C3 NUMERICAL METHODS — Answers - Worksheet A

1 **a** $f(1) = -3$ $f(2) = 7$
sign change, $f(x)$ continuous $\therefore$ root

b $f(0.5) = 2.89$ $f(1) = -0.298$
sign change, $f(x)$ continuous $\therefore$ root

c $f(-6) = -0.995$ $f(-5) = 0.0135$
sign change, $f(x)$ continuous $\therefore$ root

d $f(2.1) = -1.60$ $f(2.2) = 0.226$
sign change, $f(x)$ continuous $\therefore$ root

e $f(0.4) = -0.351$ $f(0.5) = 0.25$
sign change, $f(x)$ continuous $\therefore$ root

f $f(10) = 6.00$ $f(11) = -9.00$
sign change, $f(x)$ continuous $\therefore$ root

2 **a** $f(0) = -4$
$f(3) = 17.8$
$f(1) = -6$
$f(2) = -0.243$
$\therefore N = 2$

b $f(1) = -12$
$f(5) = 5.65$
$f(3) = -0.704$
$f(4) = 2.55$
$\therefore N = 3$

c $f(0) = 15$
$f(-2) = -57$
$f(-1) = 9$
$\therefore N = -2$

d $f(0) = -1.63$
$f(1) = 3$
$\therefore N = 0$

e $f(0) = 1$
$f(-5) = -2.87$
$f(-4) = -2.25$
$f(-3) = 0.473$
$\therefore N = -4$

f $f(0) = -6$
$f(4) = -1.58$
$f(5) = -0.454$
$f(6) = 0.684$
$\therefore N = 5$

3 **a** let $f(x) = x^3 - 12 + \frac{x}{4}$
$f(2) = -3.5$ $f(3) = 15.75$
sign change, $f(x)$ continuous $\therefore$ root

b let $f(x) = 12e^x - 9 + 4x$
$f(-1) = -8.59$ $f(0) = 3$
sign change, $f(x)$ continuous $\therefore$ root

c let $f(x) = 10 \ln 3x - 5 + 7x^2$
$f(0.47) = -0.0178$ $f(0.48) = 0.259$
sign change, $f(x)$ continuous $\therefore$ root

d let $f(x) = \sin 4x - 7e^x$
$f(-6.5) = -0.773$ $f(-6) = 0.888$
sign change, $f(x)$ continuous $\therefore$ root

e let $f(x) = 4^x - 3x - 10$
$f(-4) = 2.00$ $f(-3) = -0.984$
sign change, $f(x)$ continuous $\therefore$ root

f let $f(x) = \tan(\frac{1}{2}x) - 2x + 1$
$f(2.6) = -0.598$ $f(2.7) = 0.0552$
sign change, $f(x)$ continuous $\therefore$ root

4 **a** $f(1) = -1$
$f(2) = 12.5$
$f(1.1) = -0.809$
$f(1.2) = -0.426$
$f(1.3) = 0.164$
$\therefore a = 12$

b $f(2) = -0.303$
$f(3) = 0.292$
$f(2.5) = -0.00553$
$f(2.6) = 0.0537$
$\therefore a = 25$

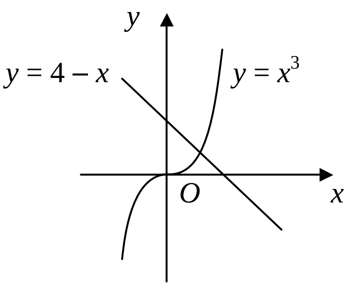
c $f(-2) = -41$
$f(-1) = 3$
$f(-1.1) = 0.715$
$f(-1.2) = -1.96$
$\therefore a = -12$

d $f(11) = 0.723$
$f(12) = -0.177$
$f(11.7) = 0.0362$
$f(11.8) = -0.0425$
$\therefore a = 117$

e $f(5) = 1.19$
$f(6) = -1.13$
$f(5.5) = 0.928$
$f(5.8) = 0.256$
$f(5.9) = -0.246$
$\therefore a = 58$

f $f(-3) = 6.42$
$f(-2) = -15.0$
$f(-2.7) = 2.60$
$f(-2.6) = 1.03$
$f(-2.5) = -0.75$
$\therefore a = -26$

5 **a**



b $x^3 + x - 4 = 0 \;\Rightarrow\; x^3 = 4 - x$

the graphs $y = x^3$ and $y = 4 - x$

intersect at exactly one point

$\therefore$ one real root

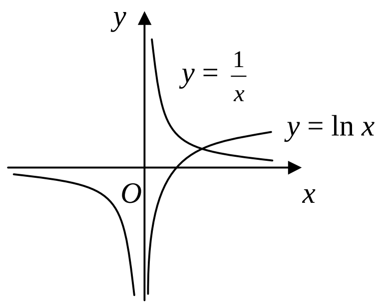
c let $f(x) = x^3 + x - 4$

$f(1) = -2$

$f(1.5) = 0.875$

sign change, $f(x)$ continuous $\therefore$ root

6 **a**



b $x \ln x - 1 = 0 \;\Rightarrow\; x \ln x = 1 \;\Rightarrow\; \ln x = \frac{1}{x}$

the graphs $y = \ln x$ and $y = \frac{1}{x}$

intersect at exactly one point

$\therefore$ one real root

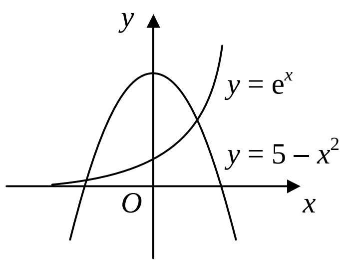
c $f(1) = -1$

$f(2) = 0.386$

$\therefore\; 1 < \alpha < 2$

$\therefore\; n = 1$

7 **a**



b $e^x + x^2 - 5 = 0 \;\Rightarrow\; e^x = 5 - x^2$

the graphs $y = e^x$ and $y = 5 - x^2$

intersect at two points,

one for $x < 0$ and one for $x > 0$

$\therefore$ one negative and one positive real root

c let $f(x) = e^x + x^2 - 5$

$f(-3) = 4.05$

$f(-2) = -0.865$

sign change, $f(x)$ continuous $\therefore$ root

d $f(1) = -1.28$

$f(2) = 6.39$

$f(1.2) = -0.240$

$f(1.3) = 0.359$

$\therefore\; 1.2 < \alpha < 1.3$

$\therefore\; n = 12$