

C3 FUNCTIONS

Answers - Worksheet H

1 a $3 + \ln(x + 2) \geq 3$

$$\ln(x + 2) \geq 0$$

$$x + 2 \geq 1$$

$$x \geq -1$$

$$\therefore k = -1$$

b $y = 3 + \ln(x + 2)$

swap $x = 3 + \ln(y + 2)$

$$y + 2 = e^{x-3}$$

$$y = e^{x-3} - 2$$

$$f^{-1}(x) = e^{x-3} - 2, \quad x \in \mathbb{R}, \quad x \geq 3$$

c $3 + \ln(x + 2) = 4 + \ln(x - 1)$

$$\ln(x + 2) - \ln(x - 1) = 1$$

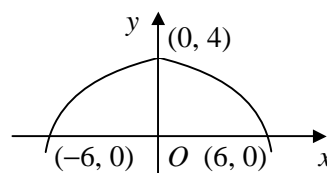
$$\frac{x+2}{x-1} = e$$

$$x + 2 = e(x - 1)$$

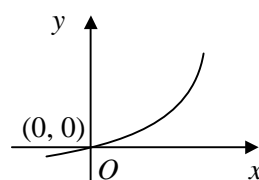
$$x(e - 1) = e + 2$$

$$x = \frac{e+2}{e-1}$$

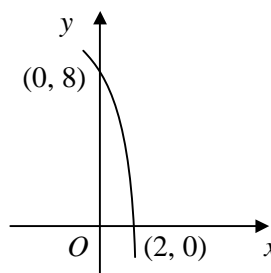
2 a



b



c



3 a $fg(x) = f\left(\frac{3}{x}\right) = \frac{\frac{3}{x}}{\frac{3}{x}+2} = \frac{3}{3+2x}$

$$\therefore \frac{3}{3+2x} = 4$$

$$3 + 2x = \frac{3}{4}$$

$$x = -\frac{9}{8}$$

b $y = \frac{x}{x+2}$

swap $x = \frac{y}{y+2}$

$$x(y + 2) = y$$

$$2x = y(1 - x)$$

$$y = \frac{2x}{1-x}$$

$$f^{-1}(x) = \frac{2x}{1-x}, \quad x \in \mathbb{R}, \quad x \neq 1$$

c $\frac{x}{x+2} = \frac{2x}{1-x}$

$$x(1 - x) = 2x(x + 2)$$

$$3x^2 + 3x = 0$$

$$3x(x + 1) = 0$$

$$x = -1, 0$$

4 a $f(x) = (x - 1)^2 - 1 - 9 = (x - 1)^2 - 10$

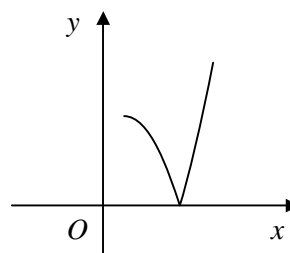
for f^{-1} to exist, f must be one-one

$$\therefore \text{min value of } k = 1$$

b $f^{-1}(x) = 4 \Rightarrow x = f(4)$

$$\therefore x = 16 - 8 - 9 = -1$$

c



d $x^2 - 2x - 9 = 6$

$$x^2 - 2x - 15 = 0$$

$$(x + 3)(x - 5) = 0$$

$$x \geq 1 \therefore x = 5$$

$$-(x^2 - 2x - 9) = 6$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

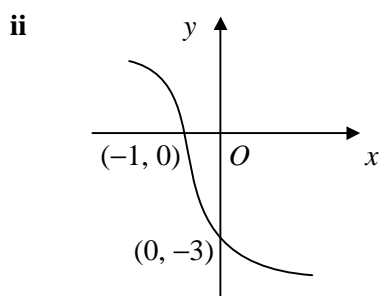
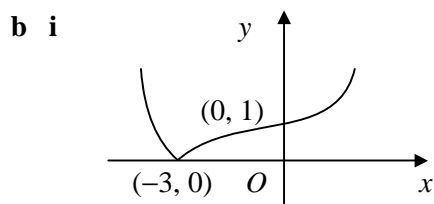
$$x \geq 1 \therefore x = 3$$

$$\therefore x = 3, 5$$

- 5 a $f(1) = 2 - 3 = -1$
 $ff(1) = f(-1) = 2 + 3 = 5$
- b $y = 2 - \frac{3}{x}$ swap $x = 2 - \frac{3}{y}$
 $\frac{3}{y} = 2 - x$
 $y = \frac{3}{2-x}$
 $f^{-1}(x) = \frac{3}{2-x}, x \in \mathbb{R}, x \neq 2$
- c $gf(x) = 1 \Rightarrow (2 - \frac{3}{x})^2 = 1$
 $2 - \frac{3}{x} = \pm 1$
 $\frac{3}{x} = 1, 3$
 $x = 1, 3$

- 6 a $f(\ln 9) = f(2 \ln 3) = e^{\ln 3} - 2 = 3 - 2 = 1$
- b $f(x) > -2$
- c $y = e^{\frac{1}{2}x} - 2$ swap $x = e^{\frac{1}{2}y} - 2$
 $\frac{1}{2}y = \ln(x + 2)$
 $y = 2 \ln(x + 2)$
 $f^{-1}(x) = 2 \ln(x + 2), x \in \mathbb{R}, x > -2$
- d $gf(x) = (e^{\frac{1}{2}x} - 2)^2 + 4(e^{\frac{1}{2}x} - 2)$
 $= e^x - 4e^{\frac{1}{2}x} + 4 + 4e^{\frac{1}{2}x} - 8$
 $gf(x) = e^x - 4$
- e $e^x - 4 + 1 = 0$
 $e^x = 3$
 $x = \ln 3$

- 7 a each value of $f(x)$ corresponds to a unique value of x



- 8 a $f(x) = \frac{5 + (2x - 3)}{(x + 1)(2x - 3)} = \frac{2x + 2}{(x + 1)(2x - 3)}$
 $= \frac{2(x + 1)}{(x + 1)(2x - 3)} = \frac{2}{2x - 3}$
- b $f(2) = 2$
 \therefore range: $0 < f(x) \leq 2$
- c $y = \frac{2}{2x - 3}$ swap $x = \frac{2}{2y - 3}$
 $2y - 3 = \frac{2}{x}$
 $f^{-1}(x) = \frac{1}{x} + \frac{3}{2}, x \in \mathbb{R}, 0 < x \leq 2$
- d $fg(x) = \frac{2}{2(\frac{1}{x-2}) - 3} = \frac{2(x-2)}{2-3(x-2)} = \frac{2x-4}{8-3x}$
 $\therefore \frac{2x-4}{8-3x} = \frac{2}{3}$
 $6x - 12 = 16 - 6x$
 $x = \frac{7}{3}$