \text{solution} = 0.390$ (3dp)